Turkish Journal of Computer and Mathematics Education Vol.12 No. 9 (2021), 3537-3544

Research Article

Nonsplit Neighbourhood Tree Domination Number In Connected Graphs

S. Muthammai[1], C. Chitiravalli[2]
[1]Principal (Retired),
Alagappa Government Arts College, Karaikudi – 630003, Tamilnadu, India.
[2]Research scholar,
Government Arts College for Women(Autonomous), Pudukkottai – 622001, Tamilnadu, India.
[1]Email: muthammai.sivakami@gmail.com, [2]Email: chithu196@gmail.com

Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 20 April 2021

Abstract: Let G = (V, E) be a connected graph. A subset D of V is called a dominating set of G if N[D] = V. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set D of a graph G is called a tree dominating set (tr - set) if the induced subgraph $\langle D\rangle$ is a tree. The tree domination number $\gamma_{tr}(G)$ of G is the minimum cardinality of a tree dominating set. A tree dominating set D of a graph G is called a neighbourhood tree dominating set (ntr - set) if the induced subgraph $\langle N(D)\rangle$ is a tree. The neighbourhood tree domination number $\gamma_{ntr}(G)$ of G is the minimum cardinality of a neighbourhood tree dominating set. A tree dominating set D of a graph G is called a nonsplit tree dominating set (nstd - set) if the induced subgraph $\langle V – D \rangle$ is connected. The nonsplit tree domination number $\gamma_{nstd}(G)$ of G is the minimum cardinality of a nonsplit tree dominating set. A neighbourhood tree dominating set D of G is called a nonsplit neighbourhood tree dominating set, if the induced subgraph $\langle V(G) – D\rangle$ is connected. The nonsplit neighbourhood tree domination number $\gamma_{nsntr}(G)$ of G is the minimum cardinality of a nonsplit neighbourhood tree dominating set of G. In this paper, bounds for $\gamma_{nsntr}(G)$ and its exact values for some particular classes of graphs and cartesian product of some standard graphs are found.

Keywords: Domination number, connected domination number, tree domination number, neighbourhood tree domination number, nonsplit domination number.

Mathematics Subject Classification: 05C69

1. INTRODUCTION

The graphs considered here are nontrivial, finite and undirected. The order and size of G are denoted by n and m respectively. If $D \subseteq V$, then $N(D) = \bigcup_{v\in D} N(v)$ and $N[D] = N(D) \cup D$ where N(v) is the set of vertices of G which are adjacent to v. The concept of domination in graphs was introduced by Ore[13]. A subset D of V is called a dominating set of G if N[D] = V. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. Xuegang Chen, Liang Sun and Alice McRac [14] introduced the concept of tree domination in graphs. A dominating set D of G is called a tree dominating set, if the induced subgraph $\langle D\rangle$ is a tree. The minimum cardinality of a tree dominating set of G is called the tree domination number of G and is denoted by $\gamma_{tr}(G)$. Kulli and Janakiram [8, 9] introduced the concept of split and nonsplit domination in graphs.

A dominating set D of a graph G is called a nonsplit dominating set if the induced subgraph $\langle V – D\rangle$ is connected. The nonsplit domination number $\gamma_{nsd}(G)$ of G is the minimum cardinality of a nonsplit dominating set. Muthammai and Chitiravalli [11, 12] defined the concept of split and nonsplit tree domination in graphs. A tree dominating set D of a graph G is called a nonsplit tree dominating set if the induced subgraph $\langle V – D\rangle$ is connected. The nonsplit tree domination number $\gamma_{nstd}(G)$ of G is the minimum cardinality of a nonsplit tree idominating set.

V.R. Kulli introduced the concepts of split and nonsplit neighbourhood connected domination in graph. A neighbourhood dominating set D of a graph G is called a nonsplit neighbourhood dominating set if the induced subgraph $\langle V – D\rangle$ is connected. The nonsplit neighbourhood domination number $\gamma_{nsntd}(G)$ of G is the minimum cardinality of a nonsplit neighbourhood dominating set.

The Cartesian product of two graphs G_1 and G_2 is the graph, denoted by $G_1 \times G_2$ with $V(G_1 \times G_2) = V(G_1)\times V(G_2)$ (where x denotes the Cartesian product of sets) and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V(G_1 \times G_2)$ are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1$ and $(u_2, v_2) \in E(G_2)]$ or $[u_2 = v_2$ and $(u_1, v_1) \in E(G_1)]$.

In this paper, bounds for $\gamma_{nsntr}(G)$ and its exact values for some particular classes of graphs and cartesian product of some standard graphs are found.

2. PRIOR RESULTS

Theorem 2.1: [2] For any graph G, $\kappa(G) \le \delta(G)$.
Theorem 2.2: [14] For any connected graph G with $n \ge 3$, $\gamma_{tr}(G) \le n - 2$.
Theorem 2.3: [14] For any connected graph G with $\gamma_{tr}(G) = n - 2$ iff $G \cong P_n$ (or) C_n.
Theorem 2.4: [11] For any connected graph G, $\gamma(G) \le \gamma_{nstd}(G)$.
Theorem 2.5: [11] For any connected graph G with n vertices, $\gamma_{nstd}(G) = 1$if and only if $G \cong H+K_1$, where H is a connected graph with $(n - 1)$ vertices.
Theorem 2.6: [11] For any graph G, $\gamma(G) \le \gamma_{ns}(G) \le \gamma_{nstd}(G)$.
Theorem 2.7: [11] For any cycle C_n on n vertices, $\gamma_{nstd}(C_n) = n - 2$, $n \ge 3$.
Theorem 2.8: [9] For any connected graph G, $\gamma_{ns}(G) \le p - 1$. Further equality holds if and only if G is a star.

3. MAIN RESULTS

In this section, nonsplit neighbourhood tree domination number is defined and studied.

3.1. Nonsplit Neighbourhood Tree Domination Number in Connected Graphs

Definition 3.1.1:

A neighbourhood tree dominating set D of G is called a nonsplit neighbourhood tree dominating set, if the induced subgraph $\langle V(G) - D\rangle$ is connected. The nonsplit neighbourhood tree domination number $\gamma_{nsntr}(G)$ of G is the minimum cardinality of a nonsplit neighbourhood tree dominating set of G.

Not all connected graphs have a nonsplit neighbourhood tree dominating set. For example, the Path $P_n(n > 5)$ has a neighbourhood tree dominating set, but no nonsplit neighbourhood tree dominating set.

If the nonsplit neighbourhood tree domination number does not exist for a given connected graph G, then $\gamma_{nsntr}(G)$ is defined to be zero.

Example 3.1.1:

G:

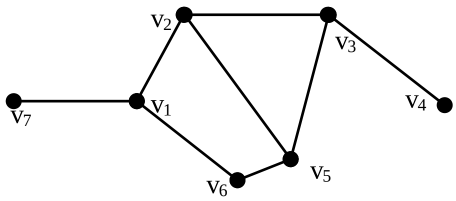


Figure 3.1

In the graph given in Figure 3.1, $D = \{v_4, v_5, v_7\}$ is a minimum nonsplit neighbourhood tree dominating set and the induced subgraph $\langle N(D)\rangle \cong P_4 = \langle\{v_3, v_2, v_6, v_1\}\rangle$ is a tree and $\langle V(G) - D\rangle$ is connected and $\gamma_{nsntr}(G) = 3$.

Remark 3.1.1:

Since $\langle V(G) - D\rangle$ is connected for any γ_{nsntr} - set D of a connected graph G, $|V(G) - D| \ge 1$.

Example 3.1.2

G:

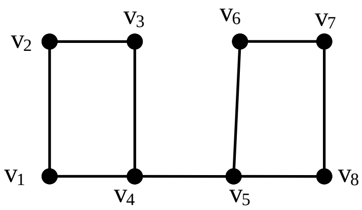


Figure 3.2

In the graph G given in Figure 3.2, $D = \{v_2, v_4, v_5, v_7\}$ is a minimum dominating set and the induced subgraph $\langle N(D)\rangle$ is a tree, but $\langle V(G) - D\rangle$ is disconnected.

Remark 3.1.2:

Every nonsplit neighbourhood tree dominating set is a dominating set and also a neighbourhood tree dominating set. Therefore, $\gamma(G) \le \gamma_{ntr}(G) \le \gamma_{nsntr}(G)$. Therefore, for any nontrivial connected graph G, $\gamma_{ntr}(G) = \min\{\gamma_{sntr}(G), \gamma_{nsntr}(G)\}$.

These are illustrated below.

Example 3.1.3:

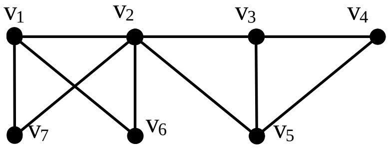


Figure 3.3(a)

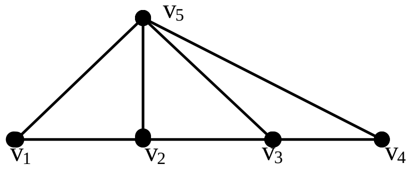


Figure 3.3(b)

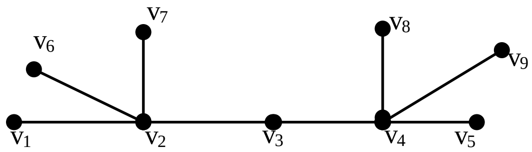


Figure 3.3(c)

In Figure 3.3(a), $D_1 = \{v_1, v_5\}$ is a minimum nonsplit neighbourhood tree dominating set. $V - D_1 = \{v_2, v_3, v_4, v_6, v_7\}$ and $\gamma(G) = \gamma_{nsntr}(G) = 2$.

In Figure 3.3(b), $D_2 = \{v_1\}$ is a minimum nonsplit neighbourhood tree dominating set. $V - D_2 = \{v_1, v_2, v_3, v_4\}$ and $\gamma(G) = \gamma_{ntr}(G) = \gamma_{nsntr}(G) = 1$.

In Figure 3.3(c), $D_3 = \{v_2, v_3, v_4\}$ is a minimum nonsplit neighbourhood tree dominating set. $V - D_3 = \{v_1, v_5, v_6, v_7, v_8, v_9\}$ and $\gamma(G) = 2$, $\gamma_{tr}(G) = 3$, $\gamma_{ntr}(G) = 3$, $\gamma_{nsntr}(G) = 7$. Here, $\gamma(G) < \gamma_{ntr}(G)$, $\gamma(G) < \gamma_{nsntr}(G)$, $\gamma_{ntr}(G) < \gamma_{nsntr}(G)$.

Example 3.1.4:

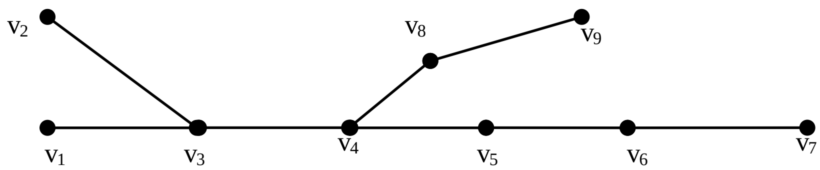


Figure 3.4

In Figure 3.4, $D_1 = \{v_3, v_4, v_7, v_9\}$ is a neighbourhood tree dominating set. $V - D_1 = \{v_1, v_2, v_5, v_6, v_8\}$, $D_2 = \{v_1, v_2, v_5\ v_6, v_7, v_8, v_9\}$ is a nonsplit neighbourhood tree dominating set and $V - D_2 = \{v_3, v_4\}$, $\gamma(G) = 3$, $\gamma_{ntr}(G) = 4$, $\gamma_{sntr}(G) = 4$, $\gamma_{nsntr}(G) = 7$. Therefore, $\gamma_{ntr}(G) = \min\{4, 7\} = 4$

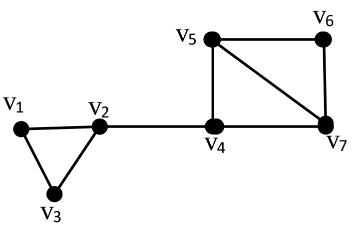
Remark 3.1.3:

If H is a spanning subgraph of a connected graph G, then $\gamma_{nsntr}(G) \leq \gamma_{nsntr}(H)$.

This is illustrated by following examples.

Example 3.1.5:

G:



H:

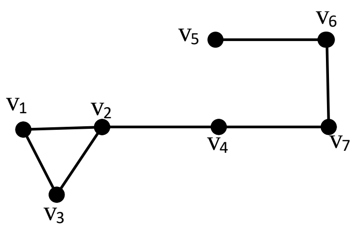
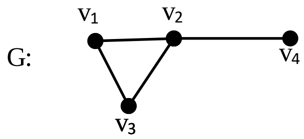


Figure 3.5

In Figure 3.5, H is a spanning subgraph of G. $D_1 = \{v_3, v_5\}$ is a minimum nonsplit neighbourhood tree dominating set of G and $\gamma_{nsntr}(G) = 2$. The set $D_2 = \{v_3, v_5, v_6, v_7\}$ is a nonsplit neighbourhood tree dominating set of H and $\gamma_{nsntr}(H) = 4$.

Therefore, $\gamma_{nsntr}(G) < \gamma_{nsntr}(H)$.

Example 3.1.6:



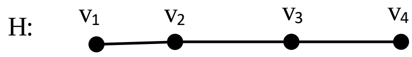


Figure 3.6

In Figure 3.6., H is a spanning subgraph of G and $\{v_3, v_4\}$ is a minimum nonsplit neighbourhood tree dominating set of G, $\gamma_{nsntr}(G) = 2$. The set $\{v_1, v_4\}$ is a minimum nonsplit neighbourhood tree dominating set of H and $\gamma_{nsntr}(H) = 2$. Therefore, $\gamma_{nsntr}(G) = \gamma_{nsntr}(H)$.

In the following, the exact values of $\gamma_{sntr}(G)$ for some standard graphs are given.

(a) For any path P_n on n vertices, $\gamma_{nsntr}(P_n) = n - 2$, $n \geq 4$.
(b) If G is a spider, then $\gamma_{nsntr}(G) = n + 1$.
(c) If G is a wounded spider, then $\gamma_{nsntr}(G) = p + 1$, where p is the number of pendant vertices which are adjacent to nonwounded legs.
(d) For any triangular cactus graph T_p whose blocks are p triangles with $p \geq 1$, $\gamma_{nsntr}(T_p) = p$ where $p > 2$ and p is odd.
(e) If $S_{m,n}$, $(1 \leq m \leq n)$ is a double star, then $\gamma_{nsntr}(S_{m,n}) = m + n$.

Theorem 3.1.1:
If T is a tree which is not a star, then $\gamma_{nsntr}(T) \leq n - 2$.

Proof:
Suppose T is not a star. Then T has two adjacent cut vertices u and v, such that deg u, deg $v \geq 2$. This implies that $D = \{V - \{u, v\}\}$ is a nonsplit neighbourhood tree dominating set of T. Therefore, $\gamma_{nsntr}(T) \leq |D| = |V(T) - \{u, v\}| = n - 2$.

3.2. Nonsplit Neighbourhood Tree Domination Number of Cartesian product of Graphs

In this section, nonsplit neighbourhood tree domination numbers of $P_2 \times C_n$, $P_3 \times C_n$, $P_2 \times P_n$, $P_3 \times P_n$ are found.

Theorem 3.2.1:

For the graph $P_2 \times P_n$ ($n \geq 5$, n is odd), $\gamma_{nsntr}(P_2 \times P_n) = \left\lceil \frac{n}{2} \right\rceil$.

Proof:

Let $G \cong P_2 \times P_n$ and let $V(G) = \bigcup_{i=1}^{n} \{v_{i1}, v_{i2}\}$ where $\langle\{v_{i1}, v_{i2}\}\rangle \cong P_2^i$, i = 1, 2 and $\langle\{v_{1j}, v_{2j}, \ldots, v_{nj}\}\rangle \cong P_n^j$, j = 1, 2, ... ,n and P_2^i is the i^{th} copy of P_2 and P_n^j is the j^{th} copy of P_n in G.

Let $D = \bigcup_{i=1}^{\left\lfloor \frac{n-3}{4} \right\rfloor + 1} \{v_{4i-1,1}\} \cup \bigcup_{i=1}^{\left\lfloor \frac{n-1}{4} \right\rfloor + 1} \{v_{4i-3,2}\}$. Then $D \subseteq V(G)$. Here, v_{11} and v_{22} are adjacent to v_{12} and v_{n1} and $v_{n-1,2}$ are adjacent to v_{n2} and $v_{2i+1,2}$ is adjacent to $v_{2i+1,1}$ $(i \geq 1)$.

Therefore, D is a dominating set of G and $\langle N(D)\rangle \cong P_{\frac{3n-1}{2}}$. Since $\langle N(D)\rangle$ is a tree and $\langle V(G) - D\rangle$ is connected, D is a nonsplit neighbourhood tree dominating set of G and is minimum.

Hence $\gamma_{nsntr}(G) = |D| = \left\lceil \frac{n}{2} \right\rceil$.

Remark 3.2.1:

$\gamma_{nsntr}(P_2 \times P_3) = 2$, the set $\{v_{31}, v_{12}\}$ is a minimum nonsplit neighbourhood tree dominating set of $P_2 \times P_n$, where v_{21}, v_{22} are the vertices of degree 3 in $P_2 \times P_3$.

Example 3.2.1:

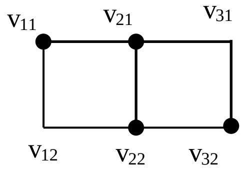


Figure 3.7

In the graph $P_2 \times P_3$ given in Figure 3.7, minimum nonsplit neighbourhood tree dominating set is D = $\{v_{11}, v_{32}\}$, where $\langle N(D)\rangle \cong P_4$ and $\gamma_{nsntr}(P_2 \times P_3) = 2$.

Theorem 3.2.2:

For the graph $P_3 \times P_n$ ($n \geq 3$), $\gamma_{nsntr}(P_3 \times P_n) = n$.

Proof:

Let $G \cong P_3 \times P_n$ and let $V(G) = \bigcup_{i=1}^{n}\{v_{i1}, v_{i2}, v_{i3}\}$ where $\langle\{v_{i1}, v_{i2}, v_{i3}\}\rangle \cong P_3^i$, i = 1, 2, 3 and $\langle\{v_{1j}, v_{2j}, \ldots, v_{nj}\}\rangle \cong P_n^j$, j = 1, 2, ... , n and P_3^i is the i^{th} copy of P_3 and P_n^j is the j^{th} copy of P_n in G.

Let $D = \bigcup_{i=1}^{\left\lceil \frac{n-1}{2} \right\rceil}\{v_{2i,3}\} \cup \bigcup_{i=1}^{\left\lceil \frac{n-2}{2} \right\rceil+1}\{v_{2i-1,1}\}$. Then $D \subseteq V(G)$. Here, $v_{2i,2}$ is adjacent to $v_{2i,3}$ ($i \geq 1$) and $v_{2i-1,2}$ is adjacent to $v_{2i-1,1}$ ($i \geq 1$). Therefore, D is a dominating set of G and $N(D)\rangle \cong P_n \circ P_1$. Since $\langle N(D)\rangle$ is a tree and $\langle V - D\rangle$ is connected, D is a nonsplit neighbourhood tree dominating set of G and is minimum.

Hence $\gamma_{nsntr}(G) = |D| = n$.

Example 3.2.2:

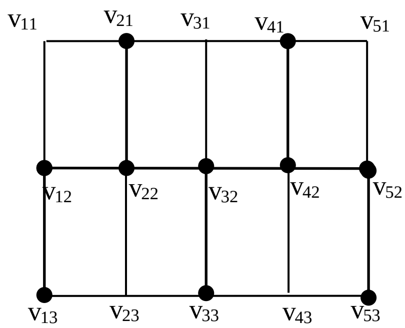


Figure 3.8

In the graph $P_3 \times P_5$ given in Figure 3.8, minimum nonsplit neighbourhood tree dominating set is D = $\{v_{12}, v_{22}, v_{32}, v_{42}, v_{52}\}$, where $\langle N(D)\rangle \cong P_5 \circ P_1$, and $\gamma_{nsntr}(P_3 \times P_5) = 5$.

Theorem 3.2.3:

For the graph $P_2 \times C_n$ ($n = 3$), $\gamma_{nsntr}(P_2 \times C_n) = 2$.

Proof:

Let $G \cong P_2 \times C_n$ and let $V(G) = \bigcup_{i=1}^{n}\{v_{i1}, v_{i2}\}$, where $\langle\{v_{i1}, v_{i2}\}\rangle \cong P_2^i$, i = 1, 2 and $\langle\{v_{1j}, v_{2j}, \ldots, v_{nj}\}\rangle \cong C_n^j$, j = 1, 2, ... ,n and P_2^i is the i^{th} copy of P_2 and C_n^j is the j^{th} copy of C_n in G.

Let D = $\{v_{31}, v_{2,2}\}$. Then $D \subseteq V(G)$. Here, v_{11}, v_{21} are adjacent to v_{31} and v_{12}, v_{32} are adjacent to $v_{2,2}$. Therefore, D is a dominating set of G and $\langle N(D)\rangle \cong P_4$. Since $\langle N(D)\rangle$ is a tree and $\langle V(G) - D\rangle$ is connected, D is a nonsplit neighbourhood tree dominating set of G and $\gamma_{ntr}(G) \leq |D| = 2$.

Let D′ be a nonsplit neighbourhood tree dominating set of $P_2 \times C_n$. Since $\gamma(P_3 \times C_3) = \left\lfloor \frac{3n}{2} \right\rfloor = 2$ and $\gamma_{ntr}(G) \geq \gamma(G)$ and $\gamma_{nsntr}(G) \geq \gamma_{ntr}(G)$. Therefore, $\gamma_{nsntr}(G) = 2$.

Example 3.2.3:

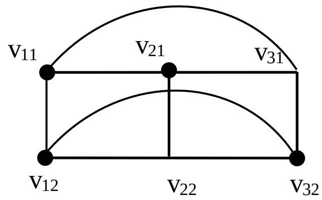


Figure 3.9

In the graph $P_2 \times C_3$ given in Figure 3.9, minimum nonsplit neighbourhood tree dominating set is D = $\{v_{31}, v_{22}\}$, where $\langle N(D)\rangle \cong P_4$ and $\gamma_{ntr}(P_2 \times C_3) = 2$.

Remark 3.2.2:

For $n \geq 4$, $\gamma_{nsntr}(P_2 \times C_n) = 0$, since there exists no nonsplit neighbourhood tree dominating set of $P_2 \times C_n$. Let D be a dominating set of $P_2 \times C_n$. If D contains two vertices, then either $\langle N(D)\rangle$ is not a tree or $\langle N(D)\rangle$ contains a cycle. If D contains atleast three vertices, then $\langle N(D)\rangle$ contains a cycle.

Theorem 3.2.4:

For the graph $P_3 \times C_n$ (n = 3), $\gamma_{nsntr}(P_3 \times C_n) = 3$.

Proof:

Let $G \cong P_3 \times C_n$, $n \geq 4$ and let $V(G) = \bigcup_{i=1}^{n}\{v_{i1}, v_{i2}, v_{i3}\}$ such that $\langle\{v_{i1}, v_{i2}, v_{i3}\}\rangle \cong P_3^i$, i = 1, 2, 3 and $\langle\{v_{1j}, v_{2j}, \ldots, v_{nj}\}\rangle \cong C_n^j$, j = 1, 2, ... ,n where P_3^i is the i[th] copy of P_3 and C_n^j is the j[th] copy of C_n in G.

Let D = $\{v_{31}, v_{12}, v_{33}\}$. Then $D \subseteq V(G)$. Here, v_{22} is adjacent to v_{12} and v_{11}, v_{21}, v_{32} are adjacent to v_{31} and v_{32}, v_{13}, v_{23} are adjacent to v_{33}. Therefore, D is a dominating set of G and $\langle N(D)\rangle$ is a connected graph obtained from P_5 by attaching a pendant edge at v_{22}. Since $\langle N(D)\rangle$ is a tree and $\langle V(G) - D\rangle$ is connected, D is a nonsplit neighbourhood tree dominating set of G and $\gamma_{nsntr}(G) \leq |D| = 3$.

Let D′ be a nonsplit neighbourhood tree dominating set of $P_3 \times C_n$. Since $\gamma(P_3 \times C_3) = \left\lceil \frac{3n}{4} \right\rceil = 3$ and $\gamma_{ntr}(G) \geq \gamma(G)$ and $\gamma_{nsntr}(G) \geq \gamma_{ntr}(G)$. Therefore, $\gamma_{ntr}(G) = 3$.

Example 3.2.4:

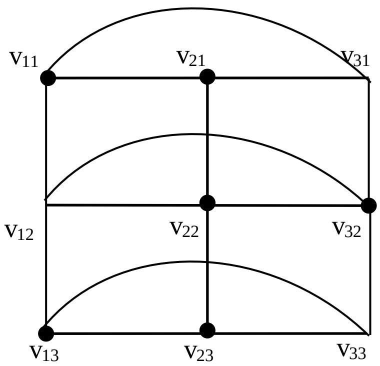


Figure 3.10

In the graph $P_3 \times C_3$ given in Figure 3.10, minimum nonsplit neighbourhood tree dominating set is D = $\{v_{21}, v_{32}, v_{13}\}$, where $N(D)\rangle \cong P_6$ and $\gamma_{nsntr}(P_3 \times C_3) = 3$.

Remark 3.2.3:

For $n \geq 4$, $\gamma_{std}(P_3 \times C_n) = 0$, since there exists no nonsplit neighbourhood tree dominating set of $P_3 \times C_n$. If a dominating set D of $P_3 \times C_n$ contains atleast three vertices, then the induced subgraph $\langle N(D)\rangle$ contains a cycle.

Theorem 3.2.5:

For the graph $P_4 \times C_n$ (n = 3), $\gamma_{nsntr}(P_4 \times C_n) = 4$.

Proof:

Let $G \cong P_4 \times C_n$, $n \geq 6$ and let $V(G) = \bigcup_{i=1}^{n} \{v_{i1}, v_{i2}, v_{i3}, v_{i4}\}$ such that $\langle\{v_{i1}, v_{i2}, v_{i3}, v_{i4}\}\rangle \cong P_4^i$, i = 1, 2, 3, 4 and $\langle\{v_{1j}, v_{2j}, \ldots, v_{nj}\}\rangle \cong C_n^j$, j = 1, 2, ... ,n, where P_4^i is the i^{th} copy of P_4 and C_n^j is the j^{th} copy of C_n in G.

Let D = $\{v_{31}, v_{22}, v_{13}, v_{34}\}$. Then $D \subseteq V(G)$. Here, v_{11}, v_{21}, v_{32} are adjacent to v_{31} and v_{12}, v_{23}, v_{33} are adjacent to v_{13} and and v_{14}, v_{24} are adjacent to v_{34}. Therefore, D is a dominating set of G and $\langle N(D)\rangle \cong P_8$. Since $\langle N(D)\rangle$ is a tree and $\langle V(G) - D\rangle$ is connected, D is a neighbourhood tree dominating set of G and $\gamma_{ntr}(G) \leq |D| =$ 4.

Let D′ be a nonsplit neighbourhood tree dominating set of $P_3 \times C_n$.

Since $\gamma(P_4 \times C_3) = \left\lceil \frac{3n}{4} \right\rceil + 1 = 4$ and $\gamma_{ntr}(G) \geq \gamma(G)$ and $\gamma_{nsntr}(G) \geq \gamma_{ntr}(G)$. Therefore, $\gamma_{ntr}(G) = 4$.

Example 3.2.5:

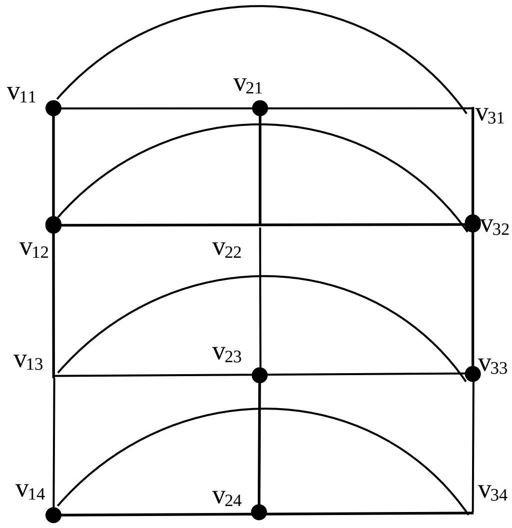


Figure 3.11

In the graph $P_4 \times C_3$ given in Figure 3.11, minimum nonsplit neighbourhood tree dominating set is D = $\{v_{31}, v_{22}, v_{13}, v_{34}\}$, where $\langle N(D)\rangle \cong P_8$, and $\gamma_{nsntr}(P_4 \times C_3) = 4$.

Remark 3.2.4:

For $n \geq 4$, $\gamma_{nsntr}(P_4 \times C_n) = 0$, since there exists no neighbourhood tree dominating set of $P_4 \times C_n$. The graph $P_4 \times C_4$ can be divided into two blocks $P_2 \times C_4$ and $P_2 \times C_4$. $\gamma_{ntr}(P_2 \times C_4) = 0$. If a dominating set D of $P_4 \times C_n$ contains three vertices, then $\langle N(D)\rangle$ contains a cycle.

Remark 3.2.5:

For $n \geq 2$, $\gamma_{nsntr}(P_n \times C_3) = n$.

REFERENCE

1. S. Arumugam and C. Sivagnanam, Neighborhood connected domination in graphs, JCMCC 73(2010), pp.55-64.
2. S. Arumugam and C. Sivagnanam, Neighborhood total domination in graphs, OPUSCULA MATHEMATICA. Vol. 31.No. 4. 2011.
3. M. El- Zahav and C.M. Pareek, Domination number of products of graphs, Ars Combin., 31 **(1991)**, 223-227.
4. F. Harary, Graph Theory, Addison- Wesley, Reading Mass, **1972**.
5. T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of domination in graphs, Marcel Dekker Inc., New York, **1998**.
6. M.S. Jacobson and L.F. Kinch, On the domination number of products of graphs I, Ars Combin., 18 (1983), 33-44.

7. M.S. Jacobson and L.F. Kinch, On the domination number of products of graphs II, J. Graph Theory, 10 (1986), 97-106.
8. V.R. Kulli, B. Janakiram, The split domination number of a graph, graph theory notes of New York Academy of Science **(1997)** XXXII, 16-19.
9. V.R. Kulli, B. Janakiram, The nonsplit domination number of a graph, Indian j. Pure Appl. Math., 31 **(2004)**, no. 4, 545-550.
10. S. Muthammai and C. Chitiravalli, Neighborhood Tree Domination in Graphs, Aryabhatta Journal of Math ematics and informatics, 8(2), (2016), 94 - 99, ISSN 2394 - 9309, (SCOUPUS INDEX ID: 73608143192), Impact factor : 4.866.
11. S. Muthammai and C. Chitiravalli, The Nonsplit Tree Domination Number of a Graph, J. of Emerging Technologies and Innovative Research (JETIR), Vol. 6, Issue 1, January 2019, pp. 1- 9.
12. S. Muthammai, C. Chitiravalli, The split tree domination number of a graph, International Journal of Pure and Applied Mathematics, volume 117 no. 12, **(2017)**, 351-357.
13. O. Ore, Theory of grap99hs, Amer. Math. Soc. Colloq. Publication, 38, **1962**.
14. Xuegang Chen, Liang Sun, Alice McRae, Tree Domination Graphs, ARS COMBBINATORIA 73**(2004)**, pp, 193-203.