Research Article

At Most Twin Outer Perfect Domination Number Of A Graph

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ABSTRACT

A Set $S \subseteq V(G)$ In A Graph G Is Said To Be At Most Twin Outer Perfect Dominating Set If For Every Vertex $v \in V-S$, $1 \leq |N(v) \cap S| \leq 2$ And $< V-S >$ Has At Least One Perfect Matching. The Minimum Cardinality Of At Most Twin Outer Perfect Dominating Set Is Called At Most Twin Outer Perfect Domination Number And Is Denoted By $\gamma_{atop}(G)$. In This Paper, We Initiate A Study Of This Parameter.

Keywords: Domination ,Complementary Perfect Domination

1. Introduction

The Concept Of Complementary Perfect Domination Number Was Introduced By Paulraj Joseph Et.Al [8]. A Set $S \subseteq V$ Is Called A Complementary Perfect Dominating Set, If S Is A Dominating Set Of G And The Induced Sub Graph $< V-S >$ Has A Perfect Matching. The Minimum Cardinality Taken Over All Complementary Perfect Dominating Sets Is Called The Complementary Perfect Domination Number And Is Denoted By $\gamma_{cp}(G)$. In [6] Mustapha Chellali Et.Al., First Studied The Concept Of $[1,2]$- Sets. A Subset $S \subseteq V$ In A Graph G Is A $[j,k]$ If, $j \leq |N(V) \cap| \leq$ K For Every Vertex , For Any Non-Negative Integer J And K. In [7], Xiaojing Yang And Baoyindureng Wu, Extended The Study Of This Parameter. A Vertex Set S In Graph G Is $[1,2]$- Set If, $1 \leq |N(V) \cap| \leq$ 2 For Every Vertex $v \in V-S$, That Is, Every Vertex $v \in V-S$ Is Adjacent To Either One Or Two Vertices In S. The Minimum Cardinality Of A $[1,2]$-Set Of G Is Denoted By $\gamma_{[1,2]}(G)$ And Is Called $[1,2]$Domination Number Of G.

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$H_{p,p}$ Is The Graph With Vertex Set $V(H_{p,p}) = \{v_1, v_2, v_3, \ldots v_p, u_1, u_2, \ldots u_p\}$ And The Edge Set $E(H_{p,p})$ = $\{v_i u_j 1 \leq i \leq p, p-i+1 \leq j \leq p\}$. We Denote L_r , P_r, C_r As Ladder Graph, Path, And Cycle Respectively On p^{th}. For $p \geq 4$, Wheel W_p Is Defined To Be The Graph $K_1 + C_{p-1}$.

By Attaching A Pendent Edge At Each Vertex Of W_p Except The Central Vertex Forms A Helm Graph.

The Square Of A Graph G Is Doneted As G^2 In Which It Has The Same Vertices As In G And The Two Vertices U And V Are Adjacent In G^2 If And Only If They Are Joined In G By A Path Of Length One Or Two.

In This Paper, We Introduce The Concept Of At Most Twin Outer Perfect Domination Number Of A Graph And Investigate This Number For Some Standard Classes Of Graphs.

2. Previous Result

Theorem 2.1[7] For Any Connected Graph G, $\gamma_{cp}(P_p) = \begin{cases} r+2, & if\ \ p = 3r \\ r+1, & if\ \ p = 3r+1 \\ r+2, & if\ \ p = 3r+2 \end{cases}$

Theorem 2.2 [7] For Any Connected Graph G, $\gamma_{cp}(C_p) = \begin{cases} r, & if\ \ p = 3r \\ r+1, & if\ \ p = 3r+1 \\ r+2, & if\ \ p = 3r+2 \end{cases}$

3. Main Result

Definition 3.1 A Set $S \subseteq V$ Is Called A At Most Twin Outer Perfect Dominating Set (Atopd-Set) In G If For Every Vertex $v \in V - S$, $1 \leq |N(v) \cap S| \leq 2$ And $< V - S >$ Has At Least One Perfect Matching. The Minimum Cardinality Taken Over All The At Most Twin Outer Perfect Dominating Set In G Is Called The At Most Twin Outer Perfect Domination Number Of G And Is Denoted By $\gamma_{atop}(G)$ And The Atopd-Set With Minimum Cardinality Is Also Called $\gamma_{atop} - set$.

Example

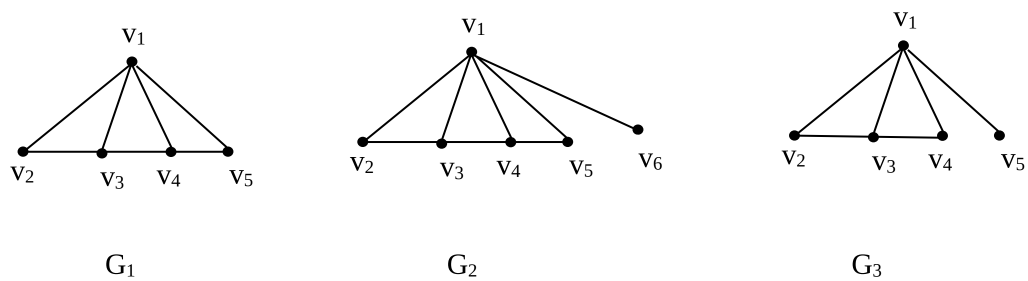

Figure 1:

In Figure 1, G_1, $\gamma_{atop}(G) = 1$. In G_2, $\gamma_{atop}(G) = 2$. In G_3, $\gamma_{atop}(G) = 3$.

Observation 3.2 For Any Connected Graph G, $\gamma(G) \leq \gamma_{cp}(G) \leq \gamma_{atop}(G)$ And The Bounds Are Sharp.

Observation 3.3 The Complement Of The Atopd - Set Need Not Be A Atopd- Set.

Observation 3.4 Every Atopd - Set Is A Dominating Set But Not The Converse.

Observation 3.5 Any At Most Twin Outer Perfect Dominating Set Must Contain All The Pendant Vertices Of G.

Theorem 3.6 For Any Connected Graph G With $p \geq 3$, We Have $1 \leq \gamma_{(atop)}(G) \leq p-2$ And The Bound Is Sharp.

Proof. Let G Be A Connected Graph With $p \geq 3$ Vertices. By Definition 3.1, $\gamma_{(atop)}(G) \neq p$. If $\gamma_{atop}(G) = p-1$, Then There Exists A Atopd- Set S With $p-1$ Vertices. In This Case $< V-S >$ Has Only One Vertex And Hence Perfect Matching Is Not Possible, Which Is A Contradiction. Hence $\gamma_{atop}(G) \neq p-1$. Thus $\gamma_{atop}(G) \leq p-2$. The Lower Bound Is Sharp For K_5 And The Upper Bound Is Sharp For C_5.

Theorem 3.7 For Any Connected Graph G With $p \geq 3$, $\lceil \frac{p}{\Delta+1} \rceil \leq \gamma_{atop}(G)$ The Bound Is Sharp.

Proof. For Any Connected Graph G, We Have $\lceil \frac{p}{\Delta+1} \rceil \leq \gamma(G)$ And Also $\gamma(G) \leq \gamma_{atop}(G)$ And The Result Follows. The Bound Is Sharp For C_4 And K_5.

Observation3.8 There Is No Graph, For Which $\gamma_{atop}(G) = p-i$ Where I Is An Odd Number.

Theorem 3.9 Let G Be A 2-Regular Graph. Then $\gamma_{atop}(G) = \chi(G)$ If And Only If $G \cong C_4 \, or \, C_5 \, or \, C_6 \, or \, C_7 \, or \, C_9$.

Proof. Let G Be A 2-Regular Graph. Assume $\gamma_{atop}(G) = \chi(G)$. Since G Is 2-Regular, $\chi = 2$ Or 3 Which Implies P = 4 Or 5 Or 6 Or 7 Or 9. If P=4, $G \cong C_4$. If P=5, $\gamma_{atop}(G) = \chi(G) = 3$ Then $G \cong C_5$. If P = 6, $\gamma_{atop}(G) = \chi(G) = 2$,Then $G \cong C_6$. If P = 7, $\gamma_{atop}(G) = \chi(G) = 3$ Then $G \cong C_7$. If P = 9, $\gamma_{atop}(G) = \chi(G) = 3$ Then $G \cong C_9$. The Converse Is Obvious.

4. Exact Values Of Atopd- Number For Some Standard Graphs

The Atopd- Number For Some Standard Graphs Are Given As Follows:

1. $\gamma_{atop}(W_p) = \begin{cases} 2 & if \ p \ is \ even \\ 1 & if \ p \ is \ odd. \end{cases}$
2. $\gamma_{atop}(K_p) = \begin{cases} 2 & if \ p \ is \ even \\ 1 & if \ p \ is \ odd. \end{cases}$
3. $\gamma_{atop}(H_{p,p}) = \begin{cases} 4 & if \ p > 2 \\ 2 & if \ p = 2. \end{cases}$
4. $\gamma_{atop}(H_p) = \begin{cases} p+1 & if \ p \ is \ even \\ p+2 & if \ p \ is \ odd. \end{cases}$
5. If G Is A Peterson Graph $\gamma_{atop}(G) = 4$.

Theorem 4.1 For Any Connected Graph G, $\gamma_{atop}(P_p) = \begin{cases} r+2 & \text{if } p = 3r \\ r+1 & \text{if } p = 3r+1 \\ r+2 & \text{if } p = 3r+2. \end{cases}$

Proof Let $P_p = (V_1, V_2, \ldots V_p)$ And Let $S = \{V_i : i \equiv 1 (Mod 3)\}$,
$S_1 = S \cup \{V_{p-1}, V_p\}$, $S_2 = S \cup \{V_p\}$. If P = 3r For Some R, Then S_1 Is A Atopd-Set. If P = 3r+1 For Some R, Then S Is A Atopd-Set. If P = 3r+2 For Some R, Then S_2 Is A Atopd-Set.

Hence $\gamma_{atop}(P_p) \leq \begin{cases} r+2 & \text{if } p = 3r \\ r+1 & \text{if } p = 3r+1 \\ r+2 & \text{if } p = 3r+2 \end{cases}$

But $\gamma_{cp}(P_p) = \begin{cases} r+2 & \text{if } p = 3r \\ r+1 & \text{if } p = 3r+1 \\ r+2 & \text{if } p = 3r+2 \end{cases}$

And $\gamma_{cp} \leq \gamma_{atop}$. Hence The Result Follows.

Theorem 4.2 For Any Connected Graph G, $\gamma_{atop}(C_p) = \begin{cases} r & \text{if } p = 3r \\ r+1 & \text{if } p = 3r+1 \\ r+2 & \text{if } p = 3r+2. \end{cases}$

Proof Let $C_p = (V_1,V_2,\ldots V_p,V_1)$ Be A Cycle And Let $S = \{V_i : I \equiv 1(Mod3)\}$, $S_1 = S \cup \{V_p\}$. If $P = 3r$ Or $P = 3r+1$ For Some R, Then S Is A Atopd-Set. If P = 3r+2 For Some R, Then S_1 Is A Atopd-Set.

Hence $\gamma_{atop}(C_p) \leq \begin{cases} r & \text{if } p = 3r \\ r+1 & \text{if } p = 3r+1 \\ r+2 & \text{if } p = 3r+2. \end{cases}$

But $\gamma_{cp}(C_p) = \begin{cases} r & \text{if } p = 3r \\ r+1 & \text{if } p = 3r+1 \\ r+2 & \text{if } p = 3r+2. \end{cases}$

And $\gamma_{cp} \leq \gamma_{atop}$. Hence The Result Follows.

5. Atopd-Number For Some Standard Square Graphs

Theorem 5.1 For A Path P_r $\gamma_{atop}(P_r^2) = \begin{cases} \left\lceil \frac{r}{5} \right\rceil + 1 & if\ r \equiv 2,4\ (mod\ 5) \\ \left\lceil \frac{r}{5} \right\rceil & otherwise. \end{cases}$

Proof Let $P_r = (V_1,V_2,\ldots V_r)$, If $R \geq 3$. Let $S = \{V_i : I \equiv 3(Mod5)\}$.

Then $S_1 = \begin{cases} S & if\ r \equiv 0,3\ (mod\ 5) \\ S \cup \{v_r\} & if\ r \equiv 1,4\ (mod\ 5) \\ S \cup \{v_1, v_r\} & if\ r \equiv 2\ (mod\ 5). \end{cases}$

Is An Atopd-Set Of P_r^2.

Hence $\gamma_{atop}(P_r^2) \leq \begin{cases} \left\lceil \frac{r}{5} \right\rceil + 1 & if\ r \equiv 2,4\ (mod\ 5) \\ \left\lceil \frac{r}{5} \right\rceil & otherwise. \end{cases}$

Let S Be Any γ_{atop}-Set Of P_r^2. Since $\gamma(P_r^2) = \left\lceil \frac{r}{5} \right\rceil$, We Have $\gamma_{atop}(P_r^2) = \left\lceil \frac{r}{5} \right\rceil$, If $r \not\equiv 2\ or\ 4\ (mod\ 5)$. Let $R \equiv 2$ Or 4 (Mod 5). Then Every γ -Set D Of P_r^2 Such That $V-D$ Contain Odd Number Of Vertices And Hence $< V-D >$ Has No Perfect Matching. Thus $\gamma_{atop}(P_r^2) \geq \left\lceil \frac{r}{5} \right\rceil + 1$. Hence The Result Follows.

Theorem 5.2 For A Cycle C_r, $r \geq 3$ $\gamma_{atop}(C_r^2) = \begin{cases} \left\lceil \frac{r}{5} \right\rceil + 1 & if\ r \equiv 2\ or\ 4\ (mod\ 5) \\ \left\lceil \frac{r}{5} \right\rceil & otherwise. \end{cases}$

Proof Let $C_r = (V_1, V_2, \ldots V_r, V_1)$, $R \geq 3$. Let $S = \{V_i : I \equiv 3(\text{Mod } 5)\}$.

Then $S_1 = \begin{cases} S & if\ r \equiv 0{,}3\ (mod\ 5) \\ S \cup \{v_r\} & if\ r \equiv 1\ (mod\ 5) \\ S \cup \{v_1, v_{r-1}\} & if\ r \equiv 2\ (mod\ 5) \\ S \cup \{v_1\} & if\ r \equiv 4\ (mod\ 5) \end{cases}$

Is An Atopd-Set Of C_r^2.

Hence $\gamma_{atop}(C_r^2) \leq \begin{cases} \left\lceil \frac{r}{5} \right\rceil + 1 & if\ r \equiv 2\ or\ 4\ (mod\ 5) \\ \left\lceil \frac{r}{5} \right\rceil & otherwise. \end{cases}$

Let S Be Any γ_{atop}-Set Of C_r^2. Since $\gamma(C_r^2) = \left\lceil \frac{r}{5} \right\rceil$, We Have $\gamma_{atop}(P_r^2) = \left\lceil \frac{r}{5} \right\rceil$ If $r \not\equiv 2\ or\ 4\ (mod\ 5)$. Let $r \equiv 2\ or\ 4\ (mod\ 5)$. Then Every γ-Set D Of C_r^2 Such That V−D Contain Odd Number Of Vertices And Hence < V−D > Has No Perfect Matching.

Thus $\gamma_{atop}(C_r^2) \geq \left\lceil \frac{r}{5} \right\rceil + 1$. Hence The Result Follows.

Theorem 5.3 Let $L_r = P_2 \times P_r$ Be A Ladder Graph

$\gamma(L_r^2) = \begin{cases} \frac{r}{4} + 1 & if\ r\ \equiv 0\ (mod\ 4) \\ \left\lceil \frac{r}{4} \right\rceil & otherwise. \end{cases}$

Proof It Can Be Verified That The Result Is True For $r \leq 4$. Now We Assume $r \geq 5$. Hence $V(L_r) = \{U_i, V_i : 1 \leq I \leq r\}$ And $E(L_r) = \{(U_i, V_i), (U_i, U_{i+1}), (V_i, V_{i+1}) (U_r, V_r) : 1 \leq I \leq r - 1\}$. Now Let

$S = \{u_i v_j : i \equiv 2\ (mod\ 8), j \equiv 6\ (mod\ 8)\}$.

Then $S_1 = \begin{cases} S & if\ r \equiv 2\ or\ 3\ (mod\ 4) \\ S \cup \{u_r\} & otherwise. \end{cases}$

Is A Dominating Set Of L_r^2 And Hence $\gamma(L_r^2) \leq$

$\begin{cases} \frac{r}{4} + 1 & if\ r\ \equiv 0\ (mod\ 4) \\ \left\lceil \frac{r}{4} \right\rceil & otherwise. \end{cases}$

Since $\Delta = 7$, We Have $\gamma \geq \left\lceil \frac{p}{\Delta + 1} \right\rceil = \left\lceil \frac{2r}{8} \right\rceil = \left\lceil \frac{r}{4} \right\rceil$. Hence If $r \not\equiv 0\ (mod\ 4)$, Then $\gamma = \left\lceil \frac{r}{4} \right\rceil$. Let $r\ \equiv 0\ (mod\ 4)$. Then Any Set $S \subseteq V$ Of Cardinality $\left\lceil \frac{r}{4} \right\rceil$ Is Not A Dominating Set And Hence $\gamma \geq \left\lceil \frac{r}{4} \right\rceil + 1 = \frac{r}{4} + 1$ And Hence $\gamma = \frac{r}{4} + 1$. Thus The Result Follows.

Theorem 5.4 Let $L_r = P_2 \times P_r$ Be A Ladder Graph

$\gamma_{atop}(L_r^2) = \begin{cases} \frac{r}{4} + 2 & if\ r\ \equiv 0\ (mod\ 8) \\ 2\left\lceil \frac{r}{8} \right\rceil & otherwise. \end{cases}$ ■

Proof Let $V(L_r) = \{U_i, V_i : 1 \leq I \leq R\}$ And $E(L_r) = \{(U_i, V_i), (U_i, U_{i+1}), (V_i, V_{i+1}), (U_r, V_r) : 1 \leq I \leq r - 1\}$.

Now Let, $S = \{u_i v_j : i \equiv 2\ (mod\ 8), j \equiv 6\ (mod\ 8)\}$.

Then $S_1 = \begin{cases} S & \text{if } r \equiv 6 \text{ or } 7 \ (mod\ 8) \\ S \cup \{u_r, v_2\} & \text{if } r \equiv 0 \text{ or } 1 \ (mod\ 8) \\ S \cup \{v_r\} & \text{otherwise.} \end{cases}$

Is An Atopd-Set Of L_r^2 And Hence

$$\gamma_{atop}(L_r^2) \leq \begin{cases} \frac{r}{4} + 2 & \text{if } r \equiv 0 \ (mod\ 8) \\ 2\left\lceil \frac{r}{8} \right\rceil & \text{otherwise.} \end{cases}$$

Let S Be Any γ_{atop}-Set Of L_r^2.

But $\gamma(L_r^2) = \begin{cases} \frac{r}{4} + 1 & \text{if } r \equiv 0 \ (mod\ 4) \\ \left\lceil \frac{r}{4} \right\rceil & \text{otherwise.} \end{cases}$

$$= \begin{cases} \frac{r}{4} + 1 & \text{if } r \equiv 0 \ (mod\ 8) \\ 2\left\lceil \frac{r}{8} \right\rceil - 1 & \text{if } r \equiv 1 \text{ or } 2 \text{ or } 3 \ (mod\ 8) \\ 2\left\lceil \frac{r}{8} \right\rceil & \text{otherwise.} \end{cases}$$

Hence If $r \equiv k \ (mod\ 8)$, $4 \leq k \leq 7$, Then $\gamma_{atop}(L_r^2) \geq 2\left\lceil \frac{r}{8} \right\rceil$. Also If $0 \leq k \leq 3$, Then Every γ- Set D Of L_r^2 Such That $V - D$ Contains An Odd Number Of Vertices And Hence $< V - D >$ Has No Perfect Matching.

Thus $\gamma_{atop}(L_r^2) \geq \begin{cases} \frac{r}{4} + 2 & \text{if } r \equiv 0 \ (mod\ 8) \\ 2\left\lceil \frac{r}{8} \right\rceil & \text{if } r \equiv 1 \text{ or } 2 \text{ or } 3 \ (mod\ 8). \end{cases}$

Hence The Result Follows.

Observation 5.5 For A Star Graph $K_{1,P-1}$, $\gamma_{atop}(K_{1,P-1}^2) = \begin{cases} 1 & \text{if } p \text{ is odd} \\ 2 & \text{if } p \text{ is even} \end{cases}$

Observation 5.6 For A Wheel Graph,

$\gamma_{atop}(W_p^2) = \begin{cases} 1 & \text{if } p \text{ is odd} \\ 2 & \text{if } p \text{ is even} \end{cases}$

Conclusion

In This Paper, We Introduced The Concept Of At Most Twin Outer Perfect Domination Number Of A Graph. We Obtain This Number For Some Standard Classes Of Graphs And Square Graphs.

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