

A Performance Analysis of Measurement Matrices used in Compressed Sensing Techniques

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Abstract

Compressed sensing technique requires three major stages: sparse representation, measurement, and sparse recovery. It performs with sparse representation of real world signals. In compressed sensing based signal acquisition, the input signal, measurement matrix and a measurement vector are required. The compressive measurements are discovered via the multiplication of random measurement matrix and input signal. The count of measurements taken here is not exactly the signal length. Hence, it utilizes a measurement matrix to test just the parts that best addresses the sparse signal. The decision of the measurement matrix influences the accomplishment of the process of sparse recovery. Consequently, the structure of a suitable measurement matrix is a significant interaction in compressive sensing. Absurd years, a few measurement matrices have been determined. Thusly, a brief survey of these measurement matrices and a correlation of their exhibitions is emphatically required. This paper gives an outline on compressed sensing featuring the measurement process. Then, it classifies the measurement matrices and compares the performances of such matrices. The performance comparison of measurement matrices is carried out using few evaluation metrics such as sparse reconstruction error, processing time and covariance.

Keywords— Compressive sensing, sparse representation, measurement matrix, restricted isometry property, sparse recovery algorithms, processing time, reconstruction error.

I. Introduction

The conventional data acquisition schemes need N numbers of a input signal x sampled at a rate at least twice the Nyquist frequency to achieve actual signal recovery. For sparse signals, which have a few nonzero elements, only a few samples are sufficient to represent these signals. Here compression takes place after acquiring the data to minimize the high number of samples. Compressed sensing is presented to reduce the processing time and the number of samples that represent a signal. This approach includes simultaneous acquisition and compression of data. Compressive sensing has shown to be a promising solution to reduce the sampling rate with effectively high efficiency and has main applications in image processing, communication, biomedical signal processing and so on. [1],[2]

Not many random measurements are used in compressed sensing techniques. compressed sensing's Acquisition model comprises of the input signal $x \in \mathbb{R}^n$ of length n , $\varphi \in \mathbb{R}^{m \times n}$ denotes a $m \times n$ measurement matrix and $y \in \mathbb{R}^m$ denotes a measurement vector having length m . The compressive measurements are gotten via the multiplication of random measurement matrix and input signal. The number of measurements considered here is not exactly the signal length .i.e $m < n$. [3]

$$y = \varphi x \quad (1)$$

The reconstruction matrix i.e $A = \varphi \psi \in \mathbb{R}^{m \times n}$ where ψ denotes the sparse basis function of the signal x and measurement vector y are considered as inputs to the model of reconstruction. The signal x is denoted as

$$x = \psi s \quad (2)$$

Here, $s \in \mathbb{R}^n$ denotes a sparse vector of length n .

The sign of interest can be remade by addressing condition (1) which is an unsure arrangement of straight conditions that prompts endless no. of potential arrangements. A selective arrangement can be acquired by taking ℓ_0 improvement issue wherein all potential mixes can be gone after for getting arrangement which is extremely dreary. Different kinds of sign recuperation calculations carrying out ℓ_1 standards and other applicable standards can be executed in order to get a gauge of scanty portrayal of x . For ideal recuperation of sign of interest, limited isometric property(RIP) and ambiguity property ought to be fulfilled.

II. MEASUREMENT MATRICES IN COMPRESSED SENSING

A fitting estimation grid ϕ ought to be picked for fruitful execution of CS. The most normally utilized irregular grids utilized in CS are Gaussian or Bernoulli, incomplete Fourier matrices, etc. Despite the fact that the likelihood of reproduction is high undoubtedly, they also have bad marks. Part of capacity will be required in the event that such networks. There is no such compelling calculation where RIP condition can be confirmed for these networks. Deterministic networks fulfill RIP just as soundness properties. The upsides of deterministic lattices incorporate less stockpiling necessity, straightforward inspecting and recuperation measures. For an exact and proficient sign recovery, deterministic networks can be utilized with the information on around deduced data about area of non-zero components. The no. of estimations needed for a few estimation lattices for amazing recuperation is given in the table1 where k is the sparsity of vector s , μ is the connection of soundness between any two components in a given pair of frameworks ϕ and ψ , m is the no. of estimations, n is the length of the info sign and c is a positive constant.[4],[5].

TABLE 1 Number of Measurements needed for various kinds of Matrices

Type of Matrix	Number of measurements needed
Deterministic	$m = O(k^2 \log n)$
Partial Fourier Matrix	$m \geq c \mu k (\log n)^4$
Gaussian and Bernouli	$m \geq ck \log n/k$
Any other matrix	$m = O(k \log n)$

A measurement matrix satisfies the RIP if there exists a constant such as:

$$(1 - \delta k) \|x\|_2 \leq \|\Phi x\|_2 \leq (1 + \delta k) \|x\|_2 \text{-----(3)}$$

Here, $\|\cdot\|_2$ denotes the ℓ_2 - norm and $\delta k \in [0,1]$ denotes the Restricted Isometry Constant (RIC) of ϕ which should be much smaller than 1.[6],[7]

The coherence estimates the greatest relationship between's any two segments of the measurement matrix Φ . In the event that ϕ is a $M \times N$ matrix with standardized section vector $\phi_1, \phi_2, \phi_3 \dots \phi_N$, each $\phi_i, (i = 1, \dots N)$ is of unit length. At that point the mutual Coherence Constant (MCC) is characterized as:

$$\mu = (|\langle \phi_i \phi_j \rangle|) / (\|\phi_i\|_2 \|\phi_j\|_2) \text{-----(4)}$$

Compressive sensing essentially manages matrices that have low rationality, which implies that a couple of tests are needed for an ideal recuperation of the sparse signal.

III. Classification of Measurement Matrices :

Measurement matrices can be arranged into two primary classes: arbitrary and deterministic. Matrices of the primary kind are produced aimlessly, simple to shape, and fulfill the RIP. Arbitrary matrices are of two kinds: unstructured and organized. Matrices of the unstructured arbitrary sort are created arbitrarily following a particular dissemination. For instance, Gaussian,

Bernoulli, and Uniform are unstructured arbitrary sort matrices that are produced following Gaussian, Bernoulli, and Uniform appropriation, individually. In organized arbitrary matrices, the passages are produced following a given capacity or explicit example. At that point the arbitrariness is made by choosing irregular lines from the created matrix. Instances of organized arbitrary matrices are - Random Partial Fourier and the Random Partial Hadamard matrices. Deterministic matrices are exceptionally attractive in light of the fact that they are developed deterministically to fulfill the RIP or to have least common cognizance. Deterministic matrices are likewise of two kinds: semi-deterministic and full deterministic. The age of semi-deterministic sort matrices are done in two stages: the initial step contains the age of the sections of the principal segment arbitrarily and the subsequent advance includes the age of passages of the remainder of the segments of this matrix dependent on the primary segment by applying straightforward change techniques on it like moving the component of the main segments. Instances of these matrices incorporate Circulant and Toeplitz matrices . Full-deterministic matrices have an unadulterated deterministic development. Double BCH, second-request Reed-Solomon, Chirp detecting, and semi cyclic low-thickness equality check code (QC-LDPC) matrices are instances of full-deterministic sort matrices.[8],[9],[10]

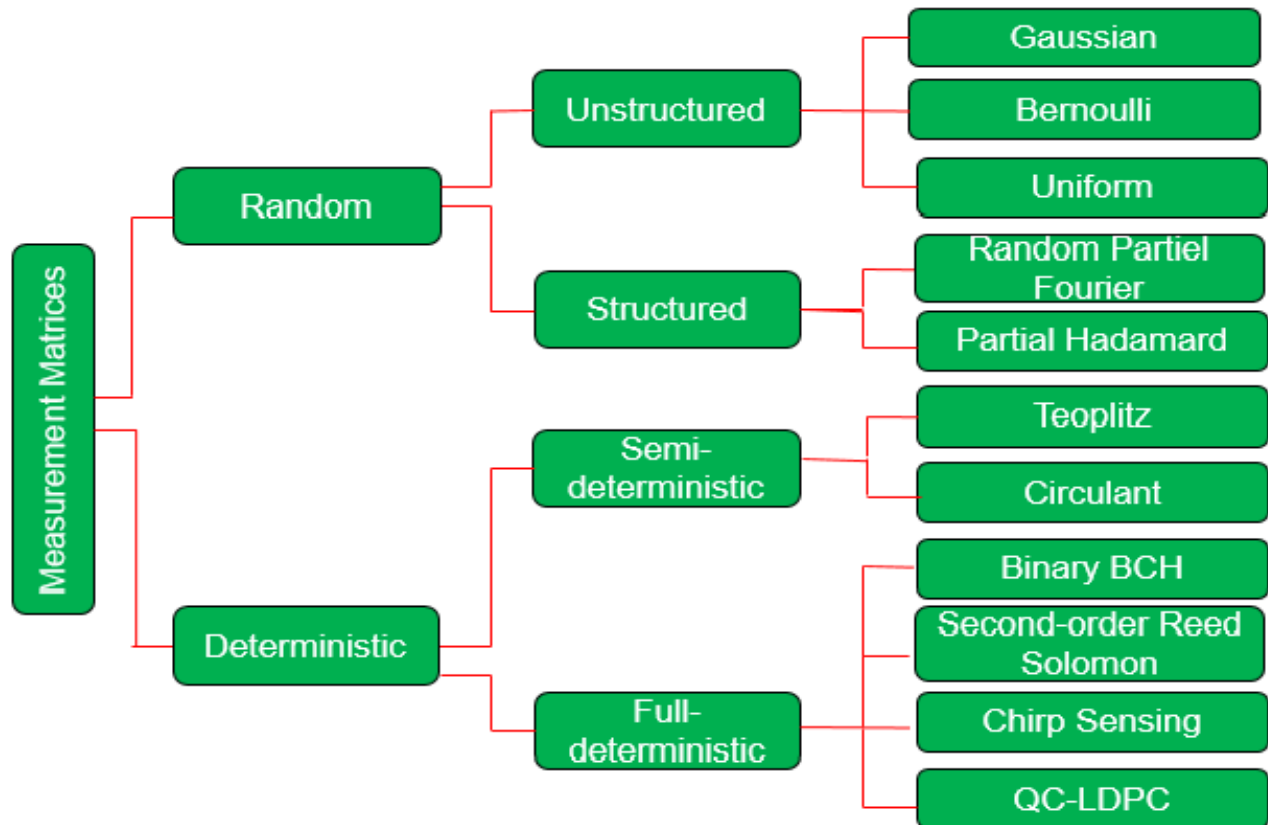


Fig 1:- Classification of Measurement Matrices

1. Random Measurement Matrices

Random matrices are produced by indistinguishable or autonomous conveyances like ordinary, Bernoulli, and arbitrary Fourier gatherings. These random matrices are of two kinds: unstructured and organized measurement irregular matrices.

1.1. Unstructured random type matrices:

Unstructured random sort measurement matrices are created randomly following a given dispersion. The created matrix is of size $M \times N$. At that point M columns is randomly chosen from N . Instances of this kind of matrices incorporate Gaussian, Bernoulli, and Uniform.

1.1.1. Random Gaussian matrix: The sections of a Gaussian matrix are autonomous and follow a normal distribution with zero mean and variance σ^2 . The likelihood thickness capacity of a normal distribution is

$$f(x) = \frac{1}{\sqrt{2\sigma^2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \text{-----(5)}$$

Here μ denotes the mean, σ denotes the standard deviation, and σ^2 denotes the variance. This random Gaussian matrix solves the restricted isometric property.[11],[12],[13]

1.1.2. Random Bernoulli matrix: A random Bernoulli matrix $B \in \mathbb{R}^{M \times N}$ is a matrix whose passages take the qualities $+1/\sqrt{M}$ or $-1/\sqrt{M}$ with equivalent probabilities. It, consequently, follows a Bernoulli conveyance which has two potential results named by $n=0$ and $n=1$. Thus, the likelihood density function is:

$$f(n) = \begin{cases} 1/2; & \text{for } n = 0 \\ 1/2; & \text{for } n = 1 \end{cases} \text{-----(6)}$$

1.2. Structured Random Type matrices:

The Gaussian or other unstructured matrices have the impediment of being moderate. Their execution as far as equipment needs critical memory space; the expense of putting away a $M \times N$ Gaussian or Bernoulli matrix is (MN) . Then again, random organized matrices are created following a given construction, which decreases the randomness, memory stockpiling, and preparing time. Two organized matrices are chosen to be implemented in this work: Random Partial Fourier and Partial Hadamard matrix. [14],[15],[16]

1.2.1. Random Partial Fourier matrix:

The Discrete Fourier matrix $F^{N \times N}$ is a matrix whose (k,j) - th entry is given by the equation:

$$(F)_{kj} = \exp(2\pi i k j / N) \text{-----(7)}$$

Where $k, = 1, 2, \dots, N$. Random Partial Fourier matrix which comprises selecting random M rows of the Discrete Fourier matrix solves the RIP with a probability of at least $1 - \epsilon$, if:

$$M \geq C.K. \log N / \epsilon \text{-----(8)}$$

Here M denotes the count of measurements, K denotes the sparsity, and N denotes the sparse signal length.

1.2.2. Random Partial Hadamard matrix

The Hadamard measurement matrix is a matrix whose sections are 1 and - 1. The segments of this matrix are symmetrical. Given a matrix H of request N , H is supposed to be a Hadamard matrix if the render of the matrix H is firmly identified with its converse.

This can be expressed by: $HH^T = NI_N$ where I_N is the $N \times N$ identity matrix, H^T is the transpose of the matrix H . [17],[18],[19]

2. Deterministic measurement matrices:

Deterministic measurement matrices will be matrices that are planned after a deterministic way to deal with fulfill the RIP or to have a low shared intelligibility. A few deterministic measurement matrices have been built to defeat the difficulties related with random matrices. These matrices are of two sorts as referenced in the past area: semi-deterministic and full-deterministic. In the accompanying, matrices from the two sorts are examined as far as cognizance and RIP. [20],[21],[22]

2.1. Semi-deterministic type matrices:

To produce a semi-deterministic sort measurement matrix, two stages are required. The initial step is random age of the primary segments and the subsequent advance is age of the full matrix by use of a straightforward change on the main segment like a revolution to produce each line of the matrix. Instances of such matrices are the Circulant and Toeplitz matrices. Albeit these two matrices are very comparative, we executed both and analyzed their exhibitions. In the accompanying, the numerical models of these two measurement matrices are depicted.

2.1.1.Circulant matrix:

For a given vector $c = (c_1, c_2, \dots, c_n) \in \mathbb{R}^N$, its associated circulant matrix $C \in \mathbb{R}^{N \times N}$ whose (i,j) entry is given by: $C_{ij} = c_{j-i}$ Where $i, j = 1, \dots, N, \dots \dots \dots (9)$

Thus, Circulantmatrix has the following form:

$$C = \begin{bmatrix} c_n & c_{n-1} & \dots & c_1 \\ c_1 & c_n & \dots & c_2 \\ c_2 & c_1 & \dots & c_3 \\ c_3 & c_2 & \dots & c_4 \\ \vdots & \vdots & \ddots & \vdots \\ c_{n-1} & c_{n-2} & \dots & c_n \end{bmatrix}$$

2.1.2.Toeplitz matrix:

The Toeplitz matrix $T \in \mathbb{R}^{N \times N}$, which is associated to a vector $t = (t_1, t_2, \dots, t_n) \in \mathbb{R}^N$

whose $(i,j) - th$ entry is given by:

$$T_{ij} = t_{j-i} \text{ where } i, j = 1, 2, \dots, N \dots \dots \dots (10)$$

The Toeplitz matrix is a matrix with a constant diagonal i.e. $T_{ij} = T_{i+1 j+1}$. Thus, the Toeplitz matrix has the following form:

$$T = \begin{bmatrix} t_n & t_{n-1} & \dots & t_1 \\ t_{n+1} & t_n & \dots & t_2 \\ t_{n+2} & t_{n+1} & \dots & t_3 \\ t_{n+3} & t_{n+2} & \dots & t_4 \\ \vdots & \vdots & \ddots & \vdots \\ t_{2n-1} & t_{2n-2} & \dots & t_n \end{bmatrix}$$

2.2. Full-deterministic Type Matrices:

Full-deterministic sort matrices will be matrices that have unadulterated deterministic developments dependent on the shared intelligence or on the RIP property. In the accompanying, two instances of deterministic development of measurement matrices are given which are the Chirp and Binary Bose-Chaudhuri-Hocquenghem (BCH) codes matrices. These two matrices are full-deterministic sort matrices. Parallel BCH is an illustration of full-deterministic dependent on cyclic codes and trill detecting matrix is as illustration of full deterministic matrix dependent on non-cyclic codes.[23],[24],[25],[26],[27]

2.2.1. Chirp Sensing Matrices:

The Chirp Sensing matrices will be matrices whose segments are loaded up with the twitter signal. A discrete tweet sign of length M has the structure:

$$A_r = 1 / \sqrt{M} \exp(2\pi i / M \omega l + 2\pi i / M r l^2), r, \omega, l \in \mathbb{Z}^M \text{ -----(11)}$$

The full chirp measurement matrix can be composed as: $A_{chirp} = [U_{r1} \ U_{r2} \ U_{r3} \ \dots]$ Where U_{rt} ($t = 1, \dots, \omega$) is a $M \times M$ matrix with segments are given by the chirp signals with a fixed rt and base recurrence ω values that differ from 0 to M-1.

2.2.2. Binary BCH Matrices:

Let n be a divisor of $2^p - 1$ for some integer $p \geq 3$ and (2) be a primitive n th root of unity and assume that p is the smallest integer for which n divides $2^p - 1$. [28],[29],[30]

If we set $\alpha = (2^{p-1})/n$, then α has order n . The BCH matrix can be written as

$$H = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \dots & \alpha^{(n-1)} \\ 1 & \alpha^3 & \alpha^6 & \alpha^9 & \dots & \alpha^{3(n-1)} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & \alpha^{(2l-1)} & \alpha^{2(2l-1)} & \alpha^{3(2l-1)} & \dots & \alpha^{(n-1)(2l-1)} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & \alpha^{(2l-1)} & \alpha^{2(2l-1)} & \alpha^{3(2l-1)} & \dots & \alpha^{(n-1)(2l-1)} \end{bmatrix}$$

IV Results:

The calculations of the matrices are created utilizing Matlab .Sparse signs are produced of a length N=1024, at that point examined utilizing eight measurement matrices. Gaussian commotion with a standard deviation $\sigma_m = 0.05$ is added to the measurement signals y . At that point, meager recuperation of the first sign is performed utilizing the Bayesian recuperation calculation. To think about the exhibition of these measurement matrices, three assessment measurements are utilized: recuperation mistake, preparing timeand covariance. [31],[32]

1. Recovery Error:

The recovery error is calculated using the following formula:

$$error = \|x - x_r\|_1 / \|x\|_1$$

Here x denotes the original sparse signal, x_r denotes the recovered signal, and $\| \cdot \|_1$ is the $\ell_1 - norm$. [17],[18]

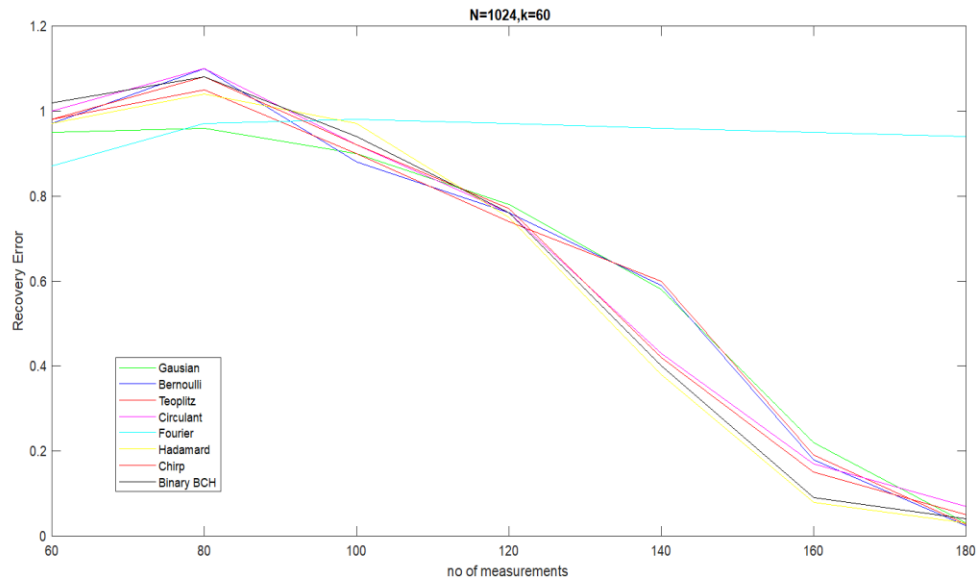


Fig.2: Recovery error with respect to no.of measurements

Fig. 2 depicts the recovery error as for the quantity of measurements. For few measurements ($M \leq 100$), Random Partial Fourier matrix permits recuperation with the littlest mistake among the wide range of various measurement matrices. For various measurements higher than 85, the recovery error s of the Gaussian, Bernoulli, Toeplitz, Circulant, Hadamard trill, and Binary BCH, the recovery error diminishes and get lesser than the Random Partial Fourier matrix. Every one of these measurement matrices, aside from the Partial Fourier matrix, have a similar exhibition, however the Binary BCH and Partial Hadamard matrices show better outcomes contrasted with different matrices.

2.Processing Time:

Processing time of a measurement matrix is a metric that ascertains the time expected to play out the measurement of a given sparse signal.

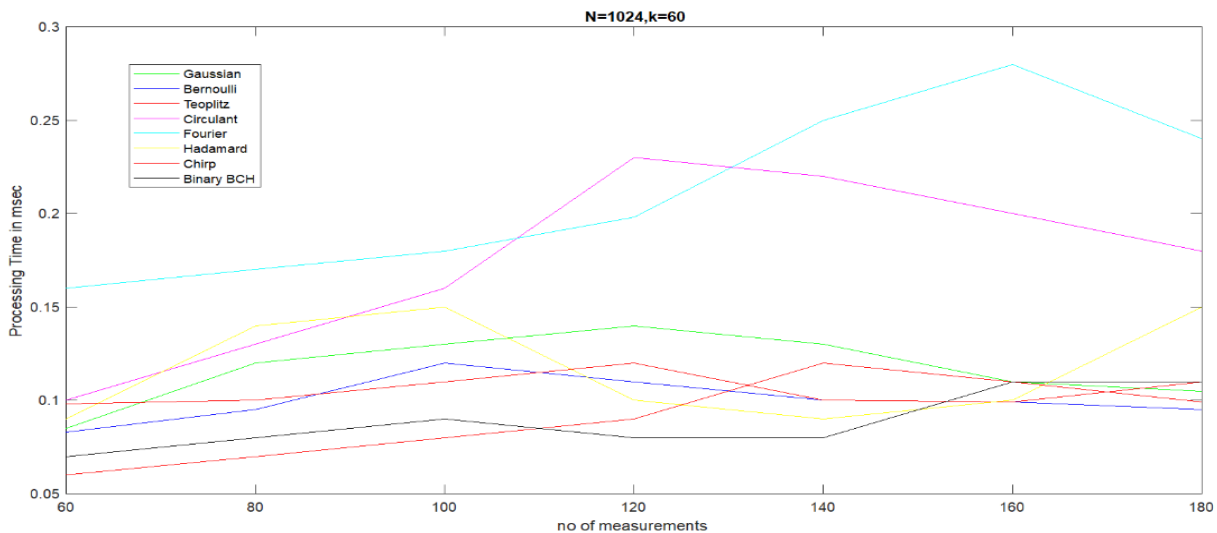


Fig.3:Processing Time with respect to no. of measurements

Fig. 3 represents the preparing season of the eight measurement matrices as an element of the quantity of measurements. As it tends to be seen from this figure, the Chirp measurement matrix has the littlest processing time, trailed by circulant, Binary BCH, the Partial Hadamard, Bernoulli, Gaussian, and afterward Toeplitz. The Partial Fourier matrix requires more preparing time than some other measurement matrices. Be that as it may, the distinction between these processing times is minuscule.

3. Covariance:

The covariance estimates the connection between's the first sign x and the recuperated signal. It is given by:

$$covariance = [E(x - E(x))(x_r - E(x_r))] \text{-----(12)}$$

Here E denotes the expectation, x denotes the original sparse signal and x_r denotes the recovered signal.

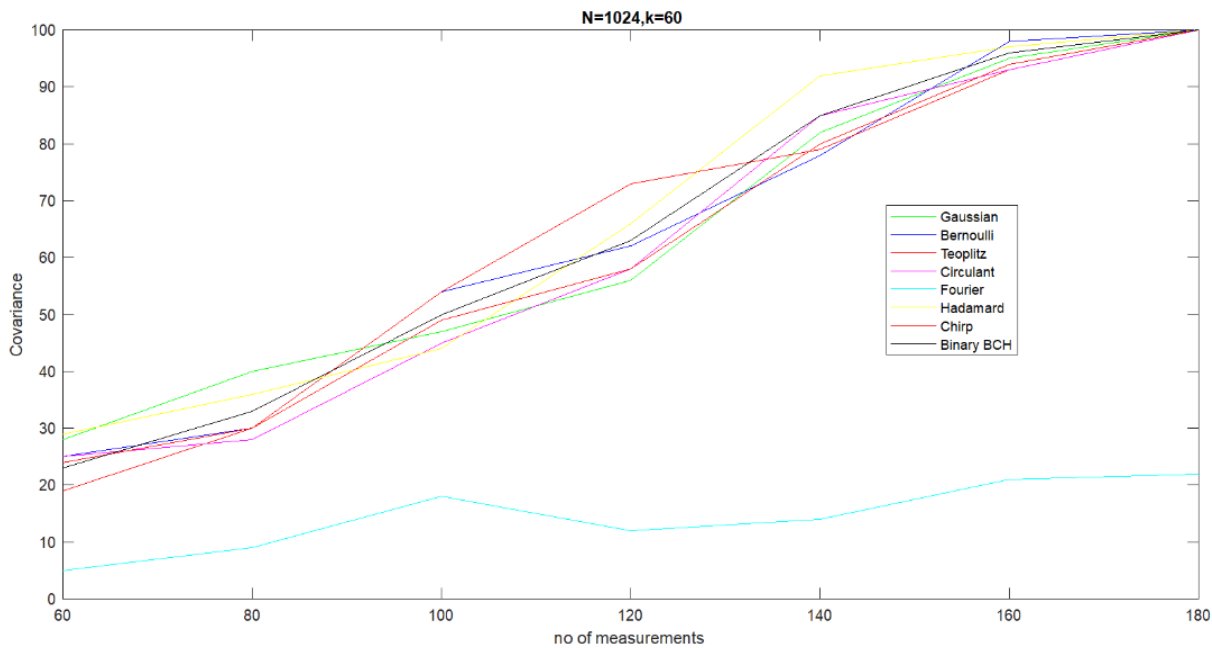


Fig 4: Covariance with respect to no. of measurements

Fig. 4 shows the covariance of the eight measurement matrices as for the quantity of measurements. As noticed, the covariance increments as the quantity of measurement increments to 100% with the exception of the Partial Fourier matrix.

V. Conclusion

In this paper, an outline on packed detecting worldview is being illustrated. Different measurement matrices are being assessed and ordered into two classifications and four sorts. Execution of every one of the eight measurement matrices, two from each kind is appeared. To look at the exhibitions of these matrices, three measurements like recuperation mistake, handling time and covariance are utilized . Test results show that deterministic measurement matrices have the very exhibition as that of Gaussian and Bernoulli matrices. The Binary BCH and Random Partial Hadamard measurement matrices misuse the remaking of inadequate signs with little blunders and decrease the preparing time. Random Partial Fourier matrix with a high number of measurement performs better compared to all the measurement matrices considered in this work. Random measurement matrices perform well, yet they also have a few disadvantages, for example, high equipment cost and high memory stockpiling prerequisite. Deterministic measurement matrices perform better compared to random matrices; be that as it may, they have some shortcomings. For example, the twitter matrix is confined to various

measurements that ought to be square the length of the sign and the Binary BCH matrix doesn't fulfill limited isometric property.

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