Exponential Synchronization On Impulsive Fractional Order Neural Networks With TimeDelay

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Abstract

This Paper Examines A Class Of Impulsive Caputo Fractional Order Neu- Ral Networks(Fonns) Having Time Varying Delays.We Investigate Exponen- Tial Synchronization For These Fonns. For This, We Take Into Account An Appropriate Lyapunov Function And Obtain The Synchronization Results' Conditions As Linear Matrix Inequalities (LMI). The Efficiency Of The Out- Come We Have Obtained Is Verified Using An Example.

Keywords— Impulsive Caputo Fractional Order Neural Network · Exponential Synchronization · Time-Varying Delays · Linear Matrix Inequality · Lyapunov Func- Tion

Introduction

Artificial Neural Networks, Or Simply, Neural Networks, Whose Performance Is Relatively Established On How Animal Neurons Function, Are Interconnected As- Sociation Of Simple Processing Units. The Processing Ability Of The Network Is Acquired By Reviewing From A Set Of Training Patterns. This Ability Is Stored In The Inter-Unit Connection Strengths, Usually Known As Weight [11]. The Last Few Decades Have Seen An Increase In The Importance Of Neural Networks Due To Its Broad Range Of Applications Like Pattern Recognition, Independent Component Analysis, Weather Prediction, Handwriting Recognition, Autopilot, Robotics And So On [1][2][20].As New Results Are Established For Neural Networks Its Different Dynamic Behaviors Are Thoroughly Studied On Many Kinds Of Neural Networks.

For Instance, The Stability [4-8],Synchronization [27-29] Passivity [15], Exponen- Tial Stabilization [3][9][12][37],Exponential Synchronization [16-20], Mittag-Leffler Stability And/Or Synchronization [30-32], Impulsive Synchronization [21]-[26],H_∞ Control Problem[33],Bifurcation [10][36] And So On. We Came Into Fractional Calculus By Simply Altering The Usual Integer Order To Non-Integer Order.[12]. Fractional-Order Model Is Preferred Over Integer-Order Model Due To Different Advantages.The Behavior Of Real-World Situations Is Accurately Noted And Prac- Tical Processes Are Analyzed By Fractional-Order Model.[5][12][27]
Usually, One Faces Time-Delay In Many Engineering Structures Such As Airliner, Control Systems, Communication Systems And So On. Time Delay Results In In- Stability, Divergence, Chaos Or Other Poor Conducts Of The System[20][14][8].In [15][31][32][34] Consequences Of Time Delays In Fonns Is Discussed.
Synchronization Primarily Signify The Dynamic Behavior Of A System Designed To Simulate Another. This Simply Means That The State Trajectory Of The Said Systems Are Similar In The End[20].Many Applications Of Synchronization Are Seen In Several Areas Namely Parallel Image Processing, Secure Communication , Trans- Mission Of Digital Signals, [22] And So On. In Order To Achieve Fast Synchronization, Exponential Synchronization Is Used[16].

In [12] Exponential Synchronization Of Fractional-Order Cohen–Grossberg Neu-Ral Networks Is Considered As A Function Of Stabilization Of Fractional Order Im- Pulse Control Systems.Along With This, We Also Consider Many Synchronization Conditions Subsequent To Impulsive Control. From [34], We Only Take The Case Where Zero Disturbance Is Taken For Fonns And We Find Solution To The Stabi- Lization Of H_∞ Control For The System.

Motivated By The Aforementioned Comments, This Paper Examines A Class Of Impulsive Caputo Fonns Having Time Varying Delays. Its Corresponding Re- Sponse System Is Taken And Hence The Error System Is Found. Our Aim Is To Establish Exponential Synchronization Of This System. The Focus Is On Achieving The Convergence In Order For The System To Be Exponentially Synchronized Faster Than The Studies Done By Now. For That, We Employ A Convex Lyapunov Function For Our System And Obtain The Synchronization Results' Conditions As Lmis .

The Remnant Sections Are Along These Lines; Section 2 Comprises Of Some Pre- Liminaries Required For The Study. Also, Our Fonns, Its Corresponding Response System And The Resulting Error System Is Introduced Here.Section 3 Covers The Main Results That We Have Obtained: I.E. The Conditions For The System Taken To Reach Exponential Synchronization In Terms Of Certain Lmis. Section 4 Contains A Numerical Example Which Verifies The Result That Was Obtained In Section 3. Section 5 Sums Up The Paper And Shows The Direction In Which The Forthcoming Study In This Topic Can Be Taken.

Notations: Let R^n And $\mathrm{R}^{n\times M}$ Imply The Set Of N-Dimensional Real Vectors And $N \times M$ Matrices, Respectively. I_n And 0_n Stands For The $N \times N$ Dimensional Iden- Tity And Zero Matrix, Respectively. $0_{n\times M}$ Designate The N M Dimensional Zero Matrix. Consider Matrices $A, B \times \mathrm{R}^{n\times N}$. $Sym(A)$ Stands For $A + A^T$. Ma- Trix A Is Symmetric Positive Definite, $\in$ With Notation $A > 0$ If $A = A^T$ And

$X^t Ax > 0$ X R^n. Matrix B $\forall$ Is Symmetric Semi-Positive-Definite, With No- Tation $B \geq 0$ If $B = B^T$ And $X^t Bx \geq 0$ X R^n. Set P^n And $\overline{\mathrm{P}}^N$ To Be The Set Of $\forall$ Symmetric Semi-Positive Definite And Set Of Symmetric Positive Definite Matrices In $\mathrm{R}^{n\times N}$ And G^n The Set Of Positive Diagonal Matrices I.E. A Matrix$\Lambda = Diag\{\Lambda_1, ...\Lambda_n\} \in \mathrm{G}^n$ If $\Lambda_i > 0(I = 1, 2, ..., N)$.

Preliminaries And Model Description

Definition 1 (12). *The Caputo Fractional Derivative* ${}^C d^\mu f(T)$ *For Differentiable* $F : [A, B] \to \mathrm{R}, \mu \in (0, 1)$ *Is Defined As;*

$$ {}^C d^\mu_t f(T) = \frac{1}{\Gamma(1-\mu)} \int_0^T \frac{F'(S)}{(T-S)^\mu} Ds \qquad T \geq 0 $$

Here $\Gamma(.)$ *Denotes The Gamma Function;*

$$ \Gamma(S) = \int_0^\infty E^{-T} t^{s-1} dt, S > 0 $$

For Our Convenience We Will Also Use $D^\mu_t f(T)$ To Denote The Caputo Fractional Derivative.

Property 1 (34). *For Any Constants* Λ_1 *And* Λ_2 *And Two Functions* $F_1(T)$ *And* $F_2(T)$,

$$ D^\mu_T(\Lambda_1 f_1(T) + \Lambda_2 f_2(T)) = \lambda_1 D^\mu_T f_1(T) + \lambda_2 D^\mu_T f_2(T) $$

Examine The Subsequent Fractional Order Neural Network Having A Time Vary-Ing Delay:

$$\begin{cases} D^{\mu}x(T) = -Lx(T) + Mh(X(T)) + Nh(X(T-G(T))) & T \neq T_k, T > 0 \\ X(T_K^+) = \left(\dfrac{D_k}{\Gamma(\mu+1)}\right)X(T_K^-) & T = T_K \\ X(T_0) = \Phi(T) & T \in [-G, 0] \end{cases} \tag{1}$$

Where $\mu \in (0, 1)$, $X(T) \in R^n$ Is The Neuron State Vector, N Is The Number Of Neu-Rons In The Fractional Order Neural Networks, (2)

$H(X(T)) = (H_1(X_1(T)), H_2(X_2(T)), \ldots, H_n(X_n(T)))^T \in R^n$ Denotes The Neuron Acti- Vation Function, $L = Diag\{M_1, \ldots, M_n\} \in G^n$, $M, N \in R^{n\times N}$ Are Known Con- Stant Matrices, $G(T)$ Is A Time Varying Delay Satisfying $0 \leq G(T) \leq G$ Where G Is A Known Positive Constant, $\Phi(T)$ Is A Vector Valued Continuous Function, $T_k < T_{k+1}$ For Each $K \in N$, And $D_k \in R^{n\times N}$ And $D_k > 0$ And $\text{Lim}_{k\to+\infty} T_k = \infty$.

If We Take (1) To Be The Drive System Its Response System Is Taken As

$$\begin{cases} D^{\mu}y(T) = -Ly(T) + Mh(Y(T)) + Nh(Y(T-G(T))) & T \neq T_k, T > 0 \\ Y(T_K^+) = \left(\dfrac{D_k}{\Gamma(\mu+1)}\right)Y(T_K^-) & T = T_K \\ Y(T_0) = \Phi(T) & T \in [-G, 0] \end{cases}$$

We Take $E(T) = Y(T) - X(T)$

To Be The Synchronization Error, Then The Error System Is Obtained From (1) And (2) As

$$\begin{cases} D^{\mu}e(T) = -Le(T) + Mh(E(T)) + Nh(E(T-G(T))) & T \neq T_k, T > 0 \\ E(T_K^+) = \left(\dfrac{D_k}{\Gamma(\mu+1)}\right)E(T_K^-) & T = T_K \\ E(T_0) = \Psi(T) = \Phi(T) - \Phi(T) \end{cases} \tag{3}$$

Assumption 1 (34)**.** *Any Activation Function $H_i(.)$ Is Continuous, Bounded And Fulfills The Condition*

$$L_i^- \leq \frac{H_i(C) - H_i(D)}{C - D} \leq L_i^+ \qquad I = 1, 2, \ldots, N$$

Where $H_i(0) = 0$, L_I^+, L_I^- Are Known Real Constants $(I = 1, 2, \ldots, N)$, $C, D \in R$ And $C \neq D$.

Lemma 1 (34)**.** *Given $\mu \in (0, 1]$, $E(T) \in R^n$ Be A Continuous Function And $U : R^n \to R^n$ Be A Convex And Differentiable Function On R^n Such That $U(0) = 0$. We Have*

$$D_t^{\mu}u(E(T)) \leq \langle \Delta U(E(T)), D_t^{\mu}e(T)\rangle \quad T \geq 0$$

Where $\Delta U(.)$ Is The Gradient Of U And $\langle , \rangle$ Is The Inner Product.

Lemma 2. *For Real Valued Function $U(T)$ On $[A, \infty)$ And $A \in R$, If There Exists A Constant Θ In A Manner That*

$$D_t^{\mu}u(T) \leq \Theta u(T), 0 < \mu \leq 1$$

Then

$$\int^{T} \frac{\Theta(T-T)^{\mu-1}D\tau}{}$$

$U(T) \leq U(A)E^{A}$ $\Gamma(A)$

$= \quad U \quad \dfrac{\Theta(T-A)\mu}{(A)E} \quad \Gamma(A+1)$

Main Result

Theorem 1. *Consider The Case Where Assumption 1 Is True. System (3) Is Expo- Nentially Synchronized If There Exists* $Q, K \in \overline{P}^N$, $Y \in P^{3n}$, $\Lambda_i = Diag\{\Lambda_{i1}, \Lambda_{i2}, \ldots, \Lambda_{in}\} \in G^n (I = 1, 2, \ldots, N)$, $K_k \in R$ *Satisfying The Subsequent LMI*

$$G^{\mu}\mu^{-1}\Upsilon^T Y \Upsilon + \sum_{I=1}^{3} \Phi_i - Bq < 0 \tag{4}$$

$$-K_k q + \left(\frac{D_k}{\Gamma(\mu+1)}\right)^T Q\left(\frac{D_k}{\Gamma(\mu+1)}\right) \leq 0 \tag{5}$$

And The Condition, If $0 < T_{k+1} - T_k \leq T$,

$$Ln(K_k) < \frac{-(N-Z)T^{\mu}}{\Gamma(\mu+1)} \tag{6}$$

$T, N, B > 0, Z \in R$

$\Pi_1 = I_n \quad 0_n \quad 0_n \quad 0_n \quad 0_n$ $\Pi_2 =$

$0_n \quad I_n \quad 0_n \quad 0_n \quad 0_n$ $\Pi_3 = 0_n \quad 0_n$

$I_n \quad 0_n \quad 0_n$ $\Pi_4 = 0_n \quad 0_n \quad 0_n$

$I_n \quad 0_n$ $\Pi_5 = 0_n \quad 0_n \quad 0_n \quad 0_n \quad I_n$

$\Theta_1 = Diag\{L_1^-, \ldots, L_n^-\}$

$\Theta_2 = Diag\{L_1^+, \ldots, L_N^+\}$

$\Upsilon = \Pi_1^T \quad \Pi_2^T \quad \Pi_6^T \quad T$

$\Upsilon_1 = \Pi_3 - \Theta_1\Pi_1$ $\Upsilon_2 =$

$\Theta_2\Pi_1 - \Pi_3$ $\Upsilon_3 = \Pi_4 -$

$\Theta_1\Pi_2 \Upsilon_4 = \Theta_2\Pi_2 - \Pi_4$

$\Upsilon_5 = \Pi_3 - \Pi_4 - \Theta_1(\Pi_1 - \Pi_2)$

$\Upsilon_6 = \Theta_2(\Pi_1 - \Pi_2) - \Pi_3 + \Pi_4$

$\Phi_1 = Sym(\Pi_1^T Q\Pi_5 - \Pi_1^T Ke_5 + \Pi_1^T KM\Pi_3 + \Pi_1^T KN\Pi_4 - \Pi_5^T KL\Pi_1 + \Pi_5^T KM\Pi_3 + \Pi_5^T KN\Pi_4)$

$\Phi_2 = \Pi_1^T[-KL - LK + Q]\Pi_1 - \Pi_2^T Q\Pi_2 + \Pi_5^T[-2K]\Pi_5$

$\Phi_3 = Sym(\Upsilon_1^T \Lambda_1 \Upsilon_2 + \Upsilon_3^T \Lambda_2 \Upsilon_4 + \Upsilon_5^T \Lambda_3 \Upsilon_6)$

Proof. First We Will Look At The Case When $T \neq T_k$. Let

$$U(T) = U(T, E(T)) = E^t(T)Qe(T) \text{ Be}$$

The Lyapunov Function For System (3)

We Observe That $U(T)$ Is A Differentiable And Convex Function On $\mathbb{R}^n$ And $U(T, 0) = 0$. Hence On Applying Lemma 1, Caputo Fractional Derivative Of The System (3) Of Order μ Is Calculated In This Way:

$$D^\mu_T u(T, E(T)) \le 2e^t(T)Qd^\mu_T e(T) = \Gamma^t(T)Sym(\Pi_1^T Q\Pi_5)\Gamma(T) \quad (7)$$

Where

$$\Gamma(T) = \begin{bmatrix} E^t(T) & E^t(T-G(T)) & H^t(E(T)) & H^t(E(T-G(T))) & (D^\mu_T e(T))^T \end{bmatrix}^t$$

For Any Matrix $Y \in \mathbb{P}^{3n}$, The Following Holds

$$G^\mu \mu^{-1}\zeta^t(T)Y Z(T) - \int_{T-G(T)}^{t} (T-S)^{\mu-1}\zeta^t(T)Y Z(T)Ds \ge 0 \quad (8)$$

Where

$$Z(T) = \begin{bmatrix} E^t(T) & E^t(T-G(T)) & (D^\mu_T e(T))^T \end{bmatrix}^t$$

Then Again, For Any $K \in \mathbb{P}^N$, The Following Equation Can Be Attained From The System (3)

$$[2e^t(T)+2(D^\mu_t e(T))^T]K \times [-D^\mu_t e(T) - Le(T) + Mh(E(T)) + Nh(E(T-G(T)))] = 0 \quad (9)$$

From Assumption 1, We Get That For Any $\Lambda_{ji} > 0$ $(I = 1, 2, \ldots, N,\ J = 1, 2, 3)$

$$2(H_i(E_i(T)) - L^-_i E_i(T))\Lambda_{1i}(L^+_i E_i(T) - H_i(E_i(T))) \ge 0$$

$$2(H_i(E_i(T-G(T))) - L^-_i E_i(T-G(T)))\Lambda_{2i}(L^+_i E_i(T-G(T)) - H_i(E_i(T-G(T)))) \ge 0$$

$$D^\mu_t U(T, E(T)) \le \Gamma^t(T)\bar{\Phi}\gamma(T) - \int_{T-G(T)}^{T} (T-S)^{\mu-1}\zeta^t(T)Y Z(T)Ds \quad (12)$$

Is Attained, Where

$$\bar{\Phi} = G^\mu \mu^{-1}\Upsilon^T X \Upsilon + \Phi_1 + \bar{\Phi}_2 + \Phi_3$$

$$\bar{\Phi}_2 = \Pi_1^T[-KL - LK + \Sigma q]\Pi_1 - \Pi_2^T Q\Pi_2 - \Pi_5^T[-2K]\Pi_5$$

Since $\Sigma > 1$ Is Arbitrary And Does Not Affect $D^\mu_t u(T, E(T))$, As $\Sigma \longrightarrow 1^+$, The Inequality (12) Becomes

$$D^\mu_t u(T, E(T)) \le \Gamma^t(T)\Phi\gamma(T) - \int_{T-G(T)}^{T}$$

$$(T - S)^{\mu-1}\zeta^{t}(T)Y\,Z(T)Ds \tag{13}$$

$$\Longrightarrow D^{\mu}_{t} u(T, E(T)) < \Gamma^{t}(T)\Phi\gamma(T)$$

Take $\Phi < Bq$ Where $B \in R$ And $B > 0$

$$\begin{aligned} D^{\mu}_{t} u(T, E(T)) &< \Gamma^{t}(T)Bq\gamma(T) \\ &< B\gamma^{t}(T)Sym(\Pi_1 Q\Pi^{T})\Gamma(T) \\ &= -Ze^{t}(T)Qe(T) \end{aligned}$$

$$2(H_i(E_i(T)) - H_i(E_i(T - H(T))) - L^{-}_{i}(E_i(T) - E_i(T - H(T))))\Lambda_{3i}(L^{+}_{i}(E_i(T) - E_i(T - H(T))) - H_i(E_i(T)) + H_i(E_i(T - G(T)))) \geq 0$$

Which Imply

$$\begin{aligned} 2\gamma^{t}(T)\Upsilon_1^{T}\Lambda_1\upsilon_2\gamma(T) &\geq 0 \\ 2\gamma^{t}(T)\Upsilon_3^{T}\Lambda_2\upsilon_4\gamma(T) &\geq 0 \\ 2\gamma^{t}(T)\Upsilon_5^{T}\Lambda_3\upsilon_6\gamma(T) &\geq 0 \end{aligned} \tag{10}$$

Since

$$U(T, E(T)) = E^{t}(T)Pe(T)$$

Taking Some Real Number $\Sigma > 1$ We Presume That

$$U(T + S, E(T + S)) < \Sigma u(T, E(T)) \quad \forall S \in [-G, 0]$$

Which Gives

$$\Sigma e^{t}(T)Qe(T) - E^{t}(T - G(T))Qe(T - G(T)) > 0 \tag{11}$$

Combining Estimates (7-11),

$$D^{\mu}_{t} u(T, E(T)) < -ZU(T, E(T)) \tag{14}$$

Where $Z \in R$

Hence

Let

$$\left(1 + \frac{Dk}{\Gamma(\mu+1)}\right)^{T} Q\left(1 + \frac{Dk}{\Gamma(\mu+1)}\right) \leq K_k Q$$

$$\begin{aligned} U(T_k, E(T_k)) &\leq E^{t}(T^{-}_{k})K_k q e(T^{-}_{k}) \\ &= K_k e^{t}(T^{-}_{K})Qe(T^{-}_{K}) \end{aligned}$$

Thus

$$U(T_k) \leq K_k u(T^{-}_{K})$$

Hence We Have

$$\begin{aligned} D^{\mu}_{t} u(T, E(T)) &< -ZU(T, E(T)) \quad T \neq T_k \\ U(T^{+}_{k}, E(T^{+}_{k})) &\leq K_k u(T_k, E(T_k)) \quad T = T_k \end{aligned} \tag{15}$$

$$U(T, E(T)) = U(T_0) \quad T = T_0$$

For Any $T \in [T_0, T_1)$ We Have From Lemma 2

$$U(T) \leq U(T_0)E^{\frac{Z(T-T_0)^{\mu}}{\Gamma(\mu+1)}}$$

Which Leads To

$$U(T_1^-) \leq U(T_0)E^{\frac{Z(T_1-T_0)^{\mu}}{\Gamma(\mu+1)}}$$

Set

$$U(T_1^+) = U(T_1)$$

Considering Any $T \in [T_1, T_2)$ We Get

$$U(T) \leq U(T_1)E^{\frac{Z(T-T_0)^{\mu}}{\Gamma(\mu+1)}}$$

$$\leq U(T_0)K_1 e^{\frac{Z((T-T_1)^{\mu}+(T_1-T_0))^{\mu})}{\Gamma(\mu+1)}}$$

Similarly For Any $T \in [T_k, T_{k+1})$,

$$U(T) \leq U(T_1)E^{\frac{Z(T-T_k)^{\mu}}{\Gamma(\mu+1)}}$$

$$\leq U(T_0)K_1K_2 \ldots K_k e^{\frac{Z((T-T_k)^{\mu}+(T_k-T_{k-1}))^{\mu}+ \ldots +(T_1-T_0)^{\mu})}{\Gamma(\mu+1)}}$$

By The Condition If

$$0 < T_{k+1} - T_k \leq T, \quad Ln(K_k) \leq \frac{-(N-Z)T^{\mu}}{\Gamma(\mu+1)}$$

Where $T, N > 0$, We Obtain

$$U(T) \leq U(T_0)E^{\frac{-(N-Z)K\tau^{\mu}}{\Gamma(\mu+1)}} E^{\frac{-Z(K+1)T^{\mu}}{\Gamma(\mu+1)}}$$

$$\leq U(T_0)E^{\frac{-Nk\tau^{\mu}}{\Gamma(\mu+1)}} E^{\frac{-Zk\tau^{\mu}}{\Gamma(\mu+1)}} E^{\frac{-Z\tau^{\mu}}{\Gamma(\mu+1)}}$$

Thus, Where

$$U(T) \leq U(T_0)E^{-(Nk+Z)T^{\mu}} \tag{16}$$

$$Nk + Z > 0$$

As $K \to \infty$ And $T \to \infty$ We Have

$$U(T) \leq 0$$

This Proves The Exponential Synchronization Of The System (3). □

Example

Given Below Is An Example To Depict The Results Proved;

Consider The Following Parameters For System (1);

$N = 2, \mu = 0.1$, L= $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$, M= $\begin{pmatrix} 2 & -0.1 \\ -5 & 0.75 \end{pmatrix}$ And N= $\begin{pmatrix} 0.15 & 0.1 \\ 0 & 0.3 \end{pmatrix}$.

$D = \begin{pmatrix} 0.95 & 0 \\ 0^k & 0.95 \end{pmatrix}$

We Take The Time Varying Function As

$$G(T) = \frac{T}{1+T}, \quad T \geq 0$$

Activation Function And Delay Term Are Given By

$$H(X(T)) = \left(\frac{1}{1+E^{-X1(T)}}, \frac{1}{1+E^{-X2(T)}}\right) \in \mathrm{R}^2$$

$$H(X(T-G(T)) = \left(\frac{1}{1+E^{-X1(T-G(T)}}, \frac{1}{1+E^{-X2(T-G(T))}}\right)$$

Initial Condition Is $X(0) = (0.15, 0.1)$

The Corresponding Response System (2) Has The Same Parameters As Above With

Initial Condition $Y(0) = (0.45, 0.3)$

Hence We Get The Error System As (3), The Same Parameters As Above And Initial Condition $E(0) = (0.3, 0.2)$

Solution:

Upon Considering The Above Parameters And Conditions For System (3) And Sub- Stituting Theses Values In The Conditions Given In The Theorem We Can See That These Conditions Are Satisfied And Hence We Get That The System Is Exponentially Synchronized.

Conclusions

In View Of This Paper, We Explored The Exponential Synchronization Of Impul- Sive Neural Networks Having Fractional Order Having A Time Delay.We Introduce A Convex Lyapunov Function For Our System And Use Certain LMI Conditions To Achieve Synchronization .The Research In The Same Problem Can Be Lead Forward By Broadening The Result Obtained Into Complex Valued Neural Networks.

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