A Comparative Study on the Numbers of Operations Used in the Computation of Value of Combinations $\binom{n}{r}$

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Abstract: Several different methods are used to compute the value of combinations. A comparison between the two methods of computing the value of combinations viz traditional method involving the use of factorial notations in the formula of $\binom{n}{r}$ and the method of computing the value of combinations by the repeated divisions with a specific lower bound 'p' associated with 'n' where n and p are positive integers is made.
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1. Introduction

1.1 Combinations

When 'r' elements of a sets which contains 'n' distinct elements are disorderly arranged it is called r-combination of 'n' elements denoted by $\binom{n}{r}$[***Rosen, K. H., &Krithivasan, K. (2012),*****Tucker, A. (2006)**].

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Or we can use an alternative formula which is used generally for the easy computation is obtained by simplifying the factorial notation is given by

$$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots r\ terms}{1.2.3\ldots r}$$

2. Review of Literature

A novel optimal method to compute the value of combinations was suggested by**Shyama, S., &Parameswaran, R. (2017).** They introduced a new technique to get the values of$\binom{n}{r}, r = 0,1,\ldots,n$. For any positive integer 'n', there exist a sufficiently larger 'p', by finding $(p+1)^n$ and then dividing it successively by p, the remainders will gives the all possible values of $\binom{n}{r}$.

A lower bound for the value of 'p' used in this computation was mentioned by **Athul, K., Allwin Antony, M.B., &Parameswaran, R. (2020, June 19-20).**For any positive integer 'n', there exists a lower bound value of 'p' which is given by

$$p = \begin{cases} \binom{n}{n/2}+1, & \text{if n is even.} \\ \binom{n}{\frac{n\pm 1}{2}}+1, & \text{if n is odd.} \end{cases}$$

Table.1. Finding combination values of $\boldsymbol{n = 6}$

For $n = 6, p = \binom{6}{3} + 1 = 20 + 1 = 21$		
$(p+1)^n = (21+1)^6 = 22^6 = 113379904$		
	Quotient	Remainder
$\frac{113379904}{21}$	5399043	$\binom{6}{0} = 1$
$\frac{5399043}{21}$	257097	$\binom{6}{1} = 6$
$\frac{257097}{21}$	12242	$\binom{6}{2} = 15$
$\frac{12242}{21}$	582	$\binom{6}{3} = 20$
$\frac{582}{21}$	27	$\binom{6}{4} = 15$
$\frac{27}{21}$	1	$\binom{6}{5} = 6$
$\frac{1}{21}$	0	$\binom{6}{6} = 1$

3. Objective

To compare the number of operations between the traditional method involving the use of factorial notations in the formula of $\binom{n}{r}$ to find the value of combinations and the method of computing the value of combinations by the repeated divisions with a specific lower bound 'p' associated with 'n' where n and p are positive integers.

In this paper we give a generalized equation for number of operations involved in the two methods specified in the aforesaid works [**Athul, K., Allwin Antony, M.B., &Parameswaran, R. (2020, June 19-20), Shyama, S., &Parameswaran, R. (2017)**].

3.1 Illustration

(a)Method of computing the value of combinations by the repeated divisions with a specific lower bound 'p' associated with 'n' where n and p are positive integers.

Table.2. Illustration for $\boldsymbol{n = 7}$ by method (a)

			Number of Operations
For $n = 7, p = \binom{7}{4} + 1 = 35 + 1 = 36$			5+1=6
$(\mathrm{p} + 1)^{\mathrm{n}} = (36 + 1)^7 = 37^7 = 94931877133$			1+6=7
	Quotient	Remainder	
$\frac{94931877133}{36}$	2636996587	$\binom{7}{0} = 1$	8
$\frac{2636996587}{36}$	73249905	$\binom{7}{1} = 7$	
$\frac{73249905}{36}$	2034719	$\binom{7}{2} = 21$	
$\frac{2034719}{36}$	56519	$\binom{7}{3} = 35$	
$\frac{56519}{36}$	1569	$\binom{7}{4} = 35$	
$\frac{1569}{36}$	43	$\binom{7}{5} = 21$	
$\frac{43}{36}$	1	$\binom{7}{6} = 7$	
$\frac{1}{36}$	0	$\binom{7}{7} = 1$	
Total number of operations			21

Table.3. Illustration for $\boldsymbol{n = 10}$ by method (a)

			Number of operations
For $n = 10, p = \binom{10}{5} + 1 = 252 + 1 = 253$			9+1=10
$(\mathrm{p} + 1)^{\mathrm{n}} = (253 + 1)^{10} = 254^{10} = 111773066554715497640857 6$			1+9=10
	Quotient	Remainder	
$\frac{1117730665547154976408576}{253}$	4417907768961086863275	$\binom{10}{0} = 1$	
$\frac{4417907768961086863275}{253}$	17462086043324454005	$\binom{10}{1} = 10$	

$\frac{17462086043324454005}{253}$	69020102938041320	$\binom{10}{2} = 45$	11
$\frac{69020102938041320}{253}$	272806730980400	$\binom{10}{3} = 120$	
$\frac{272806730980400}{253}$	1078287474230	$\binom{10}{4} = 210$	
$\frac{1078287474230}{253}$	4262005826	$\binom{10}{5} = 252$	
$\frac{4262005826}{253}$	16845872	$\binom{10}{6} = 210$	
$\frac{16845872}{253}$	66584	$\binom{10}{7} = 120$	
$\frac{66584}{253}$	263	$\binom{10}{8} = 45$	
$\frac{263}{253}$	1	$\binom{10}{9} = 10$	
$\frac{1}{253}$	0	$\binom{10}{10} = 1$	
Total number of operations			31

(b)Traditional alternative method involving the use of factorial notations in the formula of $\binom{n}{r}$to find the value of combinations

Table.4. Illustration for $\boldsymbol{n = 7}$ by method (b)

	$\binom{7}{0}$	$\binom{7}{1}$	$\binom{7}{2}$	$\binom{7}{3}$	$\binom{7}{4}$	$\binom{7}{5}$	$\binom{7}{6}$	$\binom{7}{7}$	Total number of operations
Number of operations	0	1	3	5	5	3	1	0	18

Table.5. Illustration for $\boldsymbol{n = 10}$ by method (b)

	$\binom{10}{0}$	$\binom{10}{1}$	$\binom{10}{2}$	$\binom{10}{3}$	$\binom{10}{4}$	$\binom{10}{5}$	$\binom{10}{6}$	$\binom{10}{7}$	$\binom{10}{8}$	$\binom{10}{9}$	$\binom{10}{10}$	Total number of operation s

Number of operations	0	1	3	5	7	9	7	5	3	1	0	41

Similarly for different values of n, we have

Table. 6. Number of operations for different n values.

n	Method (a)	Method (b)
3	9	2
7	21	18
8	25	25
10	31	41
30	91	421
63	189	1922
80	241	3121

3.2 Result

Generalized equation for the number of operations involved in the method of computing the value of combinations by the repeated divisions with a specific lower bound ‘p’ associated with ‘n’ where n and p are positive integers is

$$\text{Number of operations} = \begin{cases} 3n+1, & \mathit{If\ n\ is\ even.} \\ 3n, & \mathit{If\ n\ is\ odd.} \end{cases}$$

Generalized equation for the number of operations in the traditional alternative method involving the use of factorial notations in the formula of $\binom{n}{r}$ to find the value of combinations is

$$\text{Number of operations} = \begin{cases} \dfrac{n^2-2n+2}{2}, & \mathit{If\ n\ is\ even.} \\ \dfrac{n^2-2n+1}{2}, & \mathit{If\ n\ is\ odd.} \end{cases}$$

4. Conclusion

The number of operations involved in the traditional alternative method used to compute $\binom{n}{r}$is of $\boldsymbol{O}(n^2)$and that for the repeated division method as specified above is of$\boldsymbol{O}(n)$. Therefore this method of computing the value of combinations by the repeated divisions with a specific lower bound ‘p’ associated with ‘n’ where n and p are positive integers can be used to calculate values of $\binom{n}{r}, r = 0,1,\dots,n$ is much more easy compared to the traditional alternative method. This will be a great advantage in the computation of combinations of bigger numbers and also this will yield the reduction in the number of operations inthe computation which will help to reduce memory storage there by increasing the efficiency of the system.

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