Solution of a Differential Equation by Yashu's Method

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Abstract: In this article, the author has defined a new method"Yashu method" to get an approximate value of a variable by converting the differential equation first in second kind of volterra integral equation or second kind of Fredholem integral equation, after that by solving the above equation we can find approximate value. The author also compared the result by other known results e.g. Laplace transforms method, Picard's method, Euler's method and Runge-Kutte method and checked all the result with the exact value.

Keywords: Yashu method, Integral Equation, Second kind Volterra integral equation, Laplace transforms, Euler method, Picard method, Runge-Kutte method.
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1. Introduction

(i) Laplace Transform

The Laplace transform with a complex valued and real valued function $f(w)$ for $w>0$ is denoted by $L\left\{f(w);r\right\}$ or $F(r)$ or $\overline{F}(r)$ is defined as

$$L\left\{f(w);r\right\}=F(r)=\int_0^\infty e^{-rw}f(w)dw \quad (1.1)$$

Here the limit finite and exists.

(ii) Volterra Integral Equation

An integral equation with upper variable limit of integration e.g.

$$\alpha(t)\tau(t)=g(t)+\lambda\int_a^x K(t,s)\tau(s)ds \quad (1.2)$$

Here a is a constant, $g(t),\alpha(t)$ and $K(t,s)$ are knowing functions where $\lambda(s)$ is not known function, γ is a non-zero complex or real parameter. Equation (1.2) is a volterra integral equation of third kind.

When $\alpha=1$, the equation (1.2) is reduces to the volterra second kind integral equation.

(iii) Fredholm Integral Equation

An integral equation with upper fixed limit of integration e.g.

$$\alpha(r)h(r)=f(r)+\lambda\int_a^b K(r,s)h(s)ds \quad (1.3)$$

Here a is a constant, $\alpha(t)$, $f(t)$ and $K(r,s)$ are knowing functions where $h(r)$ is not known function, λ is a non-zero complex or real parameter. Equation (1.3) isa Fredholmthird kind integral equation.

When $\alpha=1$, the equation (1.3) is reduces to the Fredholm second kind integral equation.

(iv)Laplace Transform of Derivatives

Let $f(x)$ is a continuous and a function with exponential order then

$$L\left\{f'(x);k\right\}=kL\{f(x)\}-f(0) \quad (1.4)$$

(v)Inverse Laplace transform of certain elementary functions

(a) $L\{x^n; p\} = \dfrac{n!}{p^{n+1}} \Rightarrow L^{-1}\left\{\dfrac{1}{p^{n+1}}; x\right\} = \dfrac{x^n}{n!}$ (1.5)

(b) $L\{e^{ax}; t\} = \dfrac{1}{t-a} \Rightarrow L^{-1}\left\{\dfrac{1}{t-a}; x\right\} = e^{ax}$ (1.6)

2. Main Result

Here, we will use Yashu's method to solve the following differential equation.

Ex.: Find an approximate value of s , when $t = 0.1$, where $s(0) = 1$ and $\dfrac{ds}{dt} = t + s$

Sol.: Let

$$\frac{ds}{dt} = g(t) \quad (2.1)$$

Integrate (2.1) w.r.t. t from 0 to t after that by applying the beginning condition $s(0) = 1$, we have

$$s(t) - s(0) = \int_0^t g(u)du$$

$$s(t) = 1 + \int_0^t g(u)du \quad (2.2)$$

Now substituting the values of s and $\dfrac{ds}{dt}$ in the given differential equation, we find

$$g(t) = t + 1 + \int_0^t g(u)du \quad (2.3)$$

Which is a Volterra second kind integral equation?

Now $g(t) = t + 1 + c$ where

$$c = \int_0^t g(u)du = \int_0^t (u + 1 + c)du = \frac{t^2}{2} + t + ct$$

$$\because c = \frac{t^2 + t}{2(1-t)} \Rightarrow g(t) = t + 1 + \frac{t^2 + t}{2(1-t)} \quad (2.4)$$

Now substituting (2.4) in (2.1), we get the required solution

$$s(t) = \frac{t^2}{2} + t + \frac{t^2 + 2t}{2(1-t)}$$

Now, put $t = 0.1$ in (2.5), we get $g(t) = 1.2166$.

3. Solution of the above differential equation by other known methods

In the present portion, the author will obtain the result of the same differential equation $x = 0.1$ by other methods.

(i) Laplace Transform method

We apply first, the Laplace transforms of the above given differential equation after that we will use (1.4), we get

$$L\{y'(x)\} = xL\{1\} + L\{y(x)\}$$

$$rF(r) - y(0) = \frac{x}{r} + F(r) \Rightarrow F(r) = \frac{1}{r-1} + \frac{x}{r(r-1)}$$

Now by taking inverse Laplace transform of above equation and after using (1.5) and (1.6), we get

$$y(x) = (1+x)e^{x} - x$$

Now, for $x = 0.1$, we get $y = 1.121$.

(ii)Picard's Method

Let

$$\frac{dp}{ds} = f(s,p) = s + p\,; s_0 = 0, p_0 = 1$$

First approximate value is

$$p^{(1)} = p_0 + \int_{s_0}^{s} f(s,p_0)dx = 1 + \int_{0}^{s}(s+1)ds = 1 + s + \frac{s^2}{2}$$

Second approximation value is

$$p^{(2)} = p_0 + \int_{s_0}^{s} f(s,p^{(1)})ds = 1 + \int_{0}^{s} s + (1 + s + \frac{s^2}{2})ds = 1 + s + s^2 + \frac{s^3}{6}$$

Third approximate value is

$$p^{(3)} = p_0 + \int_{s_0}^{s} f(s,p^{(2)})ds = 1 + \int_{0}^{s} s + \left(1 + s + s^2 + \frac{s^3}{6}\right)ds$$

$$= 1 + s + s^2 + \frac{s^3}{3} + \frac{s^4}{24}$$

When $s = 0.1$ then $p^{(1)} = 1.105; p^{(2)} = 1.1106; p^{(3)} = 1.1103$.

(iii) Euler's Method

Here $\frac{dp}{dy} = f(y,p) = y + p\,; y_0 = 0, p_0 = 1$

Taking $h = 0.1$

$$p_1 = p_0 + hf(y_0, p_0) = 1 + 0.1(0+1) = 1.10$$

(iv) Runge-Kutta Method

Let $\frac{ds}{dy} = f(y,s) = y + s\,; y_0 = 0, s_0 = 1$

Taking $h = 0.1$

Here $p_1 = hf(y_0, s_0) = 0.1(0+1) = 0.1$

$$p_2 = hf\left(y_0 + \frac{h}{2}, s_0 + \frac{k_1}{2}\right) = (0.1)[(0+0.05)+1+0.05)] = 0.11$$

$$p_3 = p_2 = hf\left(y_0 + \frac{h}{2}, s_0 + \frac{p_2}{2}\right)(0.1)[(0+0.05)+(1+0.055)] = 0.1105$$

$$p_4 = hf\left(y_0 + h, s_0 + k_3\right) = (0.1)[(0+0.1)+(1+0.1105)] = 0.12105$$

And $p = \frac{1}{6}(p_1 + 2p_2 + 2p_3 + p_4) = \frac{1}{6}[0.1 + 2(0.11) + 2(0.1105) + 0.12105] = 0.11034$

Hence $s_1 = s_0 + p = 1 + 0.11034 = 1.11034$

Exact value of the given differential equation

$$\frac{ds}{dx} = x + s \Rightarrow \frac{ds}{dx} - s = x$$

This is a linear differential equation and it's I.F.$= e^{-x}$

$$se^{-x} = \int xe^{-x}dx + d = (-x-1)e^{-x} + d$$

$$s = -x - 1 + de^{x}$$

When $x = 0, s = 1 \Rightarrow d = 2$. Hence $s = -x - 1 + 2e^{x}$

When $x = 0.1 \Rightarrow s = -0.1 - 1 + 2e^{0.1} = 1.11034$.

The differences with exact value of the equation and all methods are as

Yashu's method=0.1063, Picard's method=0.0004, Euler's method=0.01034,

Laplace method=0.0106, Runge-Kutta method=0.0001

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