Models for Prediction of Measurement Errors using Regression Analysis and Artificial Neural Networks

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Abstract : Measurement is the act or the result of a quantitative comparison between a given quantity and a quantity of the same kind chosen as a unit. It is generally agreed that all measurements contain errors. In a measuring system where both a measuring instrument and a human being taking the measurement using a preset process, the measurement error could be due to the instrument, the process or the human being involved. The first part of the study is devoted to understanding the human errors in measurement. The second part of this research work concentrated on the characterization of errors observed during calibration done periodically (effect of time) of selected sophisticated instruments and selected standards used in legal metrology. The extent of errors due to passage of time and use, were found for some sophisticated instruments and some standards used in legal metrology. These studies have enabled the researcher to characterize errors in these instruments and thus add to the understanding of measurement errors. In order to make the data collected more useful, Regression and Artificial Neural Network [ANN] based models have been developed to predict error [extent] for instrument type and standard types studied.

Key Words : Measurement Errors,Regression Analysis,Artificial Neural Network

1. Introduction

Characterization of calibration errors of selected instruments shows that calibration errors grow with ageing. It is also noticed that the error growth is different for different types of instruments. Test and calibration engineers usually follow the prescribed rules for deciding the period between calibrations, since they do not know the extent of error growth of instruments in use before getting it calibrated. It was also found that the error prediction capability of professionals, regarding the change in extent of error in measuring instruments with use (time) was poor. Discussions with professionals revealed that, an accurate prediction of errors would help them to schedule calibration depending on requirement, and not always follow thumb rule like calibrate after twelve months. To enable this, it was decided to build models for error prediction using the calibration data collected.

Two approaches were taken, one based on regression (time series) and the other using Artificial Neural Network (ANN) [1]. Time dependent errors observed during calibration of selected sophisticated instruments, and some working standards used for calibration in legal metrology, were used to build the models to predict future value of such calibration errors in this research work.

2. Prediction of the errors using regression analysis

Regression is the determination of a statistical relationship between two or more variables. In simple regression there are only two variables, one variable (defined as independent) is the cause of the behavior of another one (defined as dependent variable). Regression can only interpret what exists physically, that is, there must be a physical way in which independent variable 'x' can affect dependent variable 'y'. The regression analysis is a statistical method to deal with the formulation of mathematical model depicting relationship amongst variables which can be used for the purpose of prediction of the values of dependent variable, given the values of the independent variable.

3. Simple Linear Model

Linear model can be derived when one variable, y called the dependent variable is 'driven by' some other variable x called the independent variable. In addition, suppose that the relationship between y and x is basically linear, but is inexact: besides its determination by x, y has a random component, u, which is called the 'disturbance' or 'error'. In this research, the independent variable is time and the dependent variable is measurement error of the instrument. The regression model therefore becomes a time series model in this case.

Let i index the observations on the data pairs(x, y). The simple linear model formalizes the ideas just stated:

$$y_1 = \beta_0 + \beta_1 x_1 + u_1$$

The parameters β0 and β1 represent the y-intercept and the slope of the relationship, respectively [2].

Measurement Errors observed during Calibration of selected sophisticated instruments and working standards used in legal metrology, over five years were used as the dependant variable, time as the independent variable, for each type of instrument and models were developed.These were used for development of the models for prediction of errors that can be observed during calibration.

4. Details of the Regression Based Models Developed for Sophisticated Equipments

Using the five year data , Regression equations, curves and R^2 values of Digital Multimeter, Digital Thermometer, CRO, Signal Generator and Pressure Guage were found out in each case. These equations can be used for prediction. Out of the study, few sample results are given below.

Digital Multimeter –DMM

DMM is basically used in six different modes of measurement. They are DC voltage, AC voltage, DC current, AC current, Resistance and Capacitance. Separate models were made for measurement error prediction in each mode. For each model, five years error observed during calibration (calibration data) of 10 instruments were used.

DMM – Direct Current Voltage (DCV)

DC Voltage mode can measure five different ranges of input voltage . Five year's calibration data of ten different instruments is used for regression analysis. Regression equations, curves and R^2 values have been found and are given in Figure 1.

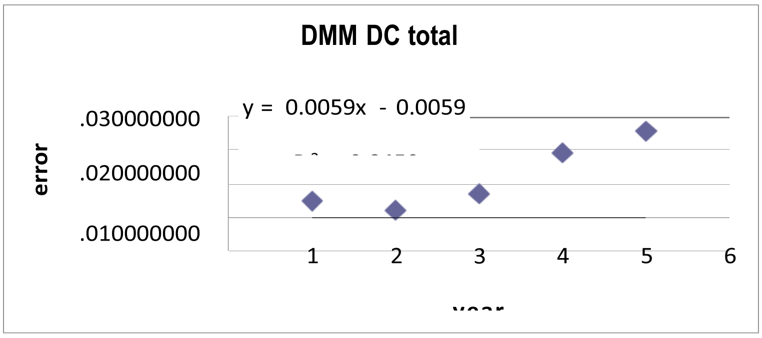

Figure 1 Graph Showing Regression Curve, Equation and R^2 Value

Similar set of regression equations, curves and R^2 values were found outfor all operating ranges of voltage.

DMM Alternating Current voltage (ACV)

Calibration of ACV mode of DMM is done by using four different groups of AC Voltages and within each group; three different voltages were applied to the DMM. Regression analysis equation with the R^2 value for a single group is given below as a sample.

Y= 0.0002 x + 0.0003; R^2 = 0.891 for 4V group

Mean residue (difference between actual and predicted measurement error value) for the four different ranges of ACV operation of DMM is obtained as 0.0008424.

DMM DC Current mode Regression equation with R^2 value for 0.004A range of operation is as follows:

Y = 0.00000016 x + 0.0000016; R^2 = 0.9817 for 0.004A

DMM AC Current mode Regression equation with R^2 value for 0.004A is as follows:

Y = 7E -07 x + 3E-06; R^2 = 0.8278 for 0.004A

DMM – Capacitance mode Regression equation and R^2 value for 4nF group is given below:

Y = 0.0664 x +0.1593; R^2 = 0.9157, for 4nF

DMM – Resistance mode Regression equation for 0.4k□ of applied input resistance for calibration is given below:

Y = 0.0002 x – 0.0001; R^2 = 0.9891 for 0.4kΩ

Digital Thermometer

Digital Thermometer using thermocouple is calibrated by applying four different temperatures. Regression equations and R^2 value for the applied temperatures are given below:

Y= 0.1309x +0.0433; R^2= 0.9567 for -50°CY = 0.0869x + 0.0758; R^2 = 0.9324 for 0°C

Y = 0.1289 x + 0.6115; R^2 = 0.9963 for 500°CY = 0.1751x + 0.6557;

R^2 = 0.9782 for 950°C

Cumulative regression equation, regression curve and R^2 value, it is inferred that actual and predicted values of errors are very close with a minimum residue. It can also be observed that the predicted error values are more accurate for higher range of applied input temperature for calibration.

Regression Analysis of Cathode Ray Oscilloscope in the vertical axis, horizontal axis or frequency axis mode by applying different voltages and square wave with different frequencies and voltages, Signal Generator with applied frequency ranging from 400Hz to 2000 KHz, Pressure Guage for different pressures were done for error prediction models.

Details of the Regression Based Models Developed for Working Standards Used in Legal Metrology

Errors observed during calibration of selected working standards such as Non Automatic Weighing Instrument, Volumetric Measures and Weight Measures were also used for error modeling in this research work. Results of Non Automatic Weighing Instrument (NAWI) are given below as a sample.

Regression equation, regression curve and R^2 value which predicts the time dependent error for the three different tests, the weighing test [up and down], eccentricity test [e-test] and repeatability test [r-Test] used for calibrating NAWI is shown separately in Figure 2. Figure shows that the residue is minimum with R-test [0.0036] and maximum with weighting test [0.453]. R^2 value is close to one except for W-test (down).

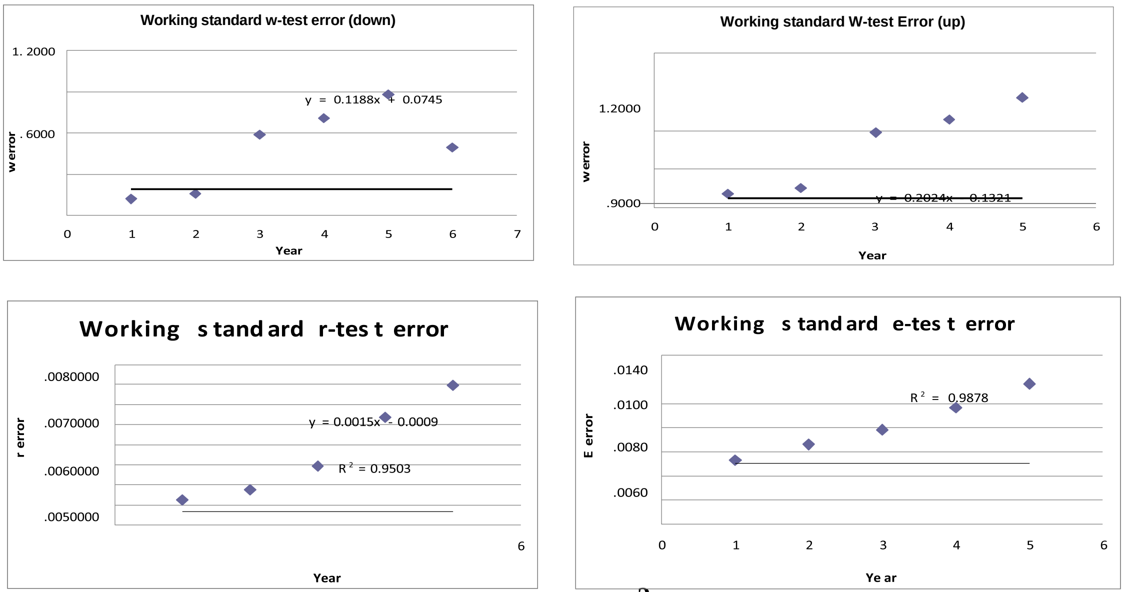

Figure 2. Regression Curve, Regression Equation and R^2 value - NAWI

5.

6. Prediction models using Artificial Neural Networks

Advances have been made in developing tools for “intelligent″ computer programs, a category of which have been inspired by biological neural networks, these are called artificial neural networks (ANNs). Researchers from various scientific disciplines are using artificial neural networks (ANNs) to solve a variety of problems in decision making, optimization, prediction, classifications and control [3]. The second set of measurement error prediction model, developed in this thesis uses ANN.

7. Network Architecture

An assembly of artificial neurons is called an Artificial Neural Network (ANN). ANNs can be viewed as weighted directed graphs in which nodes are artificial neurons and directed edges (with weights) are connections from the outputs of neurons to the inputs of neurons [4]. Feed-forward network architecture has been used in this research [5]. The most common family of feed forward networks is a layered network in which neurons are organized into layers with connections strictly in one direction from one layer to another.

Generally speaking, feed forward networks are static networks, i.e., given an input, they produce only one set of output Neural Networks values, not a sequence of values [6]. Feed forward networks are memory-less in the sense that the response of a feed forward network to an input is independent of the previous state of the network.

Recurrent networks are dynamic systems. Upon presenting a new input pattern, the outputs of the neurons are computed [7].

8. Algorithm in MATLAB for Error Models

In the supervised learning paradigm, the network is given a desired output for each input pattern. During the learning process, the actual output, y, generated by the network may not equal the desired output, d. Error-correction rules are followed in this algorithm. The basic principle of error-correction learning rules is to use the error signal (d - y) to modify the connection weights such that this error will be gradually reduced [8].

The well-known perceptron learning rule is based on this error-correction principle. A perceptron consists of a single neuron with adjustable weights, w_j; j = 1,2,….n, and threshold u. Given an input vector x = (x1; x2; …… xn) t, the net input to the neuron (before applying the threshold function) is

$$v = \sum_{j=1}^{n} w_j x_j - u$$

The output y of the perceptron is +1 if v > 0, and 0 otherwise. In a two-class classification problem, the perceptron assigns an input pattern to one class if y = 1, and to the other class if y = 0. The linear equation -

$$\sum_{j=1}^{n} w_j x_j - u = 0$$

defines the decision boundary (a hyperplane in the n-dimensional input space) which divides the space into two halves.

Note that learning occurs only when an error is made by the perceptron. Rosenblatt proved that if the training patterns are drawn from two linearly- separable classes, then the perceptron learning procedure will congregate after a finite number of iterations [9]. This is the well known perceptron convergence theorem. In practice, one does not know whether the patterns are linearly separable or not. Many variations of this learning algorithm have been proposed in the literature [10] [11]. Other activation functions can also be used, which lead to different learning characteristics. However, a single layer perceptron can only separate linearly separable patterns, as long as a monotonic activation function is used.

Back-propagation learning algorithm, which is based on the error-correction principle [12], is used in this research study.

9. Back-Propagation Algorithm

i. Initialize the weights to small random values;
ii. Randomly choose an input pattern $x^{(\mu)}$;
iii. Propagate the signal forward through the network;
iv. Compute δ_1^L in the output layer $(o_1 = y_1^L)$

$$\delta_1^L = g^1(h_1^L)[d_1^\mu - y_1^L]$$

where h_i^l represents the net input to the i^{th} unit in the $\mathbf{l}^{th}$ layer, and g` is the derivative of the activation function g.

v. Compute the deltas for the preceding layers by propagating the errorsbackwards;

$$\delta_1^l = g^1(h_i^l \sum_j w\frac{l-1}{ij}\ \delta_j^{l+1}, \text{for } l = (L-1), \dots \dots 1$$

vi. Update weights using

$$\Delta w_{ji}^l = \eta \delta_i^l y_j^{l-1}$$

vii. Go to step ii and repeat for the next pattern until the error in the output layer is below a pre-specified threshold or a maximum number of iterations is reached.

ANN based models were developed in this research work for prediction of time dependent error, in selected sophisticated measuring instruments and, in working standards used in legal metrology.

10. Prediction Model for Calibration Errors

Prediction of calibration errors (errors that is observed during calibration)in selected sophisticated instruments and in selected working standards which are used for calibration in legal metrology, were done using Artificial Neural Network [ANN]. The sophisticated instruments selected are digital multimeter, digital thermometer, cathode ray oscilloscope, signal generator and pressure gauge. Working standards selected for the study include, non-automatic weighing instruments, weight measures and volumetric measures. As in the case of regression based models, errors observed during calibration (five years) of the sophisticated instruments and the errors observed during calibration (five years) of working standards were used for modeling. Residues [difference between actual and predicted values] of prediction errors were also found out. A comparison of the residue of prediction of regression analysis and artificial neural network is also given.

Back propagation algorithm used for developing code in MAT LAB neural network for prediction of errors can be further interpreted as follows:

1) The errors observed during calibration for year1 to year 4 is the training data.
2) Year 5 data is the target data.
3) 80% of the data is taken for training the neural network and 20% for testing the neural network.
4) As the errors observed during calibration increase in ascending order, the training data and testing data were selected at random and not in sequential order.
5) In order to compare one machine with another machine separate folders were created for each machine and the .xls file inside the folders were given identical file names and sheet names.
6) Math-Lab built-in functions are used to train and test the neural network.

11. Comparison of Regression and ANN model for error prediction

Errors observed during calibration of selected sophisticated instruments were utilized for predicting errors using Regression analysis technique and Artificial Neural Network. Residue, which is an indication of error of model output, is compared for regression and artificial neural network based models. The results of the comparison are shown in Table 1.

Table 1. Comparison of Residues in Regression and ANN models

Instruments	Residues*	
	Regression Analysis	**Artificial Neural Network**
DMM – DCV	0.0003288	0.00028
DMM – ACV	0.0008424	0.00046
DMM – DCC	0.0001232	0.0000293
DMM – ACC	0.000600	0.00018
DMM – Resistance	0.0696	0.00614
DMM – Capacitance	0.2764	0.157
Digital Thermometer*	0.025764**	0.0854**
CRO – DC	0.682	0.654
CRO – AC	0.8099	0.678
CRO – Time	0.376	0.17925
Signal Generator	0.00148	0.00015
Pressure Gauge – Increasing	0.044036	0.058
Pressure Gauge – Decreasing	0.05248	0.03233

*Residue is the difference between actual and predicted measurement error value

It can be observed from the residue comparison of sophisticated instruments that except for Digital thermometer (**) in all other cases residue with artificial neural network technique is less than regression analysis technique. This indicates the superiority of ANN model in terms of accuracy of prediction.

Comparison of the prediction results of regression based model and ANN based model for sophisticated instruments and working standards used in this study shows that, the ANN based model is more accurate. At the same time, one requires the MATLAB package and the model developed using ANN on a computer, to predict the measurement error (this is expensive and requires greater expertise to use). But in the case of regression model, just by substituting the value of independent variable x in the regression equation will yield the dependent variable Y, which is the measurement error to be estimated. For example in the regression equation of a digital thermometer given below:

$$Y = 0.1289\,x + 0.6115; \qquad R^2 = 0.9963 \text{ for } 500°C$$

Y = 1.7716 for x=9. That means the digital thermometer will give a reading of 501.7716°C for a true temperature of 500°C after 9 years.

Hence it is worth noting that the regression model is simple and easy to use. Therefore, based on the requirement an appropriate model needs to be chosen. If the relationship between x and y is non-linear, regression analysis can only be successfully applied if prior knowledge of the nature of the non-linearity exists. Because, the range of study has to be divided into sections of linear segments, and regression equations made for each such segment. On the contrary, this prior knowledge of the nature of the nonlinearity is not required for ANN. In ANN, the degree of non-linearity can be also changed easily by changing the transfer function and the number of hidden layer nodes. Hence when linear relationship of the change in error with time is not seen it is safer to use ANN model for prediction.

12. Conclusion

Models for prediction of measurement errors using regression analysis and Artificial Neural Network enable us to characterize and estimate measurement errors and would also enable to quantify:

- the instrument's time in service.

The observations during calibration can be applied in the error models and if the error growth is abnormal, appropriate error correction technique can be used or the instrument can be rejected.

- Period of calibration.

An accurate estimation of Annual Average Growth (AAG) of errors by prediction of errors using the appropriate models would help to schedule calibration depending on requirement, and not always follow thumb rules like calibrate after twelve months.

- Error reduction / compensation.

Knowledge of the extent of error growth with instruments and the instrument to instrument error growth differences would help the test engineers and designers to apply appropriate error reduction and compensation techniques in instruments.

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