Research On The Modified Internal Rate Of Return

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Abstract: The internal rate of return (IRR) is one of the profitability analysis indicators which are often used in project evaluation. But it has many flaws. There are three main following flaws. The first flaw is that the implied reinvestment rate assumption of internal rate of return is unreasonable. The second flaw is that solving IRR is difficult. The third flaw is that there may be multiply IRRs or no IRR for unconventional cash flow projects. Many scholars have studied it and put forward many improved methods. Modified internal rate of return (MIRR) is one of the methods to improve IRR. This article compares IRR and MIRR. Solving MIRR is relatively simple. And it is important that MIRR is unique. An example is given to demonstrate how to compute MIRR. The reinvestment rate of MIRR usually equals to the project's cost of capital. But the value of MIRR is affected by the discount rate that often is minimum attractive rate of return(MARR).Like the IRR, ranking problem of the MIRR still exists. So MIRR is only suitable in single project evaluation. It can not be applied in mutually exclusive project evaluation. Finally an example is given to verify the correctness.

Keywords: Internal Rate of Return，Modified Internal Rate of Return

1 Introduction

In engineering economic analysis and project valuation, the present worth method (PW) and internal rate of return (IRR) are the most commonly used dynamic evaluation methods. The IRR is one of favorite indicators for companies. But the IRR method has been criticized for a long time because it is well known it has many drawbacks. C. A. Magni listed the IRR has 18 flaws which lead to debate about the IRR. [1]

Zhang Xiaoli pointed out that the internal rate of return which has many serious defects is unreliable, it only can be used as an auxiliary method. [2]

As is widely known, there are three main weaknesses.

Firstly the implicit reinvestment rate assumption of IRR is unreasonable. The IRR method assumes that net cash flows can reinvest and reinvestment rate of cash flows is the IRR. The assumption is usually unrealistic. IRR can be higher than the reinvestment rate of return or IRR can be lower than it. In practice，if IRR is higher than the reinvestment rate of return,the assumption is invalid.

Secondly the calculation of the IRR is difficult. In general, it cannot calculate directly，often needs an iterative trail-and-error procedure.

Thirdly there is the problem of multiple or nonexistent internal rates of return for unconventional cash flow's alternatives. If sign of cash flows change more than once , an alternative may has several IRRs. The IRR method is only suitable for the conventional cash flow's alternatives. When an alternative may have several IRRs，which IRR is reasonable? In this case the IRR method is commonly believed invalid or meaningless. So it is universally regarded as its fatal flaw.

Many studies have focused on these problems. A lot of improved methods have proposed to overcome its flaws. There are many improved methods, such as external rate of return[3,4], modified internal rate of return[5,6], modified external rate of return, average internal rate of return[7] , capital flow conversion method[8], generalized internal rate of return [9], generalized external rate of return[9] and so on. But these improved methods are not intensively used in practice.

Kierulff H. think MIRR is a better measure than IRR. [9]Yuan Xianfeng et al. pointed out that the modified internal rate of return method is a practical method, and recommended using the modified internal rate of return evaluate unconventional cash flow projects[10].

2 Modified Internal Rate of Return

2.1 Modified Internal Rate of Return and Internal Rate of Return

2.1.1 Internal Rate of Return，IRR

The IRR of an alternative is the discount rate at which the net present value of all cash flows from alternative is exactly equal to zero.

The IRR is calculated by the following formula.

$$\sum_{t=0}^{n}(CI-CO)_t(1+\mathrm{i}^*)^{-t}=0$$

where
t is the time of the cash flow
$(CI-CO)_t$ is net cash flow at time t
n is alternative life
i^* is internal rate of return，IRR

2.1.2 Modified Internal Rate of Return, MIRR

The investment cash flows of an alternative are discounted to time zero at finance rate,the other net cash flows of an alternative are discounted to the final period at reinvestment rate of return. The MIRR of an alternative is the discount rate at which the present value is exactly equal to the future value.
The modified internal rate of return is calculated by the following formula.

$$[\sum_{t=0}^{n}\mathrm{K_t}(1+i_c)^{-t}](1+MIRR)^n=\sum_{t=0}^{n}(\mathrm{B-C})_t(1+i_c)^{n-t} \qquad (1)$$

where
t is the time of the cash flow
i_c is the minimum attractive rate of return i.e. the rate used to discount cash flows to present value
n is the alternative life
$(B-C)_t$ is the net cash flow i.e. cash inflow-cash outflow, at time t
K_t is investment at time t

2.2 Comparison with IRR

The MIRR solves three problems above. The MIRR equation has only one real root. The MIRR method can be solved directly, without needing to resort to an iterative trail-and-error procedure. The IRR method has the problem of multiple or nonexistent internal rates of return.

When MIRR is used ,the assumed reinvestment rate is usually equal to minimum attractive rate of return. The MIRR method usually assumes that net cash flows each period are reinvested at the MARR during the study period. Minimum attractive rate of return is determined taking into account cost of capital, investment risk and other factors. The reinvestment rate of the MIRR is more practical. Compared with IRR,the assumed reinvestment rate assumption of MIRR is more reasonable.

If cash flow stream is deterministic then the IRR of an alternative is determined. The value of the IRR can not be affected by the minimum acceptable rate of return. But calculating the MIRR requires a MARR used as the discount rate. And the value of MIRR is affected by the value of assumed MARR. Here is an example to illustrate.

Suppose that initial investment is 1000 ， annual net income is 450.The sequence of cash flows of an investment alternative is shown in the table 1.

Table 1 cash flow of alternative A

End of Year	Cash flow
0	-1000
1	450
2	450
3	450
4	450
5	450

Calculate the IRR of alternative A using the following equation.
$1000=450(P/A,IRR_A,5)$
the answer is $IRR_A=34.94\%$
Calculate the MIRR of alternative A using the following equation.
$1000(1+MIRR_A)^5=450(P/A,10\%,5)$
If MARR=10% then MIRR=22.40%
If MARR=15% then MIRR=24.86%

From the above equtions,we can know calculating IRR doesn't require MARR.The root of the IRR equation is determined by cash flows of an alternative. When cash flows of an alternative is deterministic, the value of the IRR is determined. However, calculating MIRR requires MARR. When MARR is assumed different, the root of the MIRR equation can change. Calculating with different MARR, the MIRR is different. So it is important to assume a proper MARR .If a reinvestment rate is greater than the company's cost of capital, then the MIRR will underestimate an alternative's true rate of return.

2.3 Evaluating Independent alternatives by MIRR

Evaluating independent alternatives，the MIRR method is consistent with the IRR method and present worth method(PW).

Use equation (1), get equation (2)

$$\sum_{t=0}^{n}(\mathrm{B-C})_t(1+i_c)^{n-t}-[\sum_{t=0}^{n}K(1+i_c)^{-t}](1+MIRR)^n=0 \qquad (2)$$

The equation of the NFV is given by

$$NFV=\sum_{t=0}^{n}(\mathrm{B-C})_t(1+i_c)^{n-t}-[\sum_{t=0}^{n}K(1+i_c)^{-t}](1+i_c)^n \qquad (3)$$

The equation of the NPV is given by

$$NPV=\sum_{t=0}^{n}K(1+i_c)^{-t}-[\sum_{t=0}^{n}(\mathrm{B-C})_t(1+i_c)^{n-t}](1+i_c)^{-n} \qquad (4)$$

If MIRR＞MARR，then （2）＜（3），（2）＜（4），NFW＞0, NPW＞0,the alternative is economically feasible.

If MIRR＝MARR，then （2）＝（3），（2）＝（4），NFW＝0, NPW＝0,the alternative is economically feasible.

If MIRR＜MARR，then （2）＞（3），（2）＞（4），NFW＜0, NPW＜0,the alternative is economically unfeasible.

In summary, If MIRR≥MARR，then the alternative will be accepted. Otherwise, it will be refused.

2.4 Evaluating Mutually Exclusive Alternatives by MIRR

However, the MIRR method cannot be used in the selection of mutually exclusive alternatives. The MIRR method and PW method are not always consistent. Sometimes two conclusions may be contrary. An example is given as follows. Consider the two mutually exclusive alternatives. The cash flows of alternatives are shown in table 2.Negative cash flows represent capital investment. Positive cash flows represent annual net income.

Table 2　cash flow of alternatives

End of Year	alternative	
	B	C
0	-1000	-400
1	400	-600
2	400	400
3	400	400
4	400	500
5	400	500
6	400	500
7	400	500
8	400	500

At MARR = 10% ,the NPV values of alternatives are as follows.

NPV_B＝－1000＋400（P/A,10%,8）＝1133.97

NPV_C＝－400－600(P/F,10%,1)＋400(P/F,10%,2)＋400(P/F,10%,3)＋500×(P/F,10%,4)＋500(P/F,10%,5)＋500（P/F,10%,6）＋500（P/F,10%,7）＋500（P/F,10%,8）＝1109.69

Based on the PW method, alternative B is preferred to alternative C ,because alternative B has a greater NPV value.

Calculate the MIRR of alternative B using the following equation.

1000（1＋$MIRR_B$）^{8}＝400（P/A,10%,8）

the answer is $MIRR_B$＝20.93%

Calculate the MIRR of alternative C using the following equation.

[400＋600（P/F,10%,1）]（1＋$MIRR_C$）^{8}＝400（P/F,10%,6）＋400（P/F,10%,5）＋500（P/F,10%,4）＋500（P/F,10%,3）＋500（P/F,10%,2）＋500（P/F,10%,1）＋500

the answer is　$MIRR_C$＝21.21%

Based on the MIRR method, alternative C is preferred to alternative B ,because alternative C has a greater MIRR value.

The results of calculation are listed in the Table 3.

Table 3 Results of Calculation

alternative	NPV （MARR = 10%）	IRR	MIRR
B	1133.97	36.72%	20.93%
C	1109.69	34.49%	21.21%

In this case, the conclusions are contrary according different decision methods. Based on the PW method ,alternative B is selected. Base on the MIRR method, alternative C is selected. Inconsistent ranking

problem in evaluating mutually exclusive alternatives still exists. The PW method is an equivalent-worth method, but the MIRR is a rate-of-return method. In comparison of mutually exclusive alternatives MIRR method is incorrect. MIRR is only used in determining the acceptability of an alternative.

3Using Excel to calculate

In Excel there are built-in functions to calculate IRR and MIRR.They can be calculatee quickly and accurately.

IRR(values,[guess])

Where guess is the initial estimate and is optional.

MIRR(values,finance_rate,reinvest_rate)

Where finance_rate is the discount rate of funds,and reinvest_rate is the discount rate of investment.

4Conclusion

As mentioned previously, the MIRR method has three advantages over the IRR method. Calculating MIRR is relatively simple. The MIRR equation has one real root. The assumption of MIRR is more reasonable. But calculating the MIRR with different MARR，the value of MIRR varies. The problem of ranking competing alternatives still remains unsolved. When ranking competing alternatives sometimes the MIRR is inconsistent with the PW. When ranking competing alternatives, the MIRR is not applicable.

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