Research Article

Mathematical Study On Predator-Prey Holling Type-Ii Effect Of Fading Memory

L. Sahaya Amalraj[1], M. Jeyaraman [2] ,V. Ananthaswamy[3]

[1]Part-time Ph.D., Research Scholar, P.G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affliated to Alagappa University, Karaikudi, India
[2]P.G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affliated to Alagappa University, Karaikudi, India
[3]Research Centre and PG Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India
[1]amalraj11kv@gmail.com,[2] jeya.math@gmail.com, [3]ananthu9777@gmail.com

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Abstract: In this paper, we investigate the effect of Fading Memory on Predator-Prey Holling Type-II model which was previously developed. We obtain approximate analytical solution to the dimensionless prey and predator population, using the new approach to Homotopy perturbation method. The analytical solution is discussed graphically and compared with the previous study and found to be in good agreement.

Keywords: Predator-Prey, Holling Type-II, Fading Memory, Numerical simulation, New approach to Homotopy perturbation method

1. Introduction

The Predator–prey model is an appealing model, which has been developed in the recent times. The memory plays an important role for making this model vivid. The memory is an inherent characteristic of life, which enables to convey the activities of the past to predict the future. In this case, the memory of past events is accounted to plan for the future. The impact of this principle has a great effect on the growth rate of predators[1]. Jayanta Mondal framed this model and the developed model[1] is solved analytically in this presentation using the new approach to Homotopy perturbation method. The solution obtained has been compared using the numerical solution derived using MATLAB and is found to make a good agreement. The impact of each of the parameters on the dimensionless prey and predator population has been discussed.

2. Mathematical formulation of the problem

Jayanta Mondal developed a Holling–T–Tanner model with ratio-dependent functional response as given in the following equations [1]

$$\frac{du}{dt} = u(1-u) - \beta \frac{uv}{g+u} \tag{1}$$

$$\frac{dv}{dt} = \theta\beta \frac{vm}{g+m} - \delta v - qEv \tag{2}$$

$$\frac{dm}{dt} = h(u-m) \tag{3}$$

where u, v, m represent the dimensionless prey population, predator population and fading memory term respectively. The parameters β, g, δ, h, E are dimensionless parameters defined by $\beta = \frac{\beta_0 K}{\alpha_0}, g = \frac{g_0}{K}, \delta = \frac{\delta_1}{\alpha_0}, h = \frac{h_0}{\alpha_0}, E = \frac{E_0}{\alpha_0}$ where α_0 and K are the intrinsic growth rate and carrying capacity of the prey species. The parameter β_0 denotes the capturing rate of the predator on the prey and θ denotes the conversion rate of the prey to the predator. The constant g_0 is the half saturation constant for the predator. δ_1 denotes the predator's death rate in the absence of prey and E_0 represents the catchability coefficient and effort applied to harvest the individuals respectively [1]. The eqns. (1) – (3) are governed by the following initial conditions

$$u(0) = u_0, v(0) = v_0, m(0) = m_0 \tag{4}$$

3. New approach to Homotopy perturbation method

For finding approximate analytical solution of non-linear differential equations, there are many asymptotic methods, like the Variational Iteration method [2], Homotopy perturbation method [7-13], Homotopy analysis method [4-6] and Adomian decomposition method [3]. The new approach to Homotopy perturbation method [14-19] gives a better simple approximate solution in the zeroth iteration itself. The method employs a Homotopy transform to generate a convergent series solution of the given nonlinear differential equation. The main advantage of this method is that it does not need a small parameter. Hence this method is applied much in solving nonlinear differential equations.

4. Approximate analytical solution to the steady state of eqns. (1) to (4) using new approach to Homotopy perturbation method

Using the new approach to HPM, the solution of eqns. (1) to (4) are as follows:

$$u = e^{-\frac{(\beta v_0 - g - u_0)t}{g+u_0}}\left(u_0 + \frac{u_0^{\,2}(g+u_0)}{\beta v_0 - g - u_0}\right) - \frac{u_0^{\,2}(g+u_0)}{\beta v_0 - g - u_0} \tag{5}$$

$$v = e^{\frac{(\theta\beta m_0 - \delta g - \delta m_0)t}{g+m_0}}\left(v_0 - \frac{qE(g+m_0)}{\theta\beta m_0 - \delta g - \delta m_0}\right) + \frac{qE(g+m_0)}{\theta\beta m_0 - \delta g - \delta m_0} \tag{6}$$

$$m = u_0 + e^{-ht}(m_0 - u_0) \tag{7}$$

5. Numerical simulation

The eqns. (1) to (4) are solved numerically. The function graphmain3 has been used in MATLAB software to solve the initial value problem numerically. The obtained analytical results are compared with the numerical simulation. The MATLAB program is given in Appendix B.

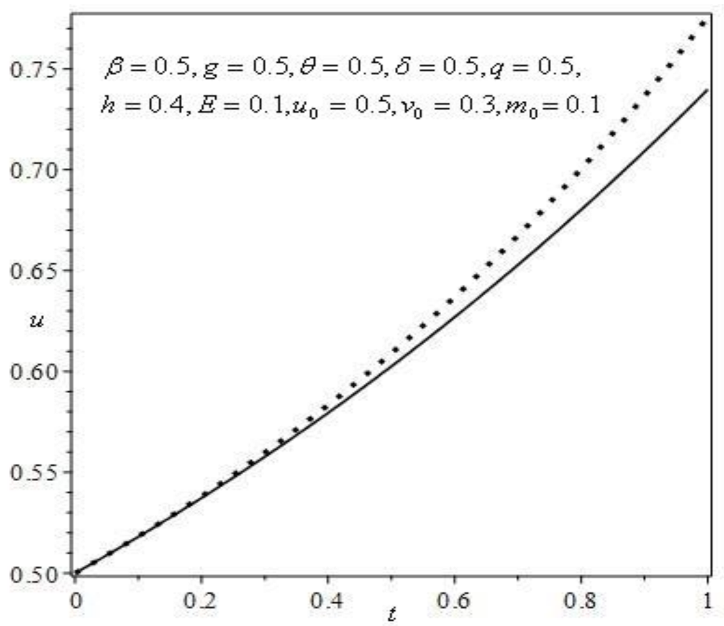

Fig. 1: Plot of dimensionless prey population versus dimensionless time. The dotted line represents the analytical solution and the solid line represents the numerical simulation.

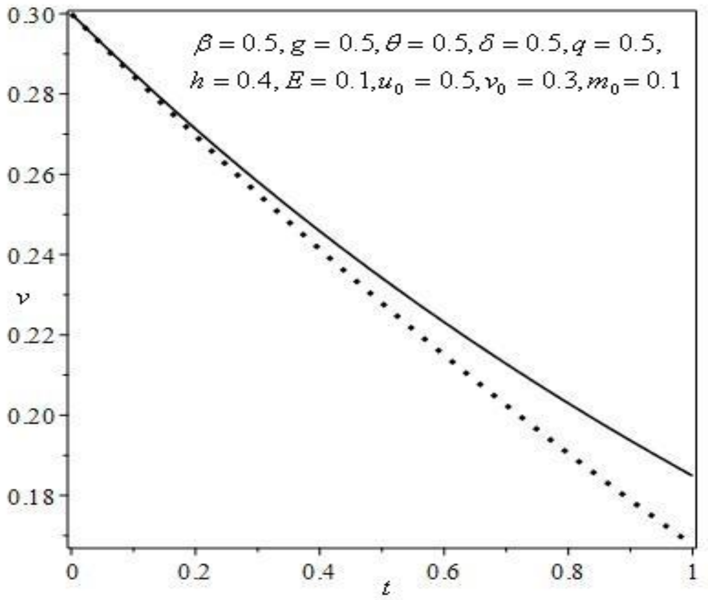

Fig.2: Plot of dimensionless predator population versus dimensionless time. The dotted line represents the analytical solution and the solid line represents the numerical simulation

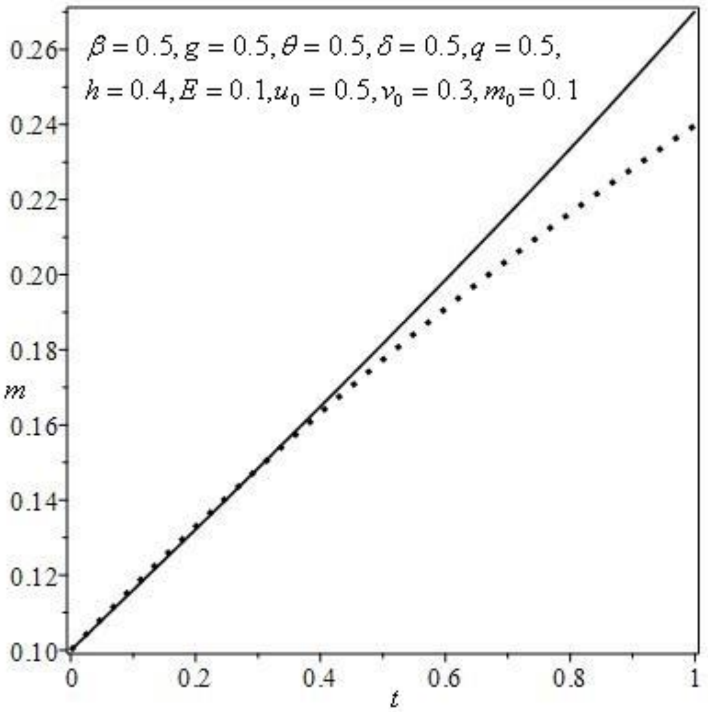

Fig.3: Plot of dimensionless fading memory term versus dimensionless time. The dotted line represents the analytical solution and the solid line represents the numerical simulation

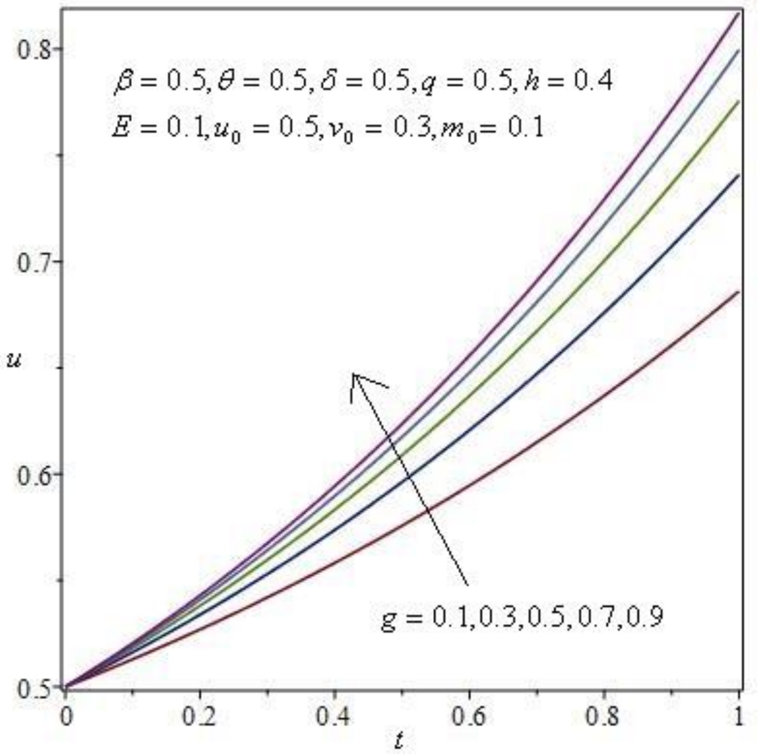

Fig.4: Plot of dimensionless prey population versus dimensionless time for various values of g.

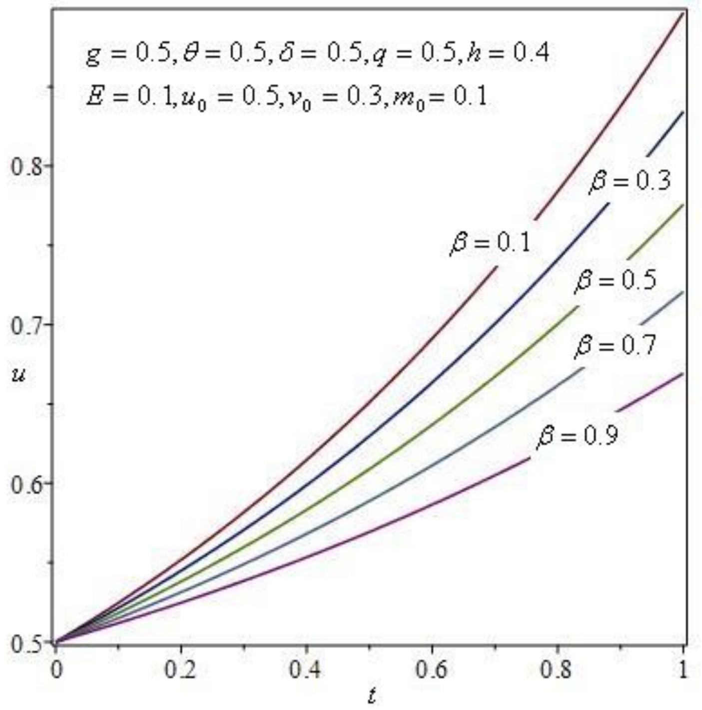

Fig.5: Plot of dimensionless prey population versus dimensionless time for various values of β.

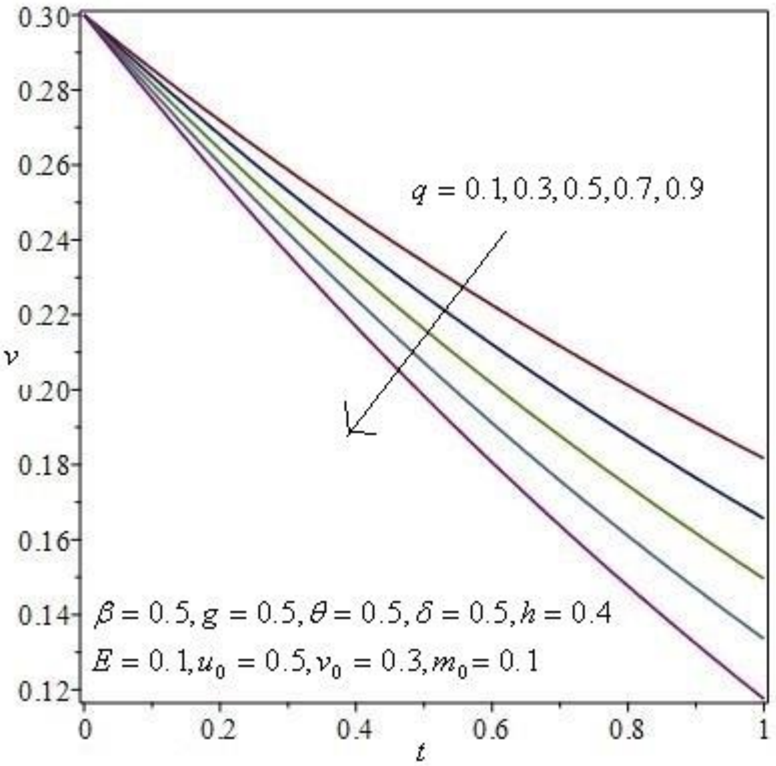

Fig.6: Plot of dimensionless predator population versus dimensionless time for various values of q.

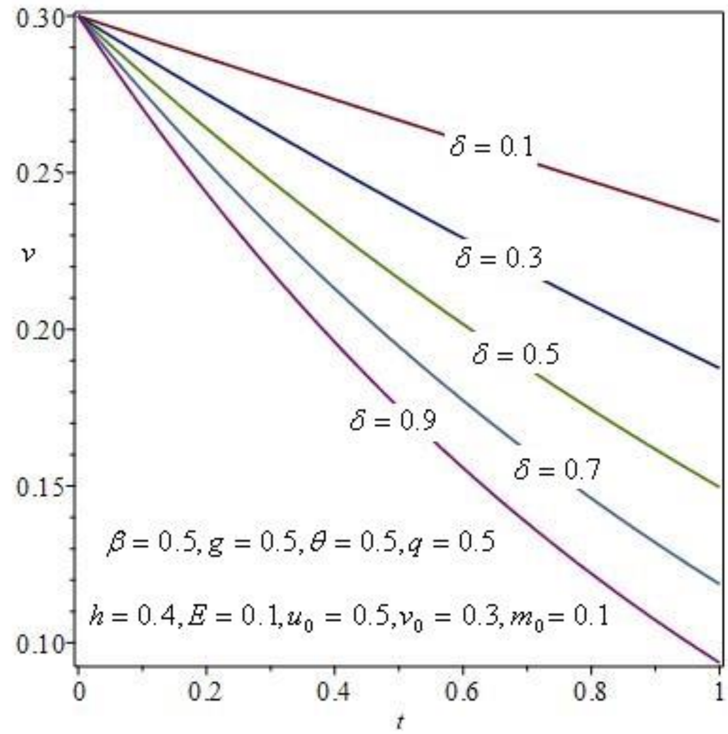

Fig.7: Plot of dimensionless predator population versus dimensionless time for various values of δ.

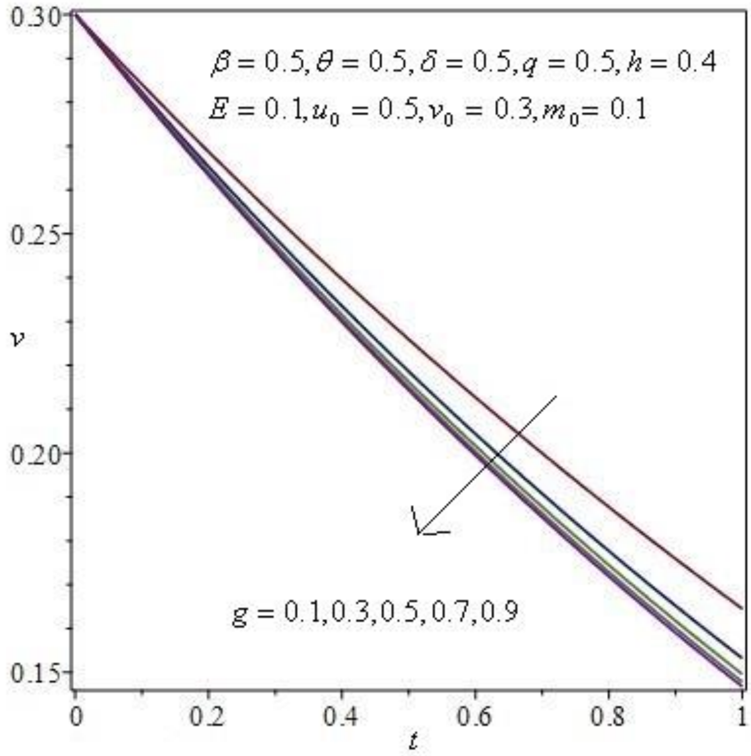

Fig.8: Plot of dimensionless predator population versus dimensionless time for various values of g.

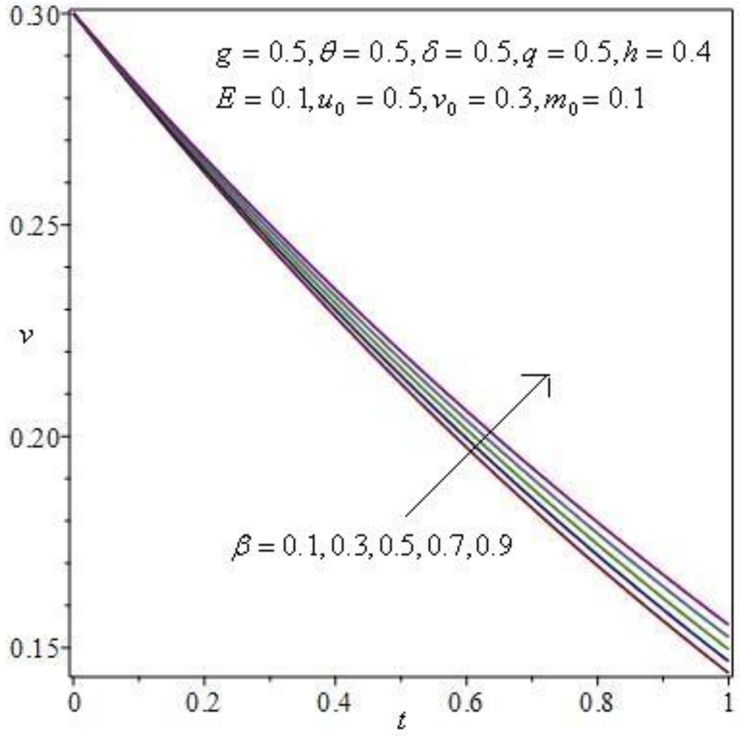

Fig.9: Plot of dimensionless predator population versus dimensionless time for various values of β.

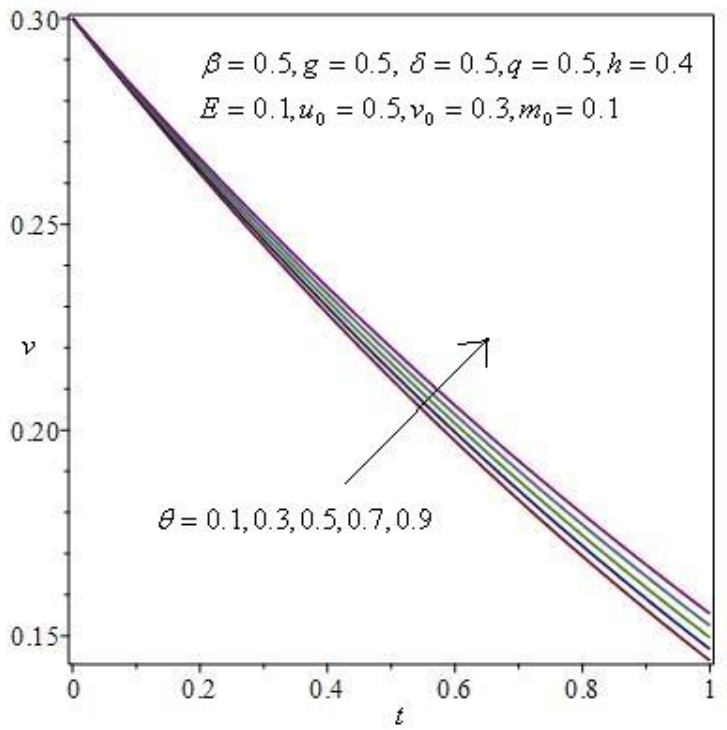

Fig.10: Plot of dimensionless predator population versus dimensionless time for various values of θ.

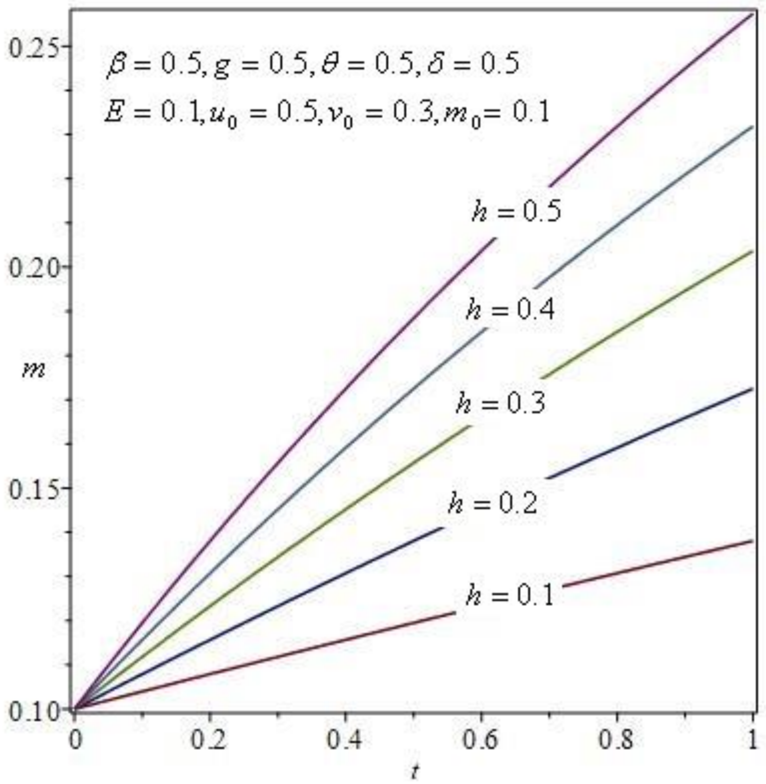

Fig.11: Plot of dimensionless fading memory term versus dimensionless time for various values of h.

Table 1. Comparison between analytical and numerical values in Fig. 1

Value of u for parameter values $\beta = 0.5, g = 0.5, \theta = 0.5, \delta = 0.5, q = 0.5, h = 0.4, E = 0.1, u_0 = 0.5, v_0 = 0.3, m_0 = 0.1$			
Value of *t*	Numerical solution	Analytical solution	Absolute deviation percentage
0	0.5	0.5	0
0.2	0.537309	0.538151	0.156767
0.4	0.579489	0.583372	0.670049
0.6	0.626965	0.636972	1.596082
0.8	0.680222	0.700504	2.981746
1	0.739807	0.77581	4.866519
Average percentage of deviation			1.71186

Table 2. Comparison between analytical and numerical values in Fig. 2 .

Value of v for parameter values $\beta = 0.5, g = 0.5, \theta = 0.5, \delta = 0.5, q = 0.5, h = 0.4, E = 0.1, u_0 = 0.5, v_0 = 0.3, m_0 = 0.1$			
Value of *t*	Numerical solution	Analytical solution	Absolute deviation percentage
0	0.3	0.3	0
0.2	0.271291	0.269602	0.62252
0.4	0.245828	0.241307	1.83929
0.6	0.223174	0.214968	3.67687
0.8	0.202964	0.190451	6.16492
1	0.18489	0.16763	9.33567
Average percentage of deviation			3.60654

Table 3. Comparison between analytical and numerical values in Fig. 3.

Value of m for parameter values $\beta = 0.5, g = 0.5, \theta = 0.5, \delta = 0.5, q = 0.5, h = 0.4, E = 0.1, u_0 = 0.5, v_0 = 0.3, m_0 = 0.1$			
Value of *t*	Numerical solution	Analytical solution	Absolute deviation percentage
0	0.1	0.1	0
0.2	0.132177	0.132962	0.594266
0.4	0.164935	0.163208	1.04714
0.6	0.198622	0.190962	3.85651
0.8	0.23359	0.216428	7.34697
1	0.270208	0.239796	11.2547
Average percentage of deviation			3.81852

6. Results and discussion

The eqns. (5) to (7) represent the simple approximate analytical expression for dimensionless prey population, predator population and fading memory term respectively. The derived analytical expressions are compared with the numerical solutions obtained using MATLAB in Figs. 1 to 3. The percentage error is given in tables 1 to 3. From the tables it is inferred that the percentage error is in the acceptable range. Hence we may say that the derived solution is an approximate analytical solution to eqns. (1) to (4). From Figs. 4 and 5, we observe that the prey population varies directly with g , while inversely with β. Further from Figs 6 to 11, we observe that the predator population varies directly with β and θ, while varies indirectly with E, q, δ and g. Fig. 12 shows that the fading memory term varies directly with *h*.

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5. Conclusion

In this paper, time dependent approximate analytical expressions for prey population, predator population and fading memory term are determined. The new Homotopy perturbation method has been used to obtain the solution. The results have a good consensus with the numerical results. The analytical results that have been found will be helpful in interpreting the effect of the different parameters over the predator-prey population.

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