Research Article

A Mathematical Loading Model in the Power Generator of Thermoelectric Generator

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Abstract

Most of the electrical energy produced today is obtained from primary energy such as oil, natural gas, and non-renewable coal. The utilization of fossil energy has caused negative impacts such as air pollution and global pollution. One of the potential sources of energy as a renewable energy source is the use of thermoelectric generators. The use of this energy source produces cheap, environmentally friendly and sustainable electrical power. The previous research results have shown that the thermoelectric generator that can produce large currents and voltages is the type of SP 1848-27145 Thermoelectric Generator. The
output current is 2 to 3 Amperes and its maximum voltage is 5 Volt dc with a load
of 5 Watt, 12 Volt DC LED lights. The measurement results show that the thermoelectric generator type SP 1848-27145 can produce current and voltage at the hot side temperature of the piltier of 74ºC and at the cold side of the piltier of 42ºC. The design results in the form of a Thermoelectric Generator Application as a load-bearing power plant in the system are as follows: 130, 140,150, 160, 170, 180, 190, 200, 210, 220, and 230 Watt. This varied loading is useful for revealing the maximum load on the system so that it works continuously or at least works for a long time. The results of calculation using the mathematical model of $\hat{y}_i = 14{,}94 - 0{,}0097x_1 + 1{,}189x_2$ reveal that the maximum allowable load is 200 Watts.

Keywords: *Thermoelectric Generator, Temperature, Power Generation by Thermoelectric Generator, Mathematical Model, Maximum Load.*

1. Introduction

Human dependence on electrical energy has become a major feature of today's modern era. Most of the electrical energy produced today is obtained from the primary energy such as oil, natural gas, and non-renewable coal. The utilization of fossil energy has caused negative impacts such as air pollution and global pollution. According to the Energy Information Administration (EIA), the electric power generated by factories using natural gas increased every year by 28% in 2014, 35% in 2018 and 36% in 2019. Furthermore, world consumption and production of liquid fuels increased from 94 million barrels per day in mid-2014 to 100 million barrels in mid-2018 which caused an increase in energy costs. To overcome the global growth in consumption of fossil fuels which is quite expensive and has an impact on world pollution, another form of environmentally friendly energy emerged at the end of this decade [1].

Nicolas Tesla once said "Electric power is everywhere in infinite quantities and can power the world machine without coal, oil or any other fuel". This statement encourages a new trend of using natural energy from the environment to produce cheap, environmentally friendly, and sustainable electric power. Today, the interest in renewable energy which can be obtained from abundant natural sources such as thermal energy, solar, motion / vibration etc. and can be converted into electrical energy to supply electronic equipment and machinery, is growing rapidly. To overcome this energy demand problem is to seek new innovations as an alternative energy source. One of the potential energy sources with new innovations is the use of a thermoelectric generator (TEG). The use of TEG as an energy source should be considered because it has not been optimally utilized. The energy produced by TEG comes from thermal energy. TEG is an active component that converts thermal energy into electrical energy with the principle that if the hot side and the cold side of the piltier have a temperature difference it will cause TEG to start working. TEG is widely used in a variety of applications, due to its advantages such as free maintenance costs and a long life time. In recent years, TEG has been in great demand in terms of energy use, both large and small scale depending on the size, power and material used [2].

The state of the art of TEG is presented in this paper. This paper differs from other writings in terms of the use of TEG in particular from the thermal energy used. Previously, the heat source for TEG came from thermal energy produced by the combustion process in a steam boiler of a power plant, motorcycle exhaust [3] [4] which is known as waste thermal energy with an open loop system. In this paper, the TEG heat source comes from a thermal source that is made separately with the concept of a closed loop system. In TEG there is a hot side and a cold side of the piltier. If the hot side of the piltier is heated with its own thermal energy, there will be a certain temperature difference with that of the cold side of the piltier (as heat dissipation). This temperature difference causes TEG to start working [5]. The greater the temperature difference, the greater the electrical energy will be produced. But if it is too large the temperature difference can cause damage to the bismuth semiconductor used [6]. In order for the Thermoelectric Generator to work continuously, a mathematical loading model is made which is used to ensure that the accumulator output voltage does not work at a critical point.

The results of the previous researches show that in general the use of TEG as a power plant utilizes thermal energy [3] [4] and mostly uses heat sources from industrial waste with an open loop process. The novelty of this research are: 1. The heat source used for heating the hot side of the TEG piltier uses the closed loop principle. 2. The voltage and the current generated from the TEG can be increased gradually.

The resulting efficiency obtained from the Thermoelectric Generator with separate heating system of closed loop is 27%.

2. Research Methods

2.1. Literature Study

Thermoelectric Generator (TEG) is a solid-state device that generates electrical energy from the temperature difference applied to the TEG. This generating technology was first introduced by Thomas Johann Seebeck in 1821 [7]. Seebeck reported that the thermoelectric potential energy could be developed in the presence of temperature differences in two different materials. As a result, this phenomenon is referred to as the "Seebeck effect".

Usually, a large number of TE elements are connected in series and parallel to increase the power of the TEG. The standard size of the TEG module varies from 40 mm × 40 mm × 3 mm to 50 mm × 50 mm × 5 mm [8]. For flexible TEGs, the thickness varies from 10 to 500 μm [12]. Standard TEG modules typically use tellurium (Te), bismuth (Bi), antimony (Sb) or selenium (Se) to form the basis of the TE system [9] [10]. Bismuth telluride (Bi_2Te_3) and antimony telluride (Sb_2Te_3) alloys are the most commonly used TE materials due to their high efficiency at room temperature. In addition, these materials are also easily stored in thin films to make flexible modules [11]. The physical form of the TEG used in this study is shown in Figure 1.

The advantages of TEG include a longer life span than that of other power generation systems, no moving parts, no emission of harmful pollutants during operation, no operational and maintenance costs, no chemical reactions with the environment (i.e. environmentally friendly), reliable operation, solid-state operation, and low potential use of thermal energy [12] [13].

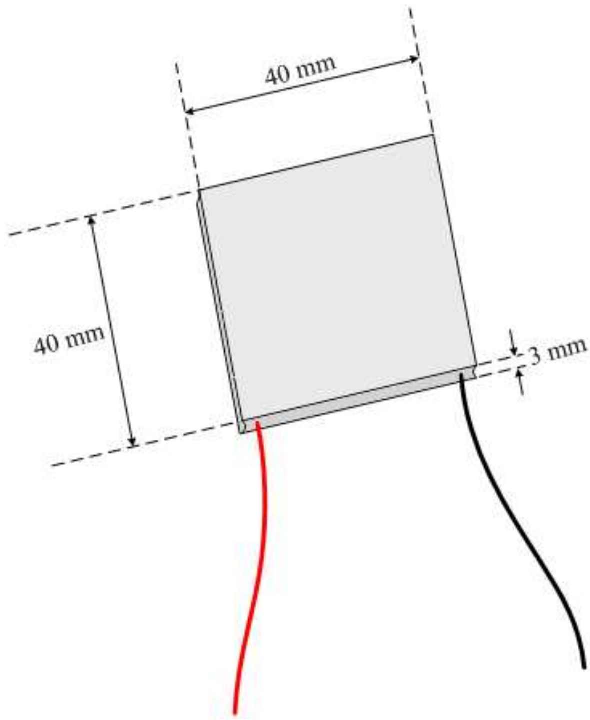

Figure 1. Physical Form of TEG [14].

2.1.1. Basic Principles

The basic principle of TEG is based on the concept of the Seebeck effect of thermoelectric materials where the resulting voltage is directly proportional to the temperature gradient as shown below [15]:

$$V = \alpha \Delta T \tag{1}$$

in which α is the Seebeck coefficient (V K^{-1}) of the thermoelectric materials (TE) and ΔT is the temperature difference between the two surfaces of the thermoelectric generator.

The TEG system consists of p-and n-type semiconductors in which the p-type has a surplus of holes and the n-type has a surplus of electrons to carry electric current (see Figure 2). When heat flows from a hot surface to a cold surface through the thermoelectric material, the free charge (electrons and holes) from the semiconductor also moves. This movement of charge converts thermal energy into electrical energy. The typical value of the Seebeck coefficient for the commercially available n-type Bismuth Telluride (Bi_2Te_3) is - 150 × 10^{-6} VK^{-1}, whereas for the p-type Antimony Telluride (Sb_2Te_3) is 101 - 161 × 10^{-6} VK^{-1} at room temperature [16] [17].

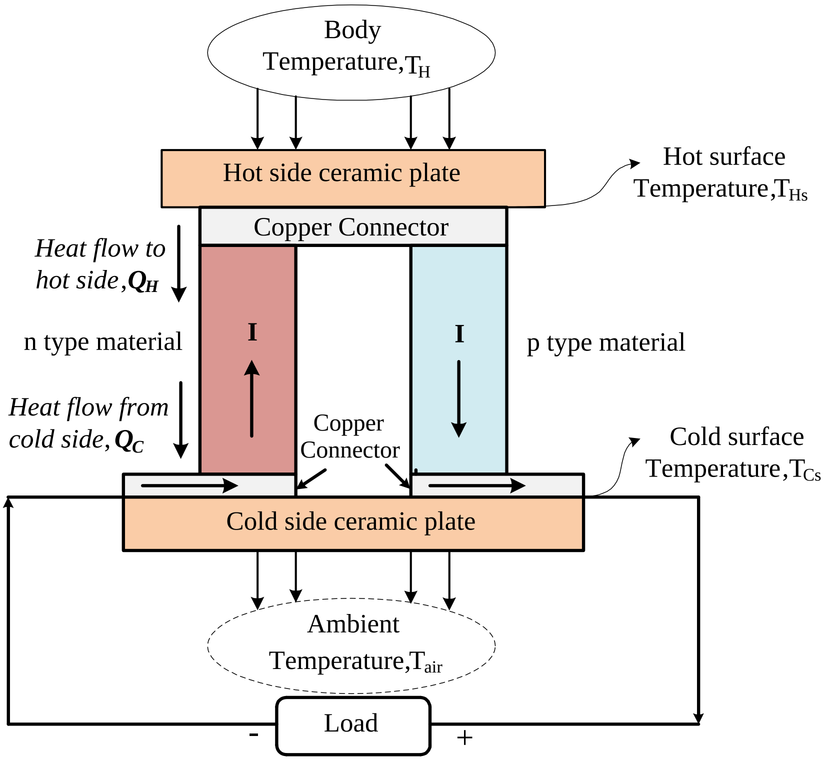

Figure 2. Single thermoelectric pair comprising of *n*-type and *p*-type material. Heat flows from hot side to top side $(Q_H \rightarrow Q_C)$ and electrical current (*I*) is flowing from *n*-type to *p*-type material due to temperature gradient ($\Delta T = T_{Hs} - T_{Cs}$) [18].

TEG works to convert heat energy into electrical energy with a certain temperature difference between the two sides of the peltier. If the metal is heated at a temperature of 80°C while the metal temperature of heat dissipation is at 50°C, the peltier experiences a difference in temperature of 30°C [19]. The temperature difference causes the TEG to work. The greater the temperature difference, the greater the electrical energy will be produced. However, if the temperature difference is too large, the bismuth semiconductor material used will be damaged [6]. This scheme can be seen in Figure 3.

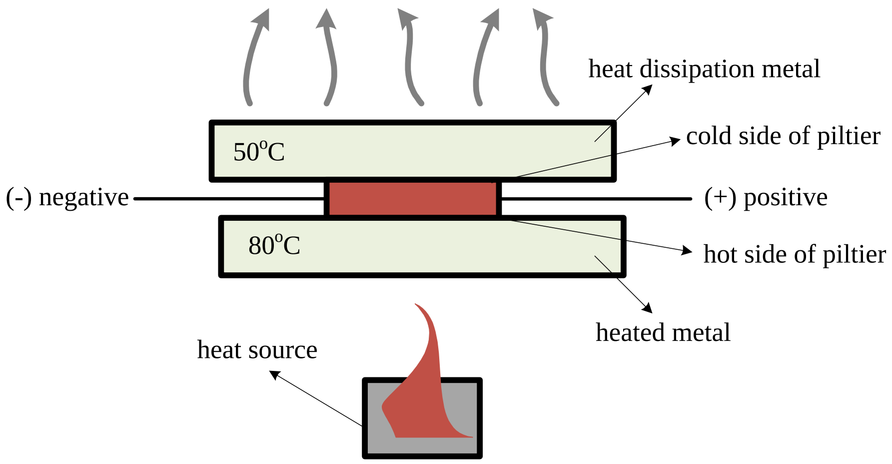

Figure 3. Seebeck effect on TEG [20].

2.1.2. Application of Thermoelectric Power Generation

TEG can be divided into low power and high power generation. Low power plants can produce power ranging from 5 µW to 1 W, and anything from 1 W up TEG is considered high power generator [21] [22]. The high power generation category consists of several TEG models linked together to produce a large amount of power. TEGs of low power generation are used in biomedical, military, aerospace and long-range applications. Others are used in the medical field such as pacemakers and hearing aids. Electrical devices incorporated in other bodies have power requirements ranging from 5 µW to 1 W. They have a life expectancy of up to 5 years [3] [23]. TEGs of high power generation are mostly used for automobile and industrial engines, iron and steel, chemical, petroleum refining, forest products, and food and beverage industries which consume enormous amounts of energy. The power plant involves the development of a TEG waste heat recovery system [24]. In 1998, Nissan built the first thermoelectric power plant based on Si-Ge elements for automobiles [25]. The Bell Solid State Thermo-electrics (BSST) group which includes BMW, Visteon, and Marlow Industries made further progress in 2004 with waste heat recovery systems from passenger vehicles [26] [27].

2.1.3. Power Calculation

For a typical TEG configuration (see Figure 3) the difference between the level of thermal energy entering the hot side (Q_H, W) and that which leaves the cold side of the thermoelectric pair (Q_C, W) is equal to the amount of power generated (P, W) by a system as given by [28]:

$$P = n(Q_H - Q_C) \tag{2}$$

In the above equation, n is the number of pairs of thermoelectric units in one module, and Q_H and Q_C are given by:

$$Q_H = \alpha I T_{Hs} + K(T_{Hs} - T_{Cs}) - 0{,}5 I^2 R \tag{3}$$

$$Q_C = \alpha I T_{Cs} + K(T_{Hs} - T_{Cs}) + 0{,}5 I^2 R \tag{4}$$

In Equations (3) and (4), $\alpha = \left(|\alpha_n| + |\alpha_p|\right)$ (V K^{-1}) in which α_n and α_p are the Seebeck coefficients of the n-type and p-type material, respectively, I is the load current (A), K is the thermal conductance (W K^{-1}) of the materials, and R is the load resistance (Ω). Thermal conductance can be expressed as:

$$K = k_n \gamma_n + k_p \gamma_p = k_n \left(\frac{A}{l}\right)_n + k_p \left(\frac{A}{l}\right)_p \tag{5}$$

$$\gamma = \left(\frac{A}{l}\right) \tag{6}$$

In which, k_n and k_p are the thermal conductivity ($Wm^{-1}K^{-1}$) of the material, A is the cross-sectional area (m^2), l is the length (m) feet, and γ is the effective length (m) of the feet.

2.1.4. Important Power Plant Parameters

The thermal efficiency of any thermoelectric material depends on many parameters; however, the most important parameter is the dimensionless feasibility rate (ZT) [29]. The typical value of ZT is -1 for most thermoelectric materials [12], whereas that of Bi_2Te_3 is 0.8 at room temperature [30] [31]. This eligibility figure can be expressed as

$$Z = \frac{\alpha^2}{\rho k} \tag{7}$$

in which α is the Seebeck coefficient for n-type and p-type materials, p is the electrical resistivity (Ω), and k is the thermal conductivity of the thermoelectric materials. The power generation of the thermoelectric module also depends on the TEG geometry, cross-sectional area (A), length of feet (l), hot (T_{Hs}) and cold (T_{Cs}) surface temperatures, material internal resistance (Rin), and number of pairs (n) in one. module. The effects of the area's and the length's changes on the power generation of thermoelectric materials are particularly sensitive. To demonstrate this effect, the n-type Bi_2Te_3 and p-type Sb_2Te_3 were selected by means of numerical analysis [32] using material properties, as given in Table 1, the effect of area and length on the modeled power plant (P, W), surface power density (Surface power density refers to power per unit cross-sectional area of feet, W cm^{-2}), and current (I, A).

Table 1. The Properties of the Materials Used for Numerical Analysis [33] [34].

type-n : Bi_2Te_3	type-p : Sb_2Te_3
α_n = -195 × 10^{-6} V K^{-1}	α_p = 230 × 10^{-6} V K^{-1}
ρ_n = 1.35 × 10^{-3} Ω cm	ρ_n = 1.75 × 10^{-3} Ω cm
z_n = 2.05 × 10^{-3} K^{-1}	Z_p = 2.5 × 10^{-3} K^{-1}

2.2. TEG Application for Self-heating Power Plants

TEG applications for Self-heating Power Plants can be made in the form of a box diagram as shown in Figure 4.

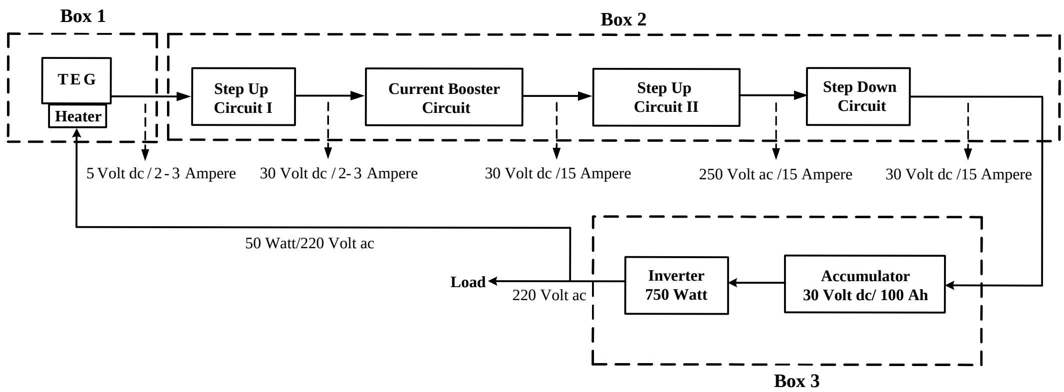

Figure 4. Diagram of TEG Application for Self-heating Power Plants.

Box 1 contains a circuit :

Heater of 50 Watt / 220 Volt ac to heat metal plates in the form of aluminum. The heat from the aluminum is then transferred to the hot side of the TEG piltier. And the cold side the TEG piltier is placed on the heatsink (as a metal heat dissipation). Half or more of the heatsink is submerged in water. The temperature of the heated metal and the temperature of the heat dissipation metal is with a certain difference. Then the temperature difference causes TEG to start working which releases a voltage and current of 5 Volts dc and 2-3 Amper.

Box 2 consists of:

• Step Up Circuit 1

The voltage generated by the two connecting copper sides, namely 5 Volts on the thermoelectric in **Box 1**, will be increased by the Step Up Circuit 1 in **Box 2**. The Step Up Circuit 1 is a circuit that can increase the input voltage (V_{in}) to a higher one. Here the input voltage V_{in} will always be smaller than V_{out} ($V_{in} < V_{out}$). This circuit is called a step-up circuit, because what is input is the dc voltage (V_{in}) and the output is also the dc voltage (V_{out}), so this circuit is called the Step Up DC To DC Converter.

• Current Booster Circuit

The Current Booster Circuit is a circuit that functions to increase a small input current into a large output current, where the output current (I_{out}) can be adjusted according to need. This circuit uses a current-increasing transistor TIP 3055. This transistor has a maximum current capacity of 15 Amper with a voltage of 60 Volts [35].

The output current of the Step Up Circuit 1 is still very small, ranging from 2 Amper to a maximum of 3 Amperes. This Current Booster Circuit is used to increase the current up to 15 Amper with a working voltage of 30 Volt dc..

• Step Up Circuit 2

The Step Up Circuit 2 or called the High Voltage Boost Converter Circuit is a circuit for increasing and changing the voltage that can increase and change the small input voltage ranging from 12 Volts dc to 30 Volts dc to output voltages ranging from 100 volts ac to 1000 volts ac depending on needs. The input and output currents are constant at 15 Amperes in a design of a high voltage boost converter with a maximum power of 500 Watts.

• Step Down Circuit

The Step Down Circuit or completely called Step Down Voltage Regulator Circuit is a device that can reduce the voltage in which the input voltage is higher than the output voltage. The output voltage will remain stable / well regulated, even though the input voltage fluctuates as long as it is within the recommended input voltage range [36]. The designing of this system involves designing a device that can reduce a stable voltage with a constant current using IC LM 2596, the input voltage of 220 Volts ac and the output voltage of 30 Volts dc with an output current of 15 Amper which are applied to battery charger of 9 Amper hour (Ah) in the system. IC LM 2596 is a monolithic regulator IC which is ideal for use as a step down switching regulator (buck converter).

Box 3 contains a circuit :

The output of the Step Down Circuit with a maximum current of 15 Amper and a voltage of 30 Volts dc in **Box 2** is the input for the accumulator in **Box 3** with a capacity of 100 Ah and a voltage of 30 Volt dc. The accumulator is connected to the inverter circuit which is used to supply power to the ac load and to heat the heater. The electric power in this Inverter Circuit is 750 Watts and a voltage of 220 Volts ac.

2.3. Linear Regression With Two Variables

Generally, the problem of a research that uses linear regression analysis is that it requires more than one independent variables. It is quite complicated to determine its mathematical model [37]. This mathematical model is useful for predicting responses to future events [38]. Look at the mathematical model of the regression analysis with two independent variables, for example, x_1 and x_2 as follows.

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i \tag{8}$$

The assumption taken in this model is that x_1 and x_2 do not have a distribution while ε_i has an N distribution $(0, \sigma^2)$. Now β_0, β_1, and β_2 will be estimated and will be represented by b_0, b_1, and b_2. By means of the least squares method in estimating the values of β_0, β_1, and β_2, they can be obtained by minimizing the quadratic form.

$$J = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2})^2 \tag{9}$$

This minimum is obtained by finding the derivative J with respect to β_0, β_1, and β_2 then equating each of these derivatives with zero. In the following calculations β_0, β_1, and β_2 are directly replaced by estimating b_0, b_1, and b_2. From the derivative with respect to β_0, β_1, β_2 by simplifying and substituting the regression coefficients in the estimate is

$$\begin{aligned} nb_0 + b_1 \sum x_{i1} + b_2 \sum x_{i2} &= \sum y_i \\ b_0 \sum x_{i1} + b_1 \sum x_{i1}^2 + b_2 \sum x_{i1} x_{i2} &= \sum y_i x_{i1} \\ b_0 \sum x_{i2} + b_1 \sum x_{i1} x_{i2} + b_2 \sum x_{i2}^2 &= \sum y_i x_{i2} \end{aligned} \qquad (10)$$

If arranged in the form of a matrix, Equation (10) takes the form of

$$\mathbf{X}'\mathbf{X}\mathbf{b} = \mathbf{X}'\mathbf{Y} \qquad (11)$$

with

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ 1 & x_{31} & x_{32} \\ \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix},$$

$$\mathbf{X}'\mathbf{X} = \begin{pmatrix} n & \sum x_{i1} & \sum x_{i2} \\ \sum x_{i1} & \sum x_{i1}^2 & \sum x_{i1} x_{i2} \\ \sum x_{i2} & \sum x_{i1} x_{i2} & \sum x_{i2}^2 \end{pmatrix},$$

$$\mathbf{X}'\mathbf{Y} = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_{11} & x_{21} & x_{31} & \cdots & x_{n1} \\ x_{12} & x_{22} & x_{32} & \cdots & x_{n2} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_{i1} y_i \\ \sum x_{i2} y_i \end{pmatrix}.$$

If X 'X is not singular then Equation (11) becomes

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{Y} \qquad (12)$$

2.3.1. Correlation Coefficient With R^2

After estimating the regression equation from the data, the next problem faced is assessing the poor fit of the regression model used with the data. Before going any further, it needs to be realized that the dependency of the model used on the least squares method. For easier understanding, see Figure 5.

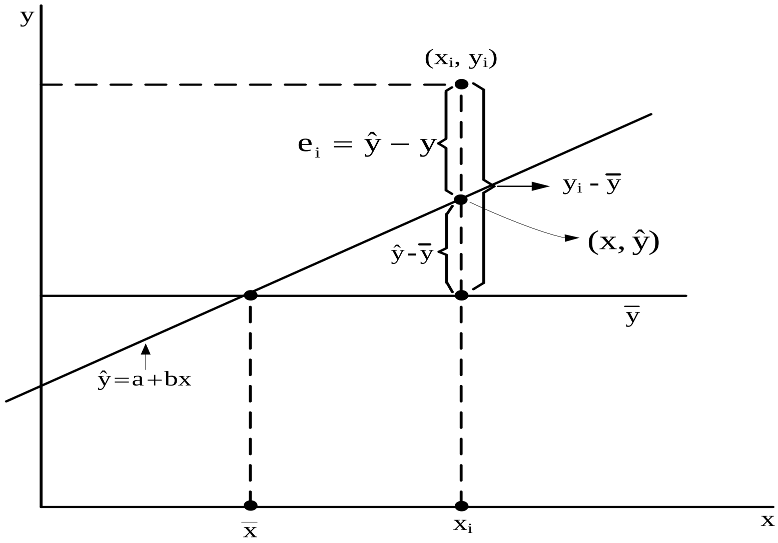

Figure 5. Making it easier to understand in finding the correlation coefficient R^2.

From Figure 5, consider the following equation

$$\left(y_i - \overline{y}\right) \equiv \left(\hat{y}_i - \overline{y}\right) + \left(y_i - \hat{y}_i\right)$$

$\uparrow$	$\uparrow$	$\uparrow$
Variation(total deviation)	due to regression	The remainder, the part that cannot be explained by regression

If the left and right segments are squared and then added, it will result in

$$\begin{aligned}\sum_{i=1}^{n}(y_i - \overline{y})^2 &\equiv \sum_{i=1}^{n}\left\{(\hat{y}_i - \overline{y}) + (y_i - \hat{y}_i)\right\}^2 \\ &= \sum_{i=1}^{n}(\hat{y}_i - y)^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + 2\sum_{i=1}^{n}(\hat{y}_i - \overline{y})(y_i - \hat{y}_i)\end{aligned} \tag{13}$$

The third part of the right-hand segment is made equal to zero, therefore

$$\sum_{i=1}^{n}(y_i - \overline{y})^2 = \sum_{i=1}^{n}(\hat{y}_i - y)^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{14}$$

if it does not raise any doubt the writing of i = 1 and n in Σ is omitted, then

$$\sum(y_i - \overline{y})^2 = \sum(\hat{y}_i - y)^2 + \sum(y_i - \hat{y}_i)^2 \tag{15}$$

Equation (15) is the **basic equation** in regression analysis and analysis of variance, the left side is called the **total number of squares** (TNS). The first part of the right-hand side is called the **sum of the squares of the regression** (SSR), and it is the variation of the response around its mean ($\overline{y}$). It is not difficult to prove that $\overline{\hat{y}}$, that is the mean of $\hat{y}_i$, is equal to mean y_i. The second part of the right side is called the **sum of the squares of the error** (remainder) and is abbreviated as SSE. This section measures the remainder of the total variation (TNS) that x cannot explain, or the part that is random in nature. Thus Equation (15) can be written as

$$R^2 = \frac{\sum(\hat{y}_i - \bar{y})^2}{\sum(y_i - \bar{y})^2} = \frac{SSR}{TNS} \quad (16)$$

Total Variation = Variation due to Regression + Variation due to Remainder

TNS is used as a comparison to determine the size of SSR or SSE. Define it R^2 is called the two-variable correlation coefficient or determinant coefficient (determination). Therefore $0 \leq SSR \leq TNS$, then of course $0 \leq R^2 \leq 1$. $R^2 = 0$ if SSR = 0, or SSE = TNS, and $R^2 = 1$ if SSR = TNS, SSE = 0. SSR = 0 if $\hat{y}_i = \bar{y}$ for every I, y_i is not dependent on or influenced by x_i. In other words, knowing about x_i does not help at all in predicting the value of y_i (see Figure 6 (a)). Conversely, if SSR = TNS then $y_i = \hat{y}_i$ for each data point. So every y_i prediction is absolutely accurate, absolutely no one is wrong (see Figure 6 (b)). So R^2 can measure the suitability of the data to the model. The closer R^2 is to 1 the better the fit of the data to the model, and conversely, the closer R^2 is to 0 the worse the fit is. R^2 is usually expressed in percent that people often use.

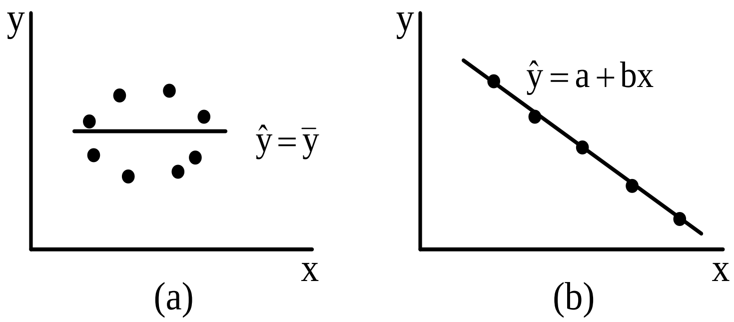

Figure 6. (a) R^2 is the smallest, and (b) R^2 is the largest.

2.3.2. Table of Variance Analysis

To determine whether the effect of an independent variable x is large or small on the response y requires a standard comparison which is not influenced by the merits of the model used. The standard benchmark is an unbiased estimator of σ^2, the variance ε. Generally σ^2 is not known, so it must be estimated. An unbiased estimate of σ^2 can be obtained from the sum of the squares of the remainder, namely SSE/(n-2), called **the mean of the squares of the remainder**. The number n-2 is called **the degrees of freedom.** However, this residual square mean will only estimate σ^2 without bias if the correct model is used. If the wrong model is used, then SSE/(n-2) will estimate σ^2 as the estimator with bias. In other words, how good the SSE/(n-2) is to estimate σ^2 depends on the exact model used. So, the use of the mean of the residual squares as an estimate for σ^2 always assumes that the model is correct. Of course, whether this assumption is reasonable or not should also be checked later, for example through an examination of the remainder. Now pay attention to

$$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 = b^2 \sum_{i=1}^{n}(x_i - \bar{x})^2 \quad (17)$$

Only b is independent information in this form because $\sum(x_i - \bar{x})^2$ is not a random variable. So there is only one piece of information that needs to be assessed in SSR, because df of SSR is 1. The degree of freedom of SSE is a little more difficult to calculate directly. The easiest way is to take the difference between df of TNS and df of SSR, so df of SSE = (n-1) - 1 = n - 2. Note that there are 2 parameters in the simple linear model used. In general, if p states the number of parameters in the model, then the df of SSE is n = p, while the df of SSR is p - 1, and the df of TNS is not model dependent, so df of TNS is still n -1. Table 2 shows the general form of the table of variance analysis for simple linear regression. The fourth column gives the sum of the squares divided by their degrees of freedom, for Regression and Remainder. The total is not written below on the Total line, because it does not apply to TNS. The last row provides the expectation of the fourth row, namely the expectation of the Average Squared Regression, E(ASR), and the expectation of the Remaining Squared Average, E (RSA). This row provides the test basis for β. If β = 0 then E(ASR)/E(RSA) = 1, but if β ≠ 0 then E(ASR)/E(RSA) > 1, because $\beta^2 \sum (x_i - \bar{x})^2 > 0$. In theory ASR/RSA has an F distribution with degrees of freedom 1 and n - 2, the statistical test can be defined as follows

Table 2. Simple regression analysis of variance.

Sources of Variations	SS (sum of square)	df (degree of freedom)	SA (squared average)	F
Regression	$SSR = \sum(\hat{y}_i - \bar{y})^2$	1	ASR = SSR/1	ASR/RSA
Remainder	$SSE = \sum(y_i - \hat{y})^2$	n-2	RSA = SSR/(n-2)	
Total	$TNS = \sum(y_i - \bar{y})^2$	n-1		

3. Results and Analysis

3.1. Tools Design

The design results of Figure 4 as shown in Figure 7.

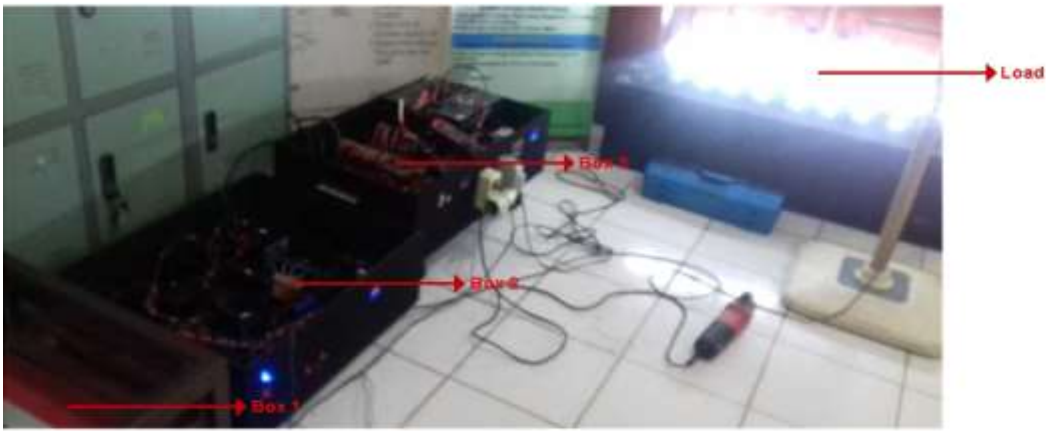

Figure 7. TEG Applications for Self-Heating Power Plants.

3.2. Measurement Results

The results of measuring the output voltage and the current on the accumulator with varying loads can be seen in Table 3.

Table 3. The Measurement Results for the Output Voltage and the Current on the Accumulator.

No.	Hours	Load (Watt)	Output Voltage (Volt)	Output Current (Ampere)	Explanation
1	07.00	130	24.66	11.17	Date 23.02.2019
2	09.00	130	24.37	10.05	
3	11.00	140	24.20	8.38	
4	13.00	140	23.76	8.08	
5	15.00	150	24.02	9.19	
6	17.00	150	23.92	8.22	
7	07.00	160	23.85	8.39	Date 24.02.2019
8	09.00	160	23.80	8.25	
9	11.00	170	23.99	9.52	
10	13.00	170	23.62	8.30	
11	15.00	180	23.93	8.72	
12	17.00	180	23.89	8.58	
13	07.00	190	23.34	8.46	Date 25.02.2019
14	09.00	190	23.16	7.69	
15	11.00	200	22.83	7.50	
16	13.00	200	22.62	7.78	
17	15.00	210	22.83	7.57	
18	17.00	210	22.25	7.76	
19	07.00	220	21.20	6.74	Date 02.03.2019
20	09.00	220	20.51	6.18	
21	11.00	230	17.18	5.52	
22	13.00	230	15.96	4.30	

3.3. Analysis

Based on the data in Table 3, a regression equation is made with the independent variable of load (Watt) as x_1 and the outflow (Ampere) as x_2, and the dependent variable of y. It is assumed that a linear model is directly used.

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$$

Now one wants to estimate β_0, β_1, and β_2 from the data of Table 3, one obtains

$$\mathbf{X} = \begin{pmatrix} & \overset{x_1}{\downarrow} & \overset{x_2}{\downarrow} \\ 1 & 130 & 11{,}17 \\ 1 & 130 & 10{,}05 \\ 1 & 140 & 8{,}38 \\ 1 & 140 & 8{,}08 \\ 1 & 150 & 9{,}19 \\ 1 & 150 & 8{,}22 \\ 1 & 160 & 8{,}39 \\ 1 & 160 & 8{,}25 \\ 1 & 170 & 9{,}52 \\ 1 & 170 & 8{,}30 \\ 1 & 180 & 8{,}72 \\ 1 & 180 & 8{,}58 \\ 1 & 190 & 8{,}46 \\ 1 & 190 & 7{,}69 \\ 1 & 200 & 7{,}50 \\ 1 & 200 & 7{,}78 \\ 1 & 210 & 7{,}57 \\ 1 & 210 & 7{,}76 \\ 1 & 220 & 6{,}74 \\ 1 & 220 & 6{,}18 \\ 1 & 230 & 5{,}52 \\ 1 & 230 & 4{,}30 \end{pmatrix}, \quad \text{dan } \mathbf{Y} = \begin{pmatrix} 24{,}66 \\ 24{,}37 \\ 24{,}20 \\ 23{,}76 \\ 24{,}02 \\ 23{,}92 \\ 23{,}85 \\ 23{,}80 \\ 23{,}99 \\ 23{,}62 \\ 23{,}93 \\ 23{,}89 \\ 23{,}34 \\ 23{,}16 \\ 22{,}83 \\ 22{,}62 \\ 22{,}83 \\ 22{,}25 \\ 21{,}20 \\ 20{,}51 \\ 17{,}18 \\ 15{,}96 \end{pmatrix}$$

One obtains the values :

$$\mathbf{X'X} = \begin{pmatrix} 20 & 3.960 & 180 \\ 3.960 & 735.800 & 30.930 \\ 180 & 30.930 & 1.460 \end{pmatrix}$$

$$\mathbf{X'Y} = \begin{pmatrix} 500 \\ 88.795 \\ 4.068 \end{pmatrix}$$

$$(\mathbf{X'X})^{-1} = \begin{pmatrix} 16{,}4248 & -0{,}0459 & -1{,}0134 \\ -0{,}0459 & 0{,}0001 & 0{,}0026 \\ -1{,}0134 & 0{,}0026 & 0{,}0689 \end{pmatrix}$$

and

$$(\mathbf{X'X})^{-1}\mathbf{X'Y} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 14{,}9401 \\ -0{,}0097 \\ 1{,}189 \end{pmatrix}$$

Its regression equation is

$$\hat{y}_i = 14{,}94 - 0{,}0097x_1 + 1{,}189x_2$$

Based on Table 3 and its regression equation above, Table 4 can be made.

Table 4. The value of y_i of Table 3, the value of $\hat{y}_i$ and other completeness.

Beban (Watt)	n_i	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$\hat{y}_i$	$\hat{y}_i - \bar{y}$	$(\hat{y}_i - \bar{y})^2$
130	1	24,66	1,94	3,76	26,96	4,23	17,89
130	2	24,37	1,65	2,72	25,63	2,90	8,41
140	3	24,20	1,48	2,19	23,55	0,82	0,67
140	4	23,76	1,04	1,08	23,19	0,46	0,21
150	5	24,02	1,30	1,69	24,41	1,68	2,82
150	6	23,92	1,20	1,44	23,26	0,53	0,28
160	7	23,85	1,13	1,28	23,36	0,63	0,40
160	8	23,80	1,08	1,17	23,20	0,47	0,22
170	9	23,99	1,27	1,61	24,61	1,88	3,53
170	10	23,62	0,90	0,81	23,16	0,43	0,19
180	11	23,93	1,21	1,46	23,56	0,83	0,69
180	12	23,89	1,17	1,37	23,40	0,67	0,45
190	13	23,34	0,62	0,38	23,16	0,43	0,19
190	14	23,16	0,44	0,19	22,24	- 0,49	0,24
200	15	22,83	0,11	0,01	21,92	- 0,81	0,66
200	16	22,62	- 0,10	0,01	22,25	- 0,48	0,23
210	17	22,83	0,11	0,01	21,90	- 0,83	0,69
210	18	22,25	- 0,47	0,22	22,13	- 0,60	0,36
220	19	21,20	- 1,52	2,31	20,82	- 1,91	3,65
220	20	20,51	- 2,21	4,88	20,15	- 2,58	6,66
230	21	17,18	- 5,54	30,96	19,27	- 3,46	11,97
230	22	15,96	- 6,76	45,70	17,82	- 4.91	24,11
	$\bar{n}_i = 11{,}5$	$\bar{y} = 22{,}72$	$\Sigma(y_i - \bar{y}) = 0{,}05$	$\Sigma(y_i - \bar{y})^2 = 76{,}66$	$\bar{\hat{y}}_i = 22{,}73$	$\Sigma(\hat{y}_i - \bar{y}) = -0{,}11$	$\Sigma(\hat{y}_i - \bar{y})^2 = 84{,}52$

The table of its variance analysis is shown as Table 5.

Table 5. Variance Analysis of Table 4.

Sources	SS	df	SA	F
Regression	84.52	1	84.62	215.06
Remainder	7.86	20	0.393	
Total	76.66	21		

From the point of view of R^2, it is clear that this model is the most perfect or the best model that can be made in predicting the loading on the Thermoelectric Generator Application system for Power Generation.

4. Conclusion

Based on the results of the calculation a mathematical model is obtained: $\hat{y}_i = 14.94 - 0.0097\, x_1 + 1.189\, x_2$ to be used to estimate the maximum load on the system, so that the system can avoid exceeding this load in order that the accumulator can work continuously or at least work for a long time. By using this mathematical model, then $x_1 = 200$ (load variable, Watt) and $x_2 = 7.78$ (current variable, Amper), then the voltage $\hat{y}_i = 22.250$ volts. If the system is loaded with more than 200 Watts, then the yellow accumulator output voltage indicator light gives a signal that the voltage is at a critical point.

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