Research Article

Linearly Deteriorating EOQ Model for Imperfect Items with Price Dependent Demand Under Different Fuzzy Environments

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Abstract

In this article, linearly deteriorating EOQ models have been developed for imperfect quality items (both crisp and fuzzy models) with linear and price dependent demand. The price depended demand is considered as two different types of fuzzy number viz. trapezoidal and cloudy fuzzy model. Defuzzification has been done using signed distance method and Yager's Ranking Index. All results are verified numerically and graphically for both models. Sensitivity analysis of the model is carried out to validate the models for optimality.

Keywords: linear deterioration, EOQ, price dependent demand, Trapezoidal fuzzy number, Signed distance, cloudy fuzzy and Yager's Ranking Index.

1. Introduction

Deterioration also known as decay, damage or spoilage in inventory models is now of immense practical importance, which is gaining attention from the researchers. Deterioration occurs with passage of time depending upon the kind of items considered. Food items, drugs, medicines, blood in blood banks are few items depending on time. Researchers, viz. Covert and Philip (1973), Giri et al. (2003), Ghosh and Chaudhari (2004), Sana et. al. (2004) are developed lot size models for deteriorating items. Mishra and Tripathy (2010), Kawale and Bansode (2012), Sharma and Chaudhary (2013), etc., considered models having deterioration rate proportional to time. A Price and ramp-type demand which also depends on time has been developed by Wang, Chuanxu, Huang, Rongbing (2014). Patro et. al. (2017) & (2018) developed EOQ models without deterioration and with deterioration using allowable proportionate discount under learning effects respectively.

A more practical and realistic EOQ model is the one considering items to be imperfect. Porteus (1986), Rosenblatt and Lee (1987), Raouf, Jain, and Sathe (1983) are few researchers who studied the basic EOQ model for influence of defective items. It is supposed that, there is no fault in the screening process of traditional inventory models that identifies the defective items, the items are screened without any inspection, i.e. zero error inspection is carried out. But in 2000 Salameh and Jaber developed model with considering after hundred % screening the imperfect quality items collect a single batch and then sold.Similar work was done by Goyal and Cardenas-Barron (2002). Inventory models developed by Pal et al. (2007), Bhunia and Shaikh (2011), were considering the effects of advertisement and variations price on rate of demand for an item. Nita Shah (2012) developed a time-proportional deterioration model without shortages and with replenishment policy for items having demands depending on price. Considering selling price dependent demand Sarkar (2013) developed a deteriorating model. For deteriorating items Chowdhury and Ghosh (2014) developed an inventory model with price and stock sensitive demand. Khana et. al. (2017) considering price dependent demand, developed a lot size deteriorating model for imperfect quality items.

Uncertainties in some situations is due to fuzziness was primarily introduced by Zadeh (1965), also some strategies for decision making in fuzzy environment was proposed by Zadeh et. al (1970). For defective items, Chang (2004) developed a model instigating the fuzziness for annual demand and rate of defective. Using triangular fuzzy number De and Rawat (2011) developed without shortage fuzzy inventory model. Considering an optimal replenishment policy and assuming fuzziness in demand, ordering and holding cost Dutta and Pawan Kumar (2013) developed an inventory fuzzy no shortage model. Kumar and Rajput (2015) have proposed fuzzy lot size models for item deteriorating items with time dependent demands respectively. Shekarian et. al. (2017) have done a comprehensive review on different fuzzy EOQ/EPQ models. Degree of learning experiences was captured by De and Beg (2016) who introduced dense fuzzy number, this idea was extended by De and Mahata (2017), who incorporated cloud-type fuzzy number to measure fuzziness in inventory cycle time. Karmakar et al. (2017) established an EPQ model with pollution-sensitive dense fuzzy having cycle time-dependent production rate. An EOQ model with fuzzy defective rate using trapezoidal fuzzy numbers and error inspection has been developed by Patro et. al. (2019).

We have considered an EOQ model with price dependent demand for deteriorating items with allowable proportional discount under crisp as well as fuzzy environments in this paper. In the crisp model, the rate of deterioration is considered depending on time in the first case and in the second case the rate of deterioration is constant. We have considered the general fuzzy environment of trapezoidal fuzzy number and also the cloudy fuzzy model for both the cases. For defuzzification, the signed distance method and Yager's ranking index has been considered respectively. Sensitivity analysis and suitable numerical examples have been considered. A table for comparison for different models has been shown below.

References	Deterioration	Demand	Imperfect/defective	Fuzzy
Chakrabarty et al. (1998)	Weibull distribution	Trend	no	no
Khan and Jaber (2011)	no	constant	yes	no
Hsu and Hsu (2013)	no	constant	yes	no
Gothi and Chaterji (2015)	no	constant	yes	no
Margatham and Lakshmidevi (2013)	constant	price dependent	no	Trapezoidal fuzzy
Jaggi et. al. (2015)	constant	Ramp type	no	Triangular Fuzzy
Shekarian et. al. (2016)	no	constant	yes	Triangular fuzzy
Khana et. al. (2017)	constant	Price dependent	yes	no
Patro et. al. (2017)	constant	constant	yes	Triangular
Kazemi et.al.(2018)	no	constant	yes	no

Sinha et.al(2020)	no	Price dependent	yes	no
Tahami et.al(2020)	no	constant	yes	fuzzy
This Paper	Time dependent	Price dependent	yes	Trapezoidal and cloudy fuzzy

2. Definitions

Definitions 2.1 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a} & ,when\ a \le x \le b; \\ 1 & ,when\ b \le x \le c; \\ R(x) = \dfrac{d-x}{d-c} & ,when\ c \le x \le d; \\ 0 & , \quad otherwise \end{cases}$$

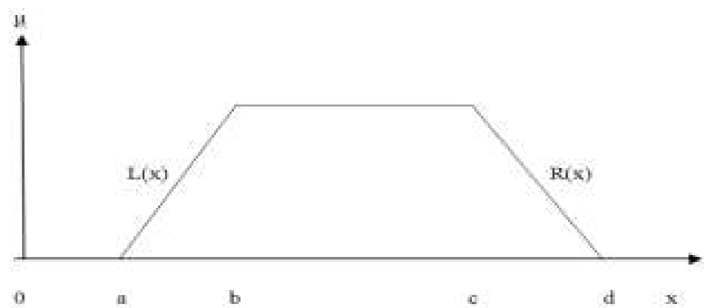


Fig.1: Trapezoidal Fuzzy Number

Definitions 2.2 Let $\tilde{A} = (a\ , b\ , c\ ,\ d)$ be a trapezoidal fuzzy number, then the signed distance method of $\tilde{A}$ is defined as $d(\tilde{A}\ , 0) = \frac{1}{2}\int_0^1 [A_L\alpha + A_R\alpha] d\ \alpha$

$$\text{Where} \quad A_\alpha = [A_L\alpha\ , A_R\alpha]$$

$$= [a + (b-a)\alpha\ , d - (d-c)\alpha], \alpha \in [0\ , 1]$$

is called alpha-cut of the trapezoidal fuzzy number $\tilde{A}$, which is a close interval $d(\tilde{A}\ , 0) = \frac{a+b+c+d}{4}$

Definitions 2.3 Let $\tilde{A} = (a, b, c)$ be a normalized general triangular fuzzy number, then its membership function defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if \quad x < a \; and \; x > b \\ \dfrac{x-a}{b-a} & if \quad a < x < b \\ \dfrac{c-x}{c-b} & if \quad b < x < c \end{cases}$$

Here $A_L = a + (b-a)\alpha$ and $A_R = c - (c-b)\alpha$ are alpha-cuts of the membership function $\mu_{\tilde{A}}(x)$. Where $\alpha \in [0\,,1]$.

Definitions 2.4 Let the left and right alpha cuts of the fuzzy number$\tilde{A}$, be considered A_L and A_R whose defuzzification rule under Yager's Ranking Index is given by

$$I(\tilde{A}) = \frac{1}{2}\int_0^1 (A_L + A_R)d\alpha$$

Definitions 2.5 A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a cloudy normalized triangular fuzzy number if after an infinite times the set its self converges to a crisp singleton. That means as time t tends infinity both $a, c \to b$. Let as consider the fuzzy number

$\tilde{A} = \left[b\left(1 - \frac{\gamma}{1+t}\right), b, b\left(1 + \frac{\delta}{1+t}\right)\right]$, for $0 < \gamma\,, \delta < 1$.

Note that $\lim_{t\to\infty} b\left(1 - \frac{\gamma}{1+t}\right) = b$ and$\lim_{t\to\infty} b\left(1 + \frac{\delta}{1+t}\right) = b$, so$\tilde{A} \to \{b\}$.

Then the membership function for $0 \le t$ is as:

$$\mu_{\tilde{A}}(x,t) = \begin{cases} 0 & if \quad x < b\left(1 - \dfrac{\gamma}{1+t}\right) \; and \; x > b\left(1 + \dfrac{\delta}{1+t}\right) \\ \left\{\dfrac{x - b\left(1 - \dfrac{\gamma}{1+t}\right)}{\dfrac{b\gamma}{1+t}}\right\} & if \quad b\left(1 - \dfrac{\gamma}{1+t}\right) \le x \le b \\ \left\{\dfrac{b\left(1 + \dfrac{\delta}{1+t}\right) - x}{\dfrac{b\delta}{1+t}}\right\} & if \quad b \le x \le \; b\left(1 + \dfrac{\delta}{1+t}\right) \end{cases}$$

3. Assumptions and Notations

3.1 Assumptions considered:

1. Price dependent demand. It is denoted by $D_R = a - bS_P$ $, where \; scale \; operator \; a \; and \; b > 0, and \; S_P$ is the selling price of good quality items.
2. The linear and time dependent rate of deterioration, that is $\theta(t) = \alpha t\,, where$ $0 < \alpha \ll 1$, t > 1 and for t =1, $\theta(t) = \alpha$.
3. Replenishment is Instantaneous.
4. Zero lead time.

5. Time horizon is considered finite.
6. Shortages are not permitted.
7. Selling price is fixed for good Quality items.
8. Batch wise 100 % inspections of items.
9. The items with defect are sold as a single batch with proportional discounted price.

3.2 Notations

3.2.1 Crisp Notations

We define the following symbols:

$\boldsymbol{Q_S}$: Order size for each cycle.

$\boldsymbol{C_V}$: Variable cost/unit.

$\boldsymbol{K_C}$: Fixed ordering cost.

$\boldsymbol{D_R}$: The rate at which demand varies.

$\boldsymbol{H_C}$: Holding cost/unit.

$\boldsymbol{P_D}$: The percentage of defective items in Q_S .

$\boldsymbol{S_P}$: Retail price of good quality items.

$\boldsymbol{S_R}$: Screening rate of the defective items.

$\boldsymbol{S_C}$: Screening cost of each item unit wise.

T : Length of one cycle.

$\boldsymbol{C_R}$: Total revenue for each cycle.

TC : Total cost for each cycle.

$\boldsymbol{C_{TP}}$: Total profit in each cycle.

$\boldsymbol{TP_U}$: Total profit made by item per unit time.

3.2.2 Fuzzy Notations

$\widetilde{D}_R$: The rate in which demand varies in fuzzy model.

$TP\widetilde{_U(Q_S)}$: Total profit by item in per unit in fuzzy sense.

$d_f\left(TP\widetilde{_U(Q_S)}\right)$: De-fuzzified the total profit.

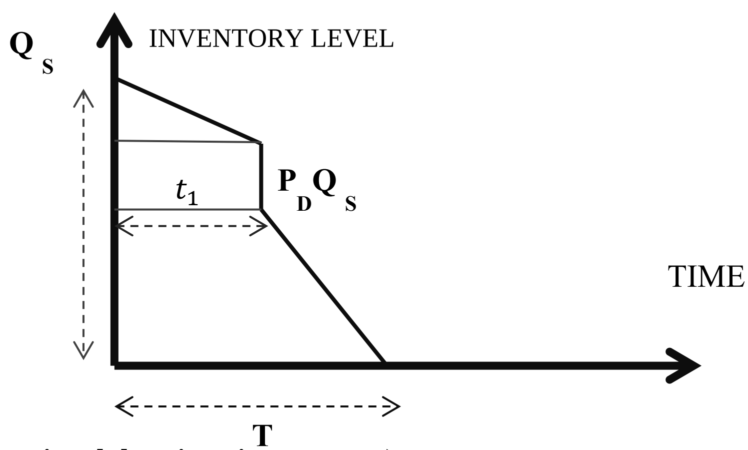
4. Model description

Salameh and Jaber (2000) and Patro et. al. (2019) considered their model that the defective items are sold at a constant and proportional discount price. But in this model we consider selling price dependent linear demand, minimum discount is considered for selling the first lot of defective items, then the next items are sold with discounts in high rate, continuing similarly and last one are sold exactly actual cost of the items.

A stock is kept for the poor quality items, which is obtained after a hundred percent screening of the lots at a rate of $\boldsymbol{S_R}$ units from which the proportional discount is estimated by approximating the selling price of the individual items with defect. These defective items are collected batch wise and sold at a proportional discounted price, by using the following formula. The unit selling price of the defective items can be calculated as follows:

$$\boldsymbol{S_P - \left(1 - \frac{Q_S P_D - i}{Q_S P_D}\right)\left(\frac{C_R(Q_S) - TC(Q_S)}{Q_S}\right)} \tag{4.1}$$

where $\boldsymbol{i = 1, 2, 3 \ldots\ldots\ldots.. Q_S P_D}$**.**
Considering a lot of size $\boldsymbol{Q_S}$ being instantaneously replenished and each of the lot containing fix proportion of defective ($\boldsymbol{P_D Q_S}$) and good quality($\boldsymbol{(1 - P_D)Q_S}$) items. After 100% screening each lot at a screening rate of S_R units/unit time with screening cost ($\boldsymbol{S_C}$) then the selling price of non-defective (good quality) items consider as $\boldsymbol{S_P}$ per unit and defective items are sold at a proportional discount price. After the inspection process, at time t_1 the inventory level $\boldsymbol{I(t)}$ becomes$\boldsymbol{(1 - P_D)Q_S - D_R t_1}$ and due to the market demand and deterioration the inventory level becomes zero at a time **T**. Within the screening time t_1, shortages are avoided, which makes the number of good items at least equal to the demand during the screening time t_1 which is given by$\boldsymbol{(1 - P_D)Q_S \geq D_R t_1}$, where $\boldsymbol{t_1 = \frac{Q_S}{S_R}}$.



5. Crisp mathematical model

5.1 Case-1 (When t > 1, time proportional deterioration rate $\boldsymbol{\alpha t}$)

The cycle initiates with an initial lot size $\boldsymbol{Q_S}$ at time t=0. During the time [0, t_1], the inventory level diminishes due to combined effect of demand and deterioration, the inventory level $\boldsymbol{I(t_1)}$ becomes $\boldsymbol{(1 - P_D)Q_S - D_R t_1}$ at time t = t_1, while due to the market demand and deterioration the inventory level becomes zero at time t=T. The instantaneous inventory level over the period [0, T] is governed by the differential equations:

$$\frac{d\boldsymbol{I(t)}}{d\boldsymbol{t}} + (\boldsymbol{\alpha t})\boldsymbol{I(t)} = -\boldsymbol{D}_{\boldsymbol{R}} \qquad , \ \boldsymbol{0 \le t \le t_1} \tag{5.1.1}$$

$$\frac{d\boldsymbol{I(t)}}{d\boldsymbol{t}} + (\boldsymbol{\alpha t})\boldsymbol{I(t)} = -\boldsymbol{D}_{\boldsymbol{R}} \qquad , \ \boldsymbol{t_1 \le t \le T} \tag{5.1.2}$$

Where $\boldsymbol{0 < \alpha << 1}$ and $\boldsymbol{D_R = a - bS_P}$

The solution of above differential equation (5.1.1) and (5.1.2) with boundary condition t=0, $I(t) = Q_S$ and t=t$_1$, $I(t_1) = (1 - P_D)Q_S - D_R t_1$ are as follows.

$$\mathrm{I(t)} = -(\mathrm{a} - \mathrm{bp})\left(\mathrm{t} + \frac{\alpha \mathrm{t}^3}{6}\right)\mathrm{e}^{\frac{-\alpha \mathrm{t}^2}{2}} + \mathrm{Q_S}\,\mathrm{e}^{\frac{-\alpha \mathrm{t}^2}{2}} \qquad , \ 0 \le t \le t_1 \tag{5.1.3}$$

$$\mathrm{I(t)} = (\mathrm{a} - \mathrm{bp})\left[(\mathrm{t}_1 - \mathrm{t}) + \frac{\alpha}{6}(\mathrm{t}_1^3 - \mathrm{t}^3)\right]\mathrm{e}^{\frac{-\alpha \mathrm{t}^2}{2}} + (1 - \mathrm{P_D})\mathrm{Q_S} - \mathrm{D_R t_1}, \mathrm{t_1} \le \mathrm{t} \le \mathrm{T} \quad (5.1.4)$$

Now the cycle wise total cost consists of sum of all cost (i.e ordering, variable, screening cost and holding cost) and is given by

$$TC(Q_S) = K_C + C_V Q_S + S_C Q_S + H_C\left[\frac{\{Q_S^2(1-P_D^2)(6D_R^2 - \alpha Q_S^2(1-P_D)^2)\}}{12D_R^3}\right] \tag{5.1.5}$$

[Holding cost during time period 0 to t$_1$ and t$_1$ to T is equal to $H_C\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right)$ after simplification we get

$$-(a - bS_P)\left(\frac{t_1^2}{2} - \frac{\alpha t_1^4}{12}\right) + Q_S\left(t_1 - \frac{\alpha t_1^3}{6}\right) \tag{5.1.6}$$

and putting $T = \frac{(1-P_D)Q_S}{D_R}$, $t_1 = \frac{Q_S}{S_R}$ and $S_R = \frac{D_R}{1-P_D}$ in equation (5.1.6), simplifying we get $H_C = \left[\frac{\{Q_S^2(1-P_D^2)(6D_R^2 - \alpha Q_S^2(1-P_D)^2)\}}{12D_R^3}\right]$

Total revenue during time period (0, T):

$$\boldsymbol{C_R(Q_S) = S_P(1 - P_D)Q_S + \sum_{i=1}^{Q_S P_D}[S_P - \left(1 - \frac{Q_S P_D - i}{Q_S P_D}\right)\left(\frac{C_R(Q_S) - TC(Q_S)}{Q_S}\right)]} \tag{5.1.7}$$

After simplification equation (5.1.7) get

$$\frac{\mathbf{2S_P(1-P_D)Q_S + (Q_S P_D + 1)\left[K_C + C_V Q_S + S_C Q_S + H_C\left\{\frac{\{Q_S^2(1-P_D^2)(6D_R^2 - \alpha Q_S^2(1-P_D)^2)\}}{12D_R^3}\right\}\right]}}{\mathbf{2Q_S + Q_S P_D + 1}} \tag{5.1.8}$$

The cycle wise total profit $C_{TP}(Q_S) = C_R(Q_S) - TC(Q_S)$

$$= \frac{2S_P Q_S^2 - 2Q_S\left[K_C + C_V Q_S + S_C Q_S + H_C\left\{\frac{\left\{Q_S^2\left(1-P_D^2\right)\left(6D_R^2 - \alpha Q_S^2(1-P_D)^2\right)\right\}}{12D_R^3}\right\}\right]}{2Q_S + Q_S P_D + 1} \qquad (5.1.9)$$

$TP_U(Q_S) = \frac{C_{TP}(Q_S)}{T}$ is the unit wise total profit given by:

$$TP_U(Q_S) = \frac{2S_P Q_S^2 - 2Q_S\left[K_C + C_V Q_S + S_C Q_S + H_C\left\{\frac{\left\{Q_S^2\left(1-P_D^2\right)\left(6D_R^2 - \alpha Q_S^2(1-P_D)^2\right)\right\}}{12D_R^3}\right\}\right]}{T(2Q_S + Q_S P_D + 1)} \qquad (5.1.10)$$

Putting $T = \frac{(1-P_D)Q_S}{D_R}$ in equation (5.1.10) and simplify $TP_U(Q_S) = \frac{2D_R(S_P Q_S - K_C - C_V Q_S - S_C Q_S)}{(1-P_D)(2Q_S + Q_S P_D + 1)} - \frac{H_C Q_S^2(1+P_D)\left(6D_R^2 - \alpha Q_S^2(1-P_D)^2\right)}{6D_R^2(2Q_S + Q_S P_D + 1)}$ (5.1.11)

The 1st and 2nd derivative of $TP_U(Q_S)$ w.r.t Q_S are as follows:

$$\frac{dTP_U(Q_S)}{dQ_S} = \frac{1}{(2Q_S + Q_S P_D + 1)^2}\left[\frac{2D_R(S_P - C_V - S_C + 2K_C + K_C P_D)}{(1-P_D)} - \frac{H_C(1+P_D)}{6D_R^2}\left\{6D_R^3 Q_S(2 + 2Q_S + Q_S P_D) - \alpha Q_S^3(1-P_D)^2(4 + 6Q_S + 3P_D Q_S)\right\}\right] \qquad (5.1.12)$$

And $\frac{d^2 TP_U(Q_S)}{dQ_S^2} < 0$ (5.1.13)

The 2nd order derivative of $TP_U(Q_S)$ is negative for all value of Q_S, which indicates that the concave function $TP_U(Q_S)$. Setting the 1st derivative equal to zero, the optimal order size that represents the maximum annual profit is determined. After some basic manipulation we get

$$(Q_S)_{max} = \sqrt{\frac{12D_R^3(S_P - C_V - S_C + 2K_C + K_C P_D)}{(1-P_D^2)H_C\left(6D_R^2 - 3\alpha Q_S^2(1-P_D)^2\right)(2+P_D)}} \qquad (5.1.14)$$

When P_D=0 , $C_V + S_C = S_P$ then $(Q_S)_{max}$ reduce to the traditional EOQ formula.

$$(Q_S)_{max} = \sqrt{\frac{2K_C D_R}{H_C}} \qquad (5.1.15)$$

5.2 Case-2 (When t = 1, deterioration rate reduces to constant deterioration.)

The instantaneous starts of $I(t)$ over period (0,T) are given by the differential equations:

$$\frac{dI(t)}{dt} + \alpha I(t) = -D_R \quad , \ 0 \le t \le t_1 \qquad (5.2.1)$$

$$\frac{dI(t)}{dt} + \alpha I(t) = -D_R \quad , \ t_1 \le t \le T \qquad (5.2.2)$$

Where $\mathbf{0} < \boldsymbol{\alpha} << \mathbf{1}$ and $\boldsymbol{D}_{\boldsymbol{R}} = \boldsymbol{a} - \boldsymbol{b}\boldsymbol{S}_{\boldsymbol{P}}$

The solution of above differential equation (5.2.1) and (5.2.2) with boundary condition t=0, $I(t) = Q_S$ and t=t₁ , $I(t_1) = (1 - P_D)Q_S - D_R t_1$ are given as follows:

$$I(t) = Q_S e^{-\alpha t} + \frac{a-bp}{\alpha}(e^{-\alpha t} - 1) \qquad , 0 \le t \le t_1 \tag{5.2.3}$$

$$I(t) = \left((1 - P_D)Q_S - (a - bS_P)t_1\right)e^{\alpha(t_1-t)} + \frac{a-bp}{\alpha}\left(e^{\alpha(t_1-t)} - 1\right) \quad , t_1 \le t \le T \tag{5.2.4}$$

The total cost obtained cycle wise is as follows:

$$TC(Q_S) = K_C + C_V Q_S + S_C Q_S + H_C\left[\int_0^{t_1} I(t) + \int_{t_1}^{T} I(t)\right] \tag{5.2.5}$$

After simplification the above equation (5.2.5) get

$$= K_C + C_V Q_S + S_C Q_S + H_C\left[\frac{Q_S}{\theta}(1 - e^{-\alpha t_1}) - \frac{a-bp}{\alpha^2}(e^{-\alpha t_1} + t_1\alpha - 1) + \frac{1}{\alpha}\left(1 - e^{\alpha(t_1-T)}\right)\left((1 - P_D)Q_S - (a - bS_P)t_1\right) - \frac{a-bp}{\alpha^2}\left(e^{\alpha(t_1-t)} + (T - t_1)\alpha - 1\right)\right] \tag{5.2.6}$$

When $T = \frac{(1-P_D)Q_S}{D_R}$, $t_1 = \frac{Q_S}{S_R}$ **and** $S_R = \frac{D_R}{1-P_D}$ and neglecting the higher degree term of α in expansion of $e^{-\alpha t}$, $0 < \alpha \ll 1$ from the following expression

$$\frac{Q_S}{\alpha}(1 - e^{-\alpha t_1}) - \frac{a - bS_P}{\alpha^2}(e^{-\alpha t_1} + t_1\alpha - 1) + \frac{1}{\alpha}\left(1 - e^{\alpha(t_1-T)}\right)\left((1 - P_D)Q_S - (a - bS_P)t_1\right) - \frac{a - bS_P}{\alpha^2}\left(e^{\alpha(t_1-t)} + (T - t_1)\alpha - 1\right)$$

reduce to $\frac{Q_S^2(1-P_D)}{D_R}$ then equation (5.2.6) becomes

$$TC(Q_S) = K_C + C_V Q_S + S_C Q_S + H_C\left[\frac{Q_S^2(1-P_D)}{D_R}\right] \tag{5.2.7}$$

Total Revenue during time period (0, T)

$$C_R(Q_S) = \frac{2S_P Q_S^2 + (Q_S P_D + 1)\left[K_C + C_V Q_S + S_C Q_S + H_C\left\{\frac{Q_S^2(1-P_D)}{D_R}\right\}\right]}{2Q_S + Q_S P_D + 1} \tag{5.2.8}$$

The cycle wise total profit $C_{TP}(Q_S) = C_R(Q_S) - TC(Q_S)$

$$= \frac{2S_P Q_S^2 - 2Q_S\left[K_C + C_V Q_S + S_C Q_S + H_C\left\{\frac{Q_S^2(1-P_D)}{D_R}\right\}\right]}{2Q_S + Q_S P_D + 1} \tag{5.2.9}$$

The Unit wise total profit is obtained as:

$$\boldsymbol{TP_U(Q_S)} = \frac{C_{TP}(Q_S)}{T} = \frac{2S_P Q_S^2 - 2Q_S\left[K_C + C_V Q_S + S_C Q_S + H_C\left\{\frac{Q_S^2(1-P_D)}{D_R}\right\}\right]}{T(2Q_S + Q_S P_D + 1)} \tag{5.2.10}$$

putting $T = \frac{(1-P_D)Q_S}{D_R}$, $t_1 = \frac{Q_S}{S_R}$ **and** $S_R = \frac{D_R}{1-P_D}$ and simplify equation (5.2.10) we get

$$\boldsymbol{TP_U(Q_S)} = \frac{2D_R(S_P Q_S - K_C - C_V Q_S - S_C Q_S)}{(1-P_D)(2Q_S + Q_S P_D + 1)} - \frac{2H_C Q_S^2}{(2Q_S + Q_S P_D + 1)} \tag{5.2.11}$$

The 1st and 2nd derivative of $\boldsymbol{TP_U(Q_S)}$ with respect to $\boldsymbol{Q_S}$ are as follows:

$$\frac{dTP_U(Q_S)}{dQ_S} = \frac{1}{(2Q_S + Q_S P_D + 1)^2}\left[\frac{2D_R(S_P - C_V - S_C + 2K_C + K_C P_D)}{(1-P_D)} - 2H_C\left(2Q_S + Q_S^2 P_D + 2Q_S\right)\right] \tag{5.2.12}$$

And $$\frac{d^2TP_U(Q_S)}{dQ_S^2} < 0 \tag{5 .2.13}$$

Again, as the 2nd order derivative of $TP_U(Q_S)$ is negative for all value of Q_S, it implies that $TP_U(Q_S)$ is concave function. So, the maximum annual profit is determined by setting the 1st order derivative equal to zero, which after some basic manipulation gives

$$(\boldsymbol{Q_S})_{max} = \sqrt{\frac{D_R(S_P - C_V - S_C + 2K_C + K_C P_D)}{H_C(2+P_D)(1-P_D)}} \tag{5.2.14}$$

When P_D=0 , $S_P - C_V - S_C = 2K_C$ then $(Q_S)_{max}$ reduce to the traditional EOQ formula.

$$(\boldsymbol{Q_S})_{max} = \sqrt{\frac{2K_C D_R}{H_C}} \tag{5.2.15}$$

6.1. Model with trapezoidal Fuzzy Price dependent demand Rate.

6.1.1 Case-1 (When t > 1, time proportional deterioration rate$\boldsymbol{\theta t}$)

It is not easy to define all the parameters preciously, due to an uncertain environment. Therefore, it might be assumed that some of the parameter changes with some limit. Here trapezoidal fuzzy number is being considered to fuzzify the price dependent demand and defuzzified by signed distance method. We consider trapezoidal fuzzy numbers $\widetilde{D}_R = (D_1, D_2, D_3, D_4)$.

Unit time wise total profit is given by

$$\widetilde{\boldsymbol{TP_U(Q_S)}} = \frac{2\widetilde{D}_R(S_P Q_S - K_C - C_V Q_S - S_C Q_S)}{(1-P_D)(2Q_S + Q_S P_D + 1)} - \frac{H_C Q_S^2(1+P_D)\left(6\widetilde{D}_R^2 - \alpha Q_S^2(1-P_D)^2\right)}{6\widetilde{D}_R^2(2Q_S + Q_S P_D + 1)} \tag{6.1.1}$$

We defuzzify the fuzzy total profit $\widetilde{\boldsymbol{TP_U(Q_S)}}$ by using signed distance method. The defuzzified value is

$$\boldsymbol{d_f}\left(\widetilde{\boldsymbol{TP_U(Q_S)}}\right) = \frac{1}{4}\left[\widetilde{TP}_{U1} + \widetilde{TP}_{U2} + \widetilde{TP}_{U3} + \widetilde{TP}_{U4}\right]$$

$$= \frac{1}{4}\left[\frac{2(D_1+D_2+D_3+D_4)(S_P Q_S-K_C-C_V Q_S-S_C Q_S)}{(1-P_D)(2Q_S+Q_S P_D+1)} - \frac{H_C Q_S^2(1+P_D)\left(24-\alpha Q_S^2(1-P_D)^2\left(\frac{1}{D_1^2}+\frac{1}{D_2^2}+\frac{1}{D_3^2}+\frac{1}{D_4^2}\right)\right)}{6(2Q_S+Q_S P_D+1)}\right]$$

The first and second derivative of $d_f\left(\widetilde{TP_U(Q_S)}\right)$ with respect to Q_S are obtained to find optimal value of Q_S and maximum profit by solving the

$$\frac{d\left(d_f\left(\widetilde{TP_U(Q_S)}\right)\right)}{dQ_S} = 0 \quad (6.1.2)$$

Provided

$$\frac{d^2\left(d_f\left(\widetilde{TP_U(Q_S)}\right)\right)}{dQ_S^2} < 0 \quad (6.1.3)$$

After simplification we get

$$(Q_S)_{max} = \sqrt{\frac{12(D_1+D_2+D_3+D_4)(S_P-C_V-S_C+2K_C+K_C P_D)}{(1-P_D^2)H_C(2+P_D)\left(24-3\alpha Q_S^2(1-P_D)^2\left(\frac{1}{D_1^2}+\frac{1}{D_2^2}+\frac{1}{D_3^2}+\frac{1}{D_4^2}\right)\right)}} \quad (6.1.4)$$

Now if we consider the special case of the model in which we neglect all the constraints to reach the traditional EOQ model i.e. b y considering P_D=0 , $C_V + S_C = S_P$, equation (6.1.4) reduces to the traditional EOQ formula.

$$(Q_S)_{max} = \sqrt{\frac{2K_C D_R}{H_C}} \quad (6.1.5)$$

6.1.2 Case-2 (When t = 1, deterioration rate reduces to constant deterioration.)

Here also we take trapezoidal fuzzy number to fuzzified the price dependent demand and de-fuzzified by signed distance method .We consider trapezoidal fuzzy number$\widetilde{D}_R = (D_1, D_2, D_3, D_4)$.

The unit time wise Total profit in fuzzy sense is given by

$$\widetilde{TP_U(Q_S)} = \frac{2\widetilde{D}_R(S_P Q_S-K_C-C_V Q_S-S_C Q_S)}{(1-P_D)(2Q_S+Q_S P_D+1)} - \frac{2H_C Q_S^2}{(2Q_S+Q_S P_D+1)} \quad (6.1.6)$$

We defuzzify the fuzzy Total profit $\widetilde{TP_U(Q_S)}$ by using signed distance method. The defuzzified value is

$$d_f\left(\widetilde{TP_U(Q_S)}\right) = \frac{1}{4}\left[\widetilde{TP}_{U1} + \widetilde{TP}_{U2} + \widetilde{TP}_{U3} + \widetilde{TP}_{U4}\right]$$

$$= \frac{1}{4}\left[\frac{2(D_1+D_2+D_3+D_4)(S_P Q_S-K_C-C_V Q_S-S_C Q_S)}{(1-P_D)(2Q_S+Q_S P_D+1)} - \frac{8H_C Q_S^2}{(2Q_S+Q_S P_D+1)}\right] \quad (6.1.7)$$

The first and second derivative of $\boldsymbol{d_f}\left(\boldsymbol{\widetilde{TP_U(Q_S)}}\right)$ w.r to $\boldsymbol{Q_S}$ are get optimal value of $\boldsymbol{Q_S}$ and total maximum profit .By solving the

$$\frac{\boldsymbol{d}\left(\boldsymbol{d_f}(\boldsymbol{\widetilde{TP_U(Q_S)}})\right)}{\boldsymbol{dQ_S}} = \mathbf{0} \tag{6.1.8}$$

Provided

$$\frac{\boldsymbol{d^2}\left(\boldsymbol{d_f}(\boldsymbol{\widetilde{TP_U(Q_S)}})\right)}{\boldsymbol{dQ_S^2}} < \mathbf{0} \tag{6.1.9}$$

After simplification we get

$$(\boldsymbol{Q_S})_{\boldsymbol{max}} = \sqrt{\frac{(\boldsymbol{D_1}+\boldsymbol{D_2}+\boldsymbol{D_3}+\boldsymbol{D_4})(\boldsymbol{S_P}-\boldsymbol{C_V}-\boldsymbol{S_C}+\mathbf{2}\boldsymbol{K_C}+\boldsymbol{K_C}\boldsymbol{P_D})}{\mathbf{4}\boldsymbol{H_C}(\mathbf{2}+\boldsymbol{P_D})(\mathbf{1}-\boldsymbol{P_D})}} \tag{6.1.10}$$

Again, to consider the special case of the model we neglect all the constraints to reach the traditional EOQ model i.e. by considering $\boldsymbol{P_D}$=0 , $\boldsymbol{S_P}-\boldsymbol{C_V}-\boldsymbol{S_C}=\mathbf{2}\boldsymbol{K_C}$ then $(\boldsymbol{Q_S})_{\boldsymbol{max}}$ in (6.1.10) reduces to the traditional EOQ formula given in (6.1.11)

$$(\boldsymbol{Q_S})_{\boldsymbol{max}} = \sqrt{\frac{\mathbf{2}\boldsymbol{K_C}\boldsymbol{D_R}}{\boldsymbol{H_C}}} \tag{6.1.11}$$

6.2. **Model with Cloudy Fuzzy Price dependent Demand Rate**

In this proposed model we assume rate of demand D_R as a cloudy type fuzzy number, where the amount of the items $\boldsymbol{Q_S}\left(=\frac{(\mathbf{1}-\boldsymbol{P_D})\boldsymbol{T}}{\boldsymbol{D_R}}\right)$ is related to the rate of demand.

Case-1 (When t > 1, deterioration rate is $\boldsymbol{\theta t}$)

So from equation (5.1.11) the fuzzy problem becomes

$$Max\ \tilde{z} = \frac{A\widetilde{D}_R\widetilde{Q}_S}{B\widetilde{Q}_S+1} - \frac{2\widetilde{D}_R K_C A'}{B\widetilde{Q}_S+1} - \frac{\left(A''\widetilde{Q}_S^2 6D_R^2 - \alpha\widetilde{Q}_S^2(1-P_D)^2\right)}{6\widetilde{D}_S^2(B\widetilde{Q}_S+1)} \tag{6.2.1}$$

where, $R = \frac{2S_P-2C_V-2S_C}{1-P_D}$, $S = 2+P_D$, $\mathrm{R}' = \frac{1}{1-P_D}$,$R'' = H_C(1+P_D)$

Subject to $\widetilde{Q}_S = \frac{\widetilde{D}_R T}{1-P_D}$ (6.2.2)

The rate of demand D_R have membership function as

$$\mu_{\tilde{A}}(\widetilde{D}_R, T) = \begin{cases} 0 & if \quad D_R < D_{R2}\left(1-\frac{\gamma}{1+T}\right) \; and \; D_R > D_{R2}\left(1+\frac{\delta}{1+T}\right) \\ \left\{\frac{D_R - D_{R2}\left(1-\frac{\gamma}{1+T}\right)}{\frac{D_{R2}\gamma}{1+T}}\right\} & if D_{R2}\left(1-\frac{\gamma}{1+T}\right) \le D_R \le D_{R2} \\ \left\{\frac{D_{R2}\left(1+\frac{\delta}{1+T}\right) - D_R}{\frac{D_{R2}\delta}{1+T}}\right\} & if \quad D_{R2} \le D_R \le D_{R2}\left(1+\frac{\delta}{1+T}\right) \end{cases} \quad (6.2.3)$$

By using the subject to constraint $\widetilde{Q}_S = \frac{\widetilde{D}_R T}{1-P_D}$ the fuzzy order quantity membership function $\widetilde{Q}_S$ is obtained as

$$\mu_{\tilde{A}}(\widetilde{Q_S}, T) = \begin{cases} 0 & if \quad \frac{(1-P_D)Q_S}{T} < D_{R2}\left(1-\frac{\gamma}{1+T}\right) \\ & and \; \frac{(1-P_D)Q_S}{T} > D_{R2}\left(1+\frac{\delta}{1+T}\right) \\ \left\{\frac{\frac{(1-P_D)Q_S}{T} - D_{R2}\left(1-\frac{\gamma}{1+T}\right)}{\frac{D_{R2}\gamma}{1+T}}\right\} & if D_{R2}\left(1-\frac{\gamma}{1+T}\right) \le \frac{(1-P_D)Q_S}{T} \le D_{R2} \\ \left\{\frac{D_{R2}\left(1+\frac{\delta}{1+T}\right) - \frac{(1-P_D)Q_S}{T}}{\frac{D_{R2}\delta}{1+T}}\right\} & if \quad D_{R2} \le \frac{(1-P_D)Q_S}{T} \le D_{R2}\left(1+\frac{\delta}{1+T}\right) \end{cases}$$

$$\mu_{\tilde{A}}(\widetilde{Q_S}, T) = \begin{cases} 0 & if \quad Q_S < \frac{TD_{R2}\left(1-\frac{\gamma}{1+T}\right)}{1-P_D} \\ & and \; Q_S > \frac{TD_{R2}\left(1+\frac{\delta}{1+T}\right)}{1-P_D} \\ \left\{\frac{Q_S - \frac{TD_{R2}\left(1-\frac{\gamma}{1+T}\right)}{1-P_D}}{\frac{D_{R2}T\gamma}{(1+T)(1-P_D)}}\right\} & if \frac{TD_{R2}\left(1-\frac{\gamma}{1+T}\right)}{1-P_D} \le Q_S \le \frac{TD_{R2}}{1-P_D} \\ \left\{\frac{\frac{TD_{R2}\left(1+\frac{\delta}{1+T}\right)}{1-P_D} - Q_S}{\frac{D_{R2}T\delta}{(1+T)(1-P_D)}}\right\} & if \quad \frac{TD_{R2}}{1-P_D} \le Q_S \le \frac{TD_{R2}\left(1+\frac{\delta}{1+T}\right)}{1-P_D} \end{cases} \quad (6.2.4)$$

More over alpha cut of $\mu_{\tilde{A}}(\widetilde{D}_R, T)$ and $\mu_{\tilde{A}}(\widetilde{Q_S}, T)$ are obtained by using above two equation

(6.2.3) and (6.2.4) ,we get as $\left[D_{R2}\left(1-\frac{\gamma}{1+T}\right) + \frac{\alpha D_{R2}\gamma}{(1+T)}, D_{R2}\left(1+\frac{\delta}{1+T}\right) - \frac{\alpha D_{R2}\delta}{1+T}\right]$ and

$\left[\frac{TD_{R2}\left(1-\frac{\gamma}{1+T}\right)}{1-P_D} + \frac{\alpha D_{R2}T\gamma}{(1+T)(1-P_D)}, \frac{TD_{R2}\left(1+\frac{\delta}{1+T}\right)}{1-P_D} - \frac{\alpha D_{R2}T\delta}{(1+T)(1-P_D)}\right]$.

Now the index value of $\widetilde{Q}_S$ and $\widetilde{D}_R$ are obtained as

$$I(\widetilde{Q}_S) = \frac{1}{2\tau}\int_0^\tau\int_0^1\left[\frac{TD_{R2}\left(1-\frac{\gamma}{1+T}\right)}{1-P_D} + \frac{\alpha D_{R2}T\gamma}{(1+T)(1-P_D)} + \frac{TD_{R2}\left(1+\frac{\delta}{1+T}\right)}{1-P_D} - \frac{\alpha D_{R2}T\delta}{(1+T)(1-P_D)}\right]d\alpha dT \tag{6.2.5}$$

After solving above equation (6.2.5) we get as:

$$I(\widetilde{Q}_S) = \frac{D_{R2}}{2(1-P_D)}\left[\tau - \frac{(\gamma-\delta)}{2}\left\{1-\frac{\log(1+\tau)}{\tau}\right\}\right] \tag{6.2.6}$$

$$I(\widetilde{D}_R) = \frac{1}{\tau}\int_0^\tau\int_0^1\left[D_{R2}\left(1-\frac{\gamma}{1+T}\right) + \frac{\alpha D_{R2}\gamma}{(1+T)} + \frac{TD_{R2}\left(1+\frac{\delta}{1+T}\right)}{1-P_D} - \frac{\alpha D_{R2}T\delta}{(1+T)(1-P_D)}\right]\mathrm{d}\alpha dT \tag{6.2.7}$$

After solving above equation (6.2.7) we get as:

$$I(\widetilde{D}_R) = D_{R2}\left[1+\frac{\gamma-\delta}{4}\left(\frac{\log(1+\tau)}{\tau}\right)\right] \tag{6.2.8}$$

Therefore, utilizing (6.2.6) and (6.2.8) the index value of the fuzzy objective function is given by

$$I(\tilde{Z}) = I\left[\frac{R\widetilde{D}_R\widetilde{Q}_S}{S\widetilde{Q}_S+1} - \frac{2\widetilde{D}_R K_C R'}{B\widetilde{Q}_S+1} - \frac{\left(R''\widetilde{Q}_S^2 6D_R^2 - \alpha \widetilde{Q}_S^2(1-P_D)^2\right)}{6\widetilde{D}_S^2(S\widetilde{Q}_S+1)}\right] \tag{6.2.9}$$

Solving equation/(6.2.9) get as :

$$I(\tilde{Z}) = \left[\frac{1}{\frac{SD_{R2}}{(1-P_D)}\left[\frac{\tau}{2}-\frac{(\gamma-\delta)}{4}\left\{1-\frac{\log(1+\tau)}{\tau}\right\}\right]+1}\right]\left[D_{R2}\left\{1+\frac{(\gamma-\delta)\log(1+\tau)}{4\tau}\right\}\left\{\frac{RD_{R2}}{(1-P_D)}\left(\frac{\tau}{2}-\frac{(\gamma-\delta)}{4}\left\{1-\frac{\log(1+\tau)}{\tau}\right\}\right) - 2K_C R'\right\} - \left\{R''\frac{D_{R2}^2}{(1-P_D)^2}\left(\frac{\tau}{2}-\frac{(\gamma-\delta)}{4}\left(1-\frac{\log(1+\tau)}{\tau}\right)\right)^2\right\}\left\{1-\frac{\alpha\frac{D_{R2}^2}{(1-P_D)^2}\left(\frac{\tau}{2}-\frac{(\gamma-\delta)}{4}\left(1-\frac{\log(1+\tau)}{\tau}\right)\right)^2(1-P_D)^2}{6D_R^2\left(1+\frac{\gamma-\delta}{4}\left(\frac{\log 1+\tau}{\tau}\right)\right)^2}\right\}\right] \tag{6.2.10}$$

A particular case arises if $(\gamma-\delta)\to 0$ then $Z = \frac{RD_RQ_S}{SQ_S+1} - \frac{2D_RK_CR'}{SQ_S+1} - \frac{R''Q_S^2\left(6D_R^2-\alpha Q_S^2(1-P_D)^2\right)}{6D_R^2(BQ_S+1)}$ which reduces to crisp objective function.

Case-2 (When t = 1, deterioration rate reduces to a constant deterioration.)

So from equation (5.2.11) the fuzzy problem becomes

$$Max\ \tilde{z} = \frac{R\widetilde{D}_R\widetilde{Q}_S}{S\widetilde{Q}_S+1} - \frac{2\widetilde{D}_RK_CR'}{S\widetilde{Q}_S+1} - \frac{R''\widetilde{Q}_S^2}{S\widetilde{Q}_S+1} \tag{6.2.11}$$

where, $R = \frac{2S_P-2C_V-2S_C}{1-P_D}$, $S = 2+P_D$, $R' = \frac{1}{1-P_D}$,$R'' = 2H_C$

subject to $\widetilde{Q}_S = \frac{\widetilde{D}_RT}{1-P_D}$

Therefore, utilizing (6.2.6) and (6.2.8) the index value of the fuzzy objective function is given by

$$I(\tilde{Z}) = I\left[\frac{R\widetilde{D}_R\widetilde{Q}_S}{S\widetilde{Q}_S+1} - \frac{2\widetilde{D}_R K_C R'}{S\widetilde{Q}_S+1} - \frac{R''\widetilde{Q}_S^2}{S\widetilde{Q}_S+1}\right] \quad (6.2.12)$$

Solving equation (6.2.12) get as:

$$I(\tilde{Z}) = \left[\frac{1}{\frac{CD_{R2}}{(1-P_D)}\left[\frac{\tau}{2}-\frac{(\gamma-\delta)}{4}\left\{1-\frac{\log(1+\tau)}{\tau}\right\}\right]+1}\right]\left[D_{R2}\left\{1+\frac{(\gamma-\delta)\log(1+\tau)}{4\tau}\right\}\left\{\frac{RD_{R2}}{(1-P_D)}\left(\frac{\tau}{2}-\frac{(\gamma-\delta)}{4}\left\{1-\frac{\log(1+\tau)}{\tau}\right\}\right)-2K_C R'\right\}-\left\{R''\frac{D_{R2}^2}{(1-P_D)^2}\left(\frac{\tau}{2}-\frac{(\gamma-\delta)}{4}\left(1-\frac{\log(1+\tau)}{\tau}\right)\right)^2\right\}\right] \quad (6.2.13)$$

A particular case which is similar to crisp objective function arises

If $(\gamma-\delta)\to 0$ then $Z = \frac{RD_RQ_S}{CQ_S+1} - \frac{2D_RK_CR'}{CQ_S+1} - \frac{R''Q_S^2}{CQ_S+1}$.

7. Numerical results

In order to illustrate the behavior of the optimal lot sizes of different models, let us consider the following parameters. variable cost $25/unit, fixed ordering cost $100/cycle, holding cost $5unit/ year, selling price of good quality items $50/unit, screening cost $0.5/unit , rate of deterioration 0.02, rate of defective 0.02 and scale parameters a=52000,b=65.

		$TP_U(Q_S)$	$(Q_S)_{max}$
Crisp Model	With time proportional deterioration	1,20,61,89.61/year	1499.03 units
	With constant deterioration	1,17,52,12.56/year	1070.19 units.
Trapezoidal Fuzzy Model	With time proportional deterioration	1,20,76,07.82/year	1498.66 units.
	With constant deterioration	1,17,53,73.39/year	1070.26 units
Cloudy Fuzzy Model	With time proportional deterioration	1,40,60,34.14/year	1500.06 units
	With constant deterioration	1,40,33,48.09/year	1107.80units

Table: 1 Total profit and lot size of different models

In the above Table- 1, it is clearly evident that the total profit per unit time in case of cloudy fuzzy model is higher than the crisp and trapezoidal fuzzy number in both the cases of time proportional as well as constant deterioration. In time proportional deterioration models, the lot size is less but profit is more in case of cloudy fuzzy model as compared to other models. It indicates the time proportional models have higher profits with lower lot size as compared to constant deterioration.

8. Sensitivity analysis for the crisp model

Sensitivity investigation is helpful for decision maker to deal with different situations. Taking all parameters in example-1, and varying one parameter at a time, maintaining the residual parameters at same value, an analysis is performed to check the sensitivity, by giving percentage change to the values of each of the parameters by 20%, 15%, 10%, 5%, -5%, -10%, -15%, and -20%.

Parameters	**% changes**	Q_S	TP_U	***% of Changes in*** TP_U
α	20%	1498.57878	1,23,12,96.856	*0.17*
	15%	1498.57845	1,23,12,96.855	*0.13*
	10%	1498.57813	1,23,12,96.855	*0.08*
	-10%	1498.57684	1,23,12,96.853	*-0.08*
	-15%	1498.57652	1,23,12,96.852	*-0.13*
	-20%	1498.57619	1,23,12,96.851	*-0.17*
K_C	20%	1626.74	1,23,06,49.67	*-0.053*
	15%	1595.66	1,23,08,06.50	*-0.04*
	10%	1563.97	1,23,09,66.63	*-0.028*
	-10%	1430.19	1,23,16,42.15	*0.028*
	-15%	1394.74	1,23,18,21.15	*0.04*
	-20%	1358.36	1,23,20,04.82	*0.053*
S_P	20%	1531.31	1,73,67,89.325	*41.05*
	15%	1523.19	1,61,04,15.874	*30.8*
	10%	1515.03	1,48,40,42.643	*20.53*
	-10%	1481.95	97,85,51.9869	*-20.53*
	-15%	1473.56	85,21,79.9089	*-30.8*

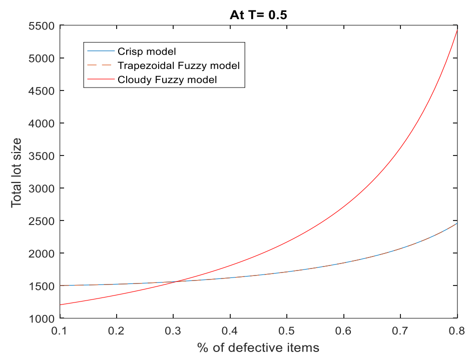
	-20%	1465.13	72,58,08.0732	*-41.05*
C_V	20%	1481.95	97,85,51.9869	*-20.53*
	15%	1486.13	1,04,17,38.116	*-15.4*
	10%	1490.29	1,10,49,24.303	*-10.26*
	-10%	1506.83	1,35,76,69.635	*10.26*
	-15%	1510.93	1,42,0,856.111	*15.4*
	-20%	1515.03	1,48,40,42.643	*20.53*
H_C	20%	1368.01	1,23,05,75.11	*-0.058*
	15%	1397.43	1,23,07,49.52	*-0.044*
	10%	1428.84	1,23,09,27.76	*-0.029*
	-10%	1579.64	1,23,16,84.92	*0.031*
	-15%	1625.44	1,23,18,87.05	*0.048*
	-20%	1675.47	1,23,20,95.23	*0.064*
S_C	20%	1498.25	1,22,62,41.948	*-0.41*
	15%	1498.33	1,22,75,05.674	*-0.307*
	10%	1498.41	1,22,87,69.401	*-0.205*
	-10%	1498.74	1,23,38,24.307	*0.205*
	-15%	1498.83	1,23,50,88.034	*0.307*
	-20%	1498.90	1,23,63,51.761	*0.41*

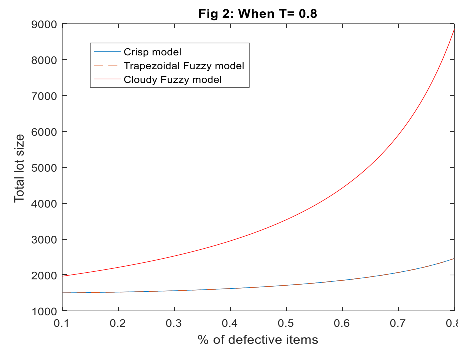
From the above Table -2, the sensitivity analysis of the crisp model, we have observed the following:

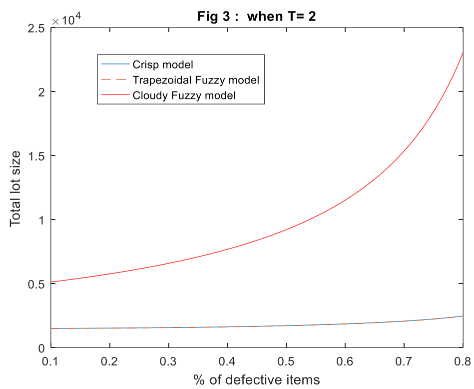
- When there is an increase in the value of α and selling price S_P from 5% to 20% there is an increase in the values of lot size Q_S as well as the total profit per unit is increased significantly. Similarly, when the parameters α and S_P values are reduced from 5% to 20%, both Q_S and TP_U values decrease.
- When the ordering cost value, K_C is increased by 5% to 20%, there is increase in the lot size Q_S but decrease in the total profit per unit TP_U. Similarly, when the parameter K_C is decreased by 5% to 20%, there is decrease in the value of Q_S and increase in the value of TP_U. It indicates increase in ordering cost, increases the lot size and total profit and vice versa.
- When the unit varying cost C_V, unit holding cost H_C and screening cost S_C values are increased from 5% to 20%, there is decrease in both Q_S and TP_U. But when the C_V, H_C and S_C values

are decreased by 5% to 20%, there is increase in both Q_S and TP_U. It indicates the total profit per unit and lot size increases if the holding cost, carrying cost and screening cost is low.

9. Graphical analysis of model.







These graphical illustrations in Fig. 1, 2 and 3 depict the % of defective items versus the total lot size at three different times, i.e. when T= 0.5, T=0.8 and T= 2 respectively. In these figs. we can notice the crisp and trapezoidal models gives almost same result. The lot size for cloudy fuzzy model increases with time whereas the lot size for crisp as well as trapezoidal fuzzy becomes constant.

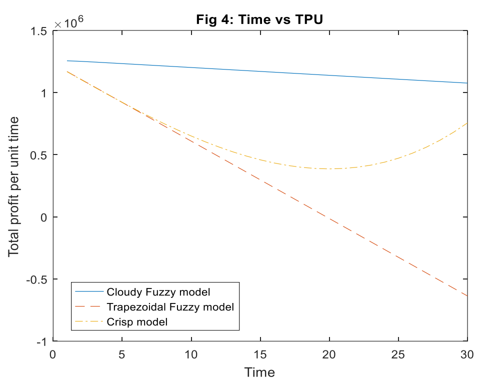
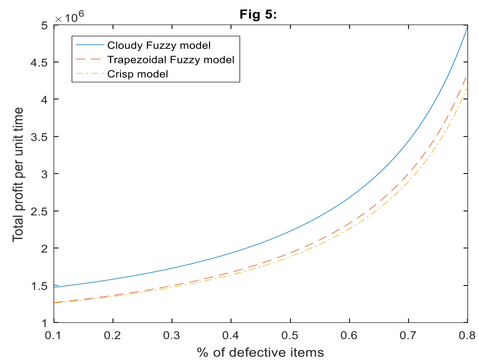
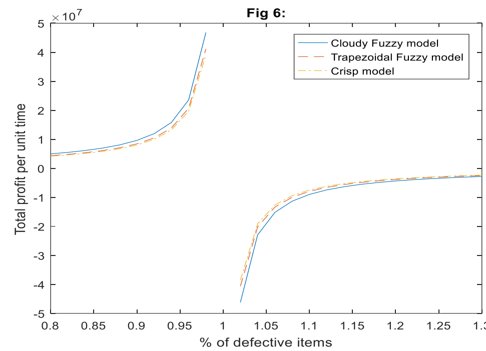


Fig. 4 is the graphical comparison of the three models, viz, crisp, trapezoidal and cloudy fuzzy for Time vs Total profit. The profit in case of cloudy fuzzy model is clearly higher than the other two models, whereas the crisp model also gives a higher profit than trapezoidal fuzzy model as time increases. Both crisp and trapezoidal model give same result at the beginning but as time increases crisp model gives better result, which even gets better with more passage of time.





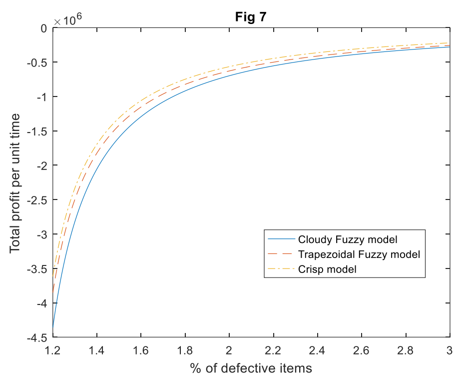


Fig. 5,6,7 all illustrates the % of defective versus the total profit per unit time in three varying ranges of defective item percent's. Fig.5 gives a clear idea of cloudy fuzzy model having better results than the other two models. Fig. 6 is in the range of 0.8 to 1.3 in which we can observe a gap when defective items percent is 1, after which there is a decrease in the profit of the models in just the opposite direction. In fig. 7 we can see that the profit again tends to rise and eventually all the three models almost give same result.

10. Conclusion

We have considered a deteriorating EOQ model with imperfect quality items with allowable proportionate discount where demand is considered to be a function of price in this paper. The decrease in price has an increase in demand. This model has been discussed over crisp, general fuzzy and cloudy fuzzy environments. The comparison in total profit and lot size has been depicted in the Table 1. It is clearly evident that the cloudy fuzzy model gives larger profit with smaller lot size compared to the other two models which is indicated in the numerical examples. The managerial insights of the paper can be summarized as follows:

1) The cloudy fuzzy model gives better profit as compared to the general fuzzy model i.e. trapezoidal fuzzy number.
2) All the cost parameters are not equally responsible for the profit of the model. Some cost parameters like ordering cost value, K_C, unit varying cost C_V, unit holding cost H_C and screening cost S_C when decreased the total profit increases, whereas for the value of α and selling price S_P if decreased the total profit decreases.
3) The choice of time in the models has a significant effect on the profit of the model.
4) The decrease in selling price of an item increases the demand of the item, resulting in higher lot sizes.

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