# Classification of pythagoras triples And How to generate them 

## A.S.Mohan Kumar

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#### Abstract

Furthermore, the Pythagorean theorem is commonly used in advanced math today. It is used in computing surface areas, volumes and perimeters of different geometric shapes, converting between polar and rectangular coordinates and computing the distance between particular points on a plane. The Pythagorean Theorem can be used in any real life scenario that involves a right triangle having two sides with known lengths. In a scenario where a certain section of a wall needs to be painted, the Pythagorean Theorem can be used to calculate the length of the ladder needed if the height of the wall and the distance of the base of the ladder from the wall are known.

The Pythagorean Theorem can be usefully applied because the relationship between the lengths of the sides in any right triangle is consistent. For example, in a baseball field, if the distance between each base is known, then the shortest distance to throw the ball from first base to third base can be calculated using the Pythagorean Theorem. When purchasing a television, the size advertised refers to the length of the diagonal of the television. If an old TV was being sold, the lengths of two of its sides can be used to calculate the length of the diagonal and thereby determine the size of the TV.


## 1. Introduction

What is the Importance of pythagorean triples?
Why do we need the Pythagorean Theorem?
How is the Pythagorean theorem used today?
The Pythagorean theorem is used any time we have a right triangle, we know the length of two sides, and we want to find the third side.

Furthermore, the Pythagorean theorem is commonly used in advanced math today. It is used in computing surface areas, volumes and perimeters of different geometric shapes, converting between polar and rectangular coordinates and computing the distance between particular points on a plane.

The Pythagorean Theorem can be used in any real life scenario that involves a right triangle having two sides with known lengths. In a scenario where a certain section of a wall needs to be painted, the Pythagorean Theorem can be used to calculate the length of the ladder needed if the height of the wall and the distance of the base of the ladder from the wall are known.

The Pythagorean Theorem can be usefully applied because the relationship between the lengths of the sides in any right triangle is consistent. For example, in a baseball field, if the distance between each base is known, then the shortest distance to throw the ball from first base to third base can be calculated using the Pythagorean Theorem. When purchasing a television, the size advertised refers to the length of the diagonal of the television. If an old TV was being sold, the lengths of two of its sides can be used to calculate the length of the diagonal and thereby determine the size of the TV.

Computer monitor sizes are determined the same way. Suitcase sizes are given in terms of the diagonal and the height, using which the length of the suitcase can be calculated using the Pythagorean Theorem.

The Pythagorean theorem is used today in construction and various other professions and in numerous day-today activities. In construction, this theorem is one of the methods builders use to lay the foundation for the corners of a building. This special triangle is useful when they do not have a carpenter's square, which is a tool for constructing right angles.

The Pythagorean theorem is applicable any time there is a right triangle. When a person knows the length of two sides of a triangle and wants to find the third side, this theorem is used. For example, a person sees an entertainment set at a furniture store and does not have the time to go home and measure his TV set. He knows the measurement of his TV screen and thus calculates the diagonal measurement of the TV space in the entertainment
set using the Pythagorean theorem. Through this simple method, he is able to determine whether his TV set fits or not.

Furthermore, the Pythagorean theorem is commonly used in advanced math today. It is used in computing surface areas, volumes and perimeters of different geometric shapes, converting between polar and rectangular coordinates and computing the distance between particular points on a plane. The distance formula is one of its most frequent applications.

Professionals that typically use the Pythagorean theorem include computer and mathematical experts, engineers, architects, surveyors, cartographers, carpenters, construction and building inspectors, electricians, glaziers, electrical installers, machinists, and managers in the construction and business industries.

## How to Use Trigonometry in Architecture

The Pythagorean Theorem is a statement in geometry that shows the relationship between the lengths of the sides of a right triangle - a triangle with one 90 -degree angle. The right triangle equation is $\mathrm{a} 2+\mathrm{b} 2=\mathrm{c} 2$. Being able to find the length of a side, given the lengths of the two other sides makes the Pythagorean Theorem a useful technique for construction and navigation.

## Architecture and Construction

Given two straight lines, the Pythagorean Theorem allows you to calculate the length of the diagonal connecting them. This application is frequently used in architecture, woodworking, or other physical construction projects. For instance, say you are building a sloped roof.

If you know the height of the roof and the length for it to cover, you can use the Pythagorean Theorem to find the diagonal length of the roof's slope. You can use this information to cut properly sized beams to support the roof, or calculate the area of the roof that you would need to shingle.

## Laying Out Square Angles

The Pythagorean Theorem is also used in construction to make sure buildings are square. A triangle whose side lengths correspond with the Pythagorean Theorem - such as a 3 foot by 4 foot by 5 foot triangle - will always be a right triangle. When laying out a foundation, or constructing a square corner between two walls, construction workers will set out a triangle from three strings that correspond with these lengths. If the string lengths were measured correctly, the corner opposite the triangle's hypotenuse will be a right angle, so the builders will know they are constructing their walls or foundations on the right lines.

## Navigation

The Pythagorean Theorem is useful for two-dimensional navigation. You can use it and two lengths to find the shortest distance. For instance, if you are at sea and navigating to a point that is 300 miles north and 400 miles west, you can use the theorem to find the distance from your ship to that point and calculate how many degrees to the west of north you would need to follow to reach that point. The distances north and west will be the two legs of the triangle, and the shortest line connecting them will be the diagonal. The same principles can be used for air navigation. For instance, a plane can use its height above the ground and its distance from the destination airport to find the correct place to begin a descent to that airport.

## Surveying

Surveying is the process by which cartographers calculate the numerical distances and heights between different points before creating a map. Because terrain is often uneven, surveyors must find ways to take measurements of distance in a systematic way. The Pythagorean Theorem is used to calculate the steepness of slopes of hills or mountains. A surveyor looks through a telescope toward a measuring stick a fixed distance away, so that the telescope's line of sight and the measuring stick form a right angle. Since the surveyor knows both the height of the measuring stick and the horizontal distance of the stick from the telescope, he can then use the theorem to find the length of the slope that covers that distance, and from that length, determine how steep it is.

Pythagoras is also credited with the discovery that the intervals between harmonious musical notes always have whole number ratios. For instance, playing half a length of a guitar string gives the same note as the open string, but an octave higher; a third of a length gives a different but harmonious note; etc. Non-whole number ratios, on the other hand, tend to give dissonant sounds. In this way, Pythagoras described the first four overtones which create the common intervals which have become the primary building blocks of musical harmony: the octave ( $1: 1$ ), the perfect fifth (3:2), the perfect fourth ( $4: 3$ ) and the major third ( $5: 4$ ). The oldest way of tuning the 12-note chromatic scale is known as Pythagorean tuning, and it is based on a stack of perfect fifths, each tuned in the ratio 3:2.

The mystical Pythagoras was so excited by this discovery that he became convinced that the whole universe was based on numbers, and that the planets and stars moved according to mathematical equations, which corresponded to musical notes, and thus produced a kind of symphony, the "Musical Universalis" or "Music of the Spheres".

In a 45-45 degree right triangle we can get the length of the hypotenuse by multiplying the length of one leg by root 2 .

In a 30-60 degree right triangle the opposite side of the 30 degree angle is half of the hypotenuse.

## Instruments

The instruments used were Laptop, Calculator, Papers and Pencils.
Keywords
Triples, Pythagoras, Generate, Classification,

## 2. Methods \& Methodology

In the first place the data about Pythagorean triples was collected from a single website.
Then the multiples of the triples were left out; the rest of the triples were the basic Pythagorean triples.
Then based on the value of ' $a$ ' segregated into two groups that is odd and even sections.
Then based on the difference between $c$ and $b$ they were further segregated into different columns from A to Q .
The triples were arranged in the columns in the increasing value of ' $a$ '.
This was how the classification of all the basic triples was done.
To prove the theorem X
If in a right angled triangle $a, b, c$ are the three sides such that $a<b<c$. Then every integer ' $a$ ' has a set of integers $b$ and $c$ which are the other two sides of the right angled triangle and follow the Pythagoras theorem.

For an odd number ' $a$ ' $a=2 n+1 ; b=2 n^{2}+2 n ; c=2 n^{2}+2 n+1$
For an even number ' $a$ ' $a=2 n ; b=n^{2}-1 ; c=n^{2}+1$
Which follow the Pythagoras theorem $\mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{c}^{2}$

## PROOF:-

For an odd number let $\mathrm{a}=2 \mathrm{n}+1 ; \mathrm{b}=2 \mathrm{n}^{2}+2 \mathrm{n} ; \mathrm{c}=2 \mathrm{n}^{2}+2 \mathrm{n}+1$
Using Pythagoras theorem $\mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{c}^{2}$, we get $(2 \mathrm{n}+1)^{2}+\left(2 \mathrm{n}^{2}+2 \mathrm{n}\right)^{2}=\left(2 n^{2}+2 \mathrm{n}+1\right)^{2}$
Expanding this Equation we get $\left(4 n^{2}+4 n+1\right)+\left(4 n^{4}+4 n^{2}+8 n^{3}\right)=\left(4 n^{4}+8 n^{2}+8 n^{3}+4 n+1\right)$
$4 n^{2}+4 n+1+4 n^{4}+4 n^{3}+4 n^{3}+4 n^{2}=4 n^{4}+8 n^{2}+8 n^{3}+4 n+1$
$4 n^{4}+8 n^{3}+8 n^{2}+4 n+1=4 n^{4}+8 n^{3}+8 n^{2}+4 n+1$
LHS=RHS
Now replacing $\mathrm{n}+1$ for n we get
$\left.(2(\mathrm{n}+1)+1)^{2}+\left(2(\mathrm{n}+1)^{2}+2(\mathrm{n}+1)\right)^{2}=(2(\mathrm{n}+1))^{2}+2(\mathrm{n}+1)+1\right)^{2}$
$(2 \mathrm{n}+3)^{2}+\left(2\left(\mathrm{n}^{2}+2 \mathrm{n}+1\right)+2 \mathrm{n}+2\right)^{2}=\left(2\left(\mathrm{n}^{2}+2 \mathrm{n}+1\right)+2 \mathrm{n}+3\right)^{2}$
$4 n^{2}+12 n+9+\left(2 n^{2}+6 n+4\right)^{2}=\left(2 n^{2}+6 n+4\right)^{2}$
$4 n^{2}+12 n+9+4 n^{4}+52 n^{2}+16+24 n^{3}+48 n+16=4 n^{4}+24 n^{3}+56 n^{2}+60 n+25$
$4 n^{4}+24 n^{3}+56 n^{2}+60 n+25=4 n^{4}+24 n^{3}+56 n^{2}+60 n+25$
LHS=RHS
Hence proved by method of mathematical induction
For an even number let $a=2 n ; b=n^{2}-1 ; c=n^{2}+1$

Substituting in Pythagoras Theorem;
We get $(2 n)^{2}+\left(n^{2}-1\right)^{2}=\left(n^{2}+1\right)^{2}$
$4 \mathrm{n}+\mathrm{n}^{4}+1-2 \mathrm{n}^{2}=\mathrm{n}^{4}+1+2 \mathrm{n}^{2}$
$n^{4}+2 n^{2}+1=n^{4}+2 n^{2}+1$
LHS $=$ RHS
Now replacing $\mathrm{n}+1$ for n we get

$$
\begin{aligned}
& (2(n+1))^{2}+\left((n+1)^{2}-1\right)^{2}=\left((n+1)^{2}+1\right)^{2} \\
& 4 n^{2}+8 n+4+n^{4}+4 n^{3}+4 n=n^{4}+4 n^{3}+8 n^{2}+8 n+4 \\
& 4 n^{2}+8 n+4+n^{4}+4 n^{3}+4 n^{2}=n^{4}+4 n^{3}+8 n^{2}+8 n+4 \\
& n^{4}+4 n^{3}+8 n^{2}+8 n+4=n^{4}+4 n^{3}+8 n^{2}+8 n+4 \\
& \text { LHS }=\text { RHS }
\end{aligned}
$$

Hence proved by method of mathematical induction.
Since any number, be it odd or even possesses two other numbers to form a triple.
So every number ' $a$ ' with a certain $b$ and c forms a Pythagoras triple.

A little proof for Fermat's last theorem
In number theory, Fermat's Theorem states that "no three positive integers $a, b$ and $c$ satisfy the equation $a^{n}+b^{n}=c^{n}$ for any integer value of $n$ greater than $2 . "$

Since by proof of mathematical induction of the theorem X every number ' $a$ ' has a triple means that every number ' $a$ ' whether odd or even has a set of numbers $b$ and $c$ which follows pythagoras theorem such that $\mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{c}^{2}$

Let's take $\mathrm{n}=\mathrm{m}+2$
Where in we assume that $\mathrm{n}>2$
So $\mathrm{a}^{\mathrm{n}}+\mathrm{b}^{\mathrm{n}}=\mathrm{c}^{\mathrm{n}}$
Taking $b^{n}$ to the other side $a^{n}=c^{n}-b^{n}$
further we split as
$\mathrm{a}^{\mathrm{m}+2}=\mathrm{c}^{\mathrm{m}+2}-\mathrm{b}^{\mathrm{m}+2}$
$\mathrm{a}^{\mathrm{m}} \mathrm{X} \mathrm{a}^{2}=\mathrm{c}^{\mathrm{m}+2}-\mathrm{b}^{\mathrm{m}+2}$
$a^{2}=(c / a)^{m}-(b / a)^{m}$
$(\mathrm{c} / \mathrm{a})^{\mathrm{m}}=(\mathrm{b} / \mathrm{a})^{\mathrm{m}}=1$
Since if $x^{y}=0$ then $y=0$
Hence $m=0$
And we have taken $\mathrm{n}=\mathrm{m}+2$
And $n$ can't be more than 2 since $m=0$
Or
If $\mathrm{c} / \mathrm{a}=\mathrm{b} / \mathrm{a}$ and therefore $\mathrm{c}=\mathrm{b}$
Which means there are only two integers involved either ' $a$ ' and ' $c$ ' or ' $a$ ' and ' $b$ ' and this result contradicts Fermat's Theorem that there should be three positive integers.

We arrive at Reductio ad absurdum.
Hence proved.
And n can't be more than 2 .
How to generate Pythagoras triples?
By Pythagoras theorem $\mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{c}^{2}$
Taking $\mathrm{b}^{2}$ to the other side we get
$a^{2}=c^{2}-b^{2}$
$\mathrm{a}^{2}=(\mathrm{c}+\mathrm{b})(\mathrm{c}-\mathrm{b})$
Let's introduce a new variable ${ }^{\text {' } d \text { ' where }} \mathrm{c}-\mathrm{b}=\mathrm{d}$
This c-b, the difference between c and b which is our (d) plays an important role in rolling out all the triples.
$\mathrm{a}^{2} / \mathrm{d}=\mathrm{c}+\mathrm{b}$
It follows that $a^{2} / d=2 b+d$ and $a^{2} / d=2 c-d$
Hence $b=\left(\left(a^{2} / d\right)-d\right) / 2$ and $c=\left(\left(a^{2} / d\right)+d\right) / 2$
So for the integer values of $b$ and $c, d$ has to be a divisor of $a^{2}$.
Take any number as ' $a$ ', square it and divide it by the corresponding ' $d$ ' which should be a perfect divisor or factor of $a^{2}$ then the product should be split into two parts with a difference of ' $d$ ' to give us the respective ' $b$ ' and ' $c$ '. Now $a, b$ and $c$ form a Pythagoras triple. Find the factors of ' $a$ ' like we do to find LCM. Now take odd factors
if ' $a$ ' is odd and even factors if ' $a$ ' is even and square them to get a set of ' $d$ 's. Divide $a^{2}$ by each of the d's. The product is the sum of $c \& b$ split this sum to two numbers with a difference of ' $d$ ' to get $b \& c$ of the triple.

The number of triples we can generate for a given single ' $a$ ' depends on the number of factors.
The triples are a function of the prime factors of the a's.
Observe the tables below. Let's call all the triples a,b,c.
Let's divide all the triples into odd and even sections.
The top rank is a series of odd no's the second rank are the squares and they are the difference between ' $b$ ' and ' $c$ ' which has to be maintained for each and every column.

Odd section
The a's in the consecutive columns A,B,C etc; are multiples / tables of 1,3,5,7,9 and so on respectively, in the odd section.

In the first column on the odd side all a's are odd. Now if you square this ' $a$ ' and split the product with a difference of one we get the first triple. That is $3,4,5$. This goes on with all the odd no's.

In the second column all the a's are multiples of 3 that is a three table. You have to divide this ' $a$ ' by 3 and square the product and split such that the difference is 9 you have all triples. 9 is the square of 3 .

The third is a 5 table and you can repeat the same to produce triples in third column which consist of a's which are multiples of 5 but the difference between ' $b$ ' and ' $c$ ' should be 25 .

If you observe the next column, the a's are multiples of 7 and after repeating the procedure of dividing the a's by 7 and squaring the product, the product should be split with a difference of 49 . For example 119 is divided by 7 to get 17 , squaring this gives 289 you split this 289 with a difference of 49 you get 120 and 169 .Now 119,120 and 169 form a triple.

The next columns would be $9,11,13$ and so on such that the difference between ' $b$ ' and' $c$ ' would be 81,121 ,169 respectively

This procedure follows for all odd numbers.
The difference between the ' $a$ 's in the columns is observed to be $2,6,10,14,18,22$ respectively and so on.

## Even section

The a's in the consecutive columns $\mathrm{A}, \mathrm{B}, \mathrm{C}$ etc; are multiples / tables of $2,4,6,8,1012$ and so on respectively, in the even section.

The vertical difference between the 'a's in the columns is observed to be $4,8,12,16,20,24$ respectively and so on.

The procedure to generate the tripes $\mathrm{a}, \mathrm{b}, \mathrm{c}$ on the even side is the same as on the odd section.
In the first column all the a's are multiples of 2 that is a two table. You have to divide this 'a' by root 2 and square the product and split such that the difference is 2 you have all triples. 2 is the square of 1 root 2 .

The second is a 4 table and you can repeat the same to produce triples in second column which consist of a's of multiples of 4 but the difference between ' $b$ ' and ' $c$ ' should be 8 which is the square of 2 root 2 .

If you observe the next column, the a's are multiples of 6 and after repeating the procedure of dividing the a's by (3root2) and squaring the product, the product should be split with a difference of 18 . For example 48 is divided by 3 root 2 to get 8 root 2 squaring this gives 128 you split this 128 with a difference of 18 you get 55 and 73.Now $48,55,73$ form a triple.

The next columns would be $(4 \text { root } 2)^{2}$, $(5 \text { root } 2)^{2},(6 \text { root } 2)^{2}$ and so on such that the difference between ' $b$ ' and 'c' would be $32,50,72,98$ respectively

This procedure follows for all even numbers.
The difference between the ' $a$ 's in the columns is observed to be $4,8,12,16,20,24$ respectively and so on.
The other triples can be got as multiples of these basic triples for example $6,8,10 ; 9,12,15$ are the multiples of $3,4,5$ by 2,3 respectively and so on and it goes on for all triples covering all the numbers.

All even numbers ' $a$ ' will have a set of numbers $b$ and $c$ with a difference (d) of (sqrt2) ${ }^{2}, \quad(2 \text { sqrt } 2)^{2},(3$ sqrt $2)^{2}$ ie; $2,8,18,32,50,72,98$ and it goes on. If ' $a$ ' is an odd number then the difference (d) between the set of numbers $b$ and $c$ will be $1^{2}, 3^{2}, 5^{2}, 7^{2}, 9^{2}, 11^{2}$ ie, $1,9,25,49,81,121$ and it goes on.

The series of differences (d) of the columns A,B,C etc; in the even section ie; 2,8,18,32,50,72,98 is similar and akin to the capacities of the shells of the electrons K,L,M,N,O in chemistry.

This is a significant finding correlating mathematics and chemistry.
For example take ' $a$ ' as 75 , an odd number. The factors are $1,3,5,15,25$ and 75 . In another way it is $1 \times 75,3 \times 25$ and $5 \times 15$.

Squaring 75 gives 5625 . Split this product into two parts with a difference of $1^{2}$ that is 1 . We get 2812 and 2813. Now 75, 2812 and 2813 form a triple.

Take this 5625 further, divide by $3^{2}$ i.e; 9 or square 25 to get 625 and split into two parts with a difference of $3^{2}$ that is 9 . We get 308 and 317 . Now 75, 308 and 317 form a triple.

Take this 5625 further, divide by 25 or square 15 to get 225 and split into two parts with a difference of $5^{2}$ that is 25 . We get 100 and 125 . Now 75,100 and 125 form a triple. This is also a multiple of the basic triple $3,4,5$ by 25.

Revert and take a step further, divide 5625 by 15 to get 375 , split this into two parts such that the difference is 15 ( $375-15$ yields $360,360 / 2$ gives 180 , plus 15 (the ' $d$ ') gets you to 195) we get 180 and 195 or look for a basic triple with ' $a$ ' value 5 that is $5,12,13$ and multiply by 15 to get the triple 75,180 and 195 .

5625 divided by 5 gives 1125 . Split this 1125 into two with a difference of 5 to arrive at 560 and 565 or look at a basic triple $15,112,113$ multiplied by 5 gives $75,560,565$.

5625 divided by 3 produces 1875. Split this product into two parts with a difference of 3 or A basic triple 25, 312,313 multiplied by 3 gives $75,936,939$.

A prime number has no factors and hence its entry in the ' $a$ 's place in the entire table is only once, that of the first column of the odd section.

## 3. Results \& Analysis

All the a's in all the columns A to Q were odd numbers in the odd section
It was observed that all the b's were even numbers and all the $c$ 's in the columns were odd numbers.
In the similar way, all the a's in all the columns A to Q were even numbers in the even section.
It was observed that all the b's were odd numbers and all the c's in the columns were odd numbers

## References

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$\frac{\frac{\text { CLASSIFICATION OF }}{\text { PYTHAGORAS TRIPLES }}}{\underline{\text { ODD }}} \underset{\underline{\text { SECTI }}}{\underline{\text { ON }}}$

| Column | A | B | C | D | E | F | G | H | I | J | K |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMB ER 'p' | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 35 |
| multiply <br> $n$ to get | 3 | 9 | 13 | 17 | 21 | 27 | 31 | 37 | 43 | 47 | 53 | 57 |  |





