Influence of radiation on MHD flow of a Casson fluid and heat transfer over a stretched surface

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Abstract: The effects of radiation on MHD Casson liquid motion and heat transport past a stretchable layer are discussed in this paper. An appropriate transformation considers transforming the model partial differential equations (PDEs) into a system of nonlinear total differential equations (NODE) by using the fourth order Runge-Kutta along shooting approach (R-S method). A comprehensive analysis of different flow terms on velocity profiles and temperature plots are deliberated and accessible diagramatically.

Keywords: Heat transfer, Casson fluid, MHD, radiation**.**

1. Introduction

In recent days, research into non-Newtonian fluids (NF) has been greatly improved due to their use of larger materials in the field of science and engineering. Casson liquid is a NF attributes that is considered with boundary pressure. The blood of humans can be preserved as Casson owing to fibrinogen, synovial liquid, blood cells etc. Vajravelu [1] analyzed viscous motion past a nonlinearly stretchable sheet. Elbashbeshy et al. [2] examined internal heat generation along heat transportation past a stretchable surface. Emad et al. [3] explored mixed convection within a continuously inclined stretchable surface with generation of heat and absorption has a suction/blowing contribution on hydromagnetic heat transportation. Subhas et al. [4] investigated non-uniform heat source, viscous dissipation, and heat transportation in a boundary layer motion and visco-elastic past a stretchable sheet. Goud et al. [5] have been presented a stretching exponentially sheet with radiation contribution on MHD motion in the layers. Nandeppanavar et al. [6] analysed viscoelastic liquid motion with heat transport owing to nonlinear stretchable layer with heat source internally. Siti et al. [7] explored the contribution of thermal radiation on hydro-magnetic boundary layer motion past a stretchable surface. Yahaya et al. [8] elucidated the contribution of buoyancy with thermal radiation on MHD motion over a stretchable penetrable layer were investigated. Several researchers [9] to [17] have investigated the contribution of thermal radiation on MHD motion past a stretchable sheet under different flow conditions..

The current model's aim is to investigate MHD Casson fluid motion and heat transport through a stretchable sheet. The motion and heat transport analysis governing equations are simplified into NODE assets and numerically solved using the R-S method.

2. Mathematical formulation

Consider 2-D heat transport over a stretched surface with an incompressible, viscous, radiating and electrically conducting Casson liquid. The same and directly opposite forces are imposed towards the to extend the surface while retaining the origin. In a motion orientation, the is parallel to the stretchable surface , while is in perpendicular direction. The surface extends nonlinearly towards with a distance of 'xn'. The surface is exposed to a variable magnetism B(x) of intensity that is imposed transversely to the direction.

The constitutive equation for an isotropic and incompressible Casson liquid motion is thought to be,

$$\tau_{ij} = \begin{cases} \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}} \right) 2e_{ij}, \ \pi > \pi_c, \\ \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}} \right) 2e_{ij}, \ \pi < \pi_c \end{cases}$$

with and are the deformation rate elements, signifies multiplication of the portion of the deformation rate by itself.

The governing equations can be witten as follows under these assumptions:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\left[B(x)\right]^2}{\rho}u,$$
(2)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{q^m}{pc_p}-\frac{1}{pc_p}\frac{\partial q_r}{\partial y} \quad (3)$$

where u and v denote the fluid's velocity components in the x and y directions, respectively.

$$q^m=\frac{ku_w(x)}{xv}\left(a\left(T_W-T_\infty\right)e^{-\eta}+b\left(T-T_\infty\right)\right) \quad (4)$$

The radiative heat flux q_r of an optically thick liquid can be simplified by the Rosseland approximation as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (5)$$

with σ^* and k^* as the Stephan-Boltzman and coefficient of mean absorption respectively. The distinct temperature between the fluids inside the boundary layer area is presumed to be so small, that 'T4' can be written as a temperature T with linear function. We obtain by simplifying 'T4' in Taylor series regarding 'T_∞' while ignoring the higher terms.

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \quad (6)$$

Eq. (3) can be simplified using Eqs. (5) and (6)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y_2}+\frac{16\sigma^*T_\infty^3}{3k^*pc_p}\frac{\partial^2 T}{\partial y^2}+\frac{q^m}{pc_p}. \quad (7)$$

The boundary constraints are

$$u=u_w(x),\quad v=0,\quad T=T_w(x)=Ax^\lambda\quad at\quad y=0,$$
$$u=0,\quad T\to T_\infty\quad y\to\infty \quad (8)$$

with λ as parameter of temperature

3. Similarity analysis

The coefficient of stream functions u and v are defined as

$$u=\frac{\partial\Psi}{\partial y},\quad v=-\frac{\partial\Psi}{\partial x} \quad (9)$$

Utilizing the similarity variable η and the dimensionless temperature θ as

$$\eta=y\sqrt{\frac{b(n+1)}{2v}}x^{\frac{n-1}{2}},\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\quad where\quad T_w-T_\infty=Ax^\lambda, \quad (10)$$

After simplifications, the coefficients of stream function resulted to:

$$u=\frac{\partial\Psi}{\partial y}=bx^nf'(\eta),\quad v=-\frac{\partial\Psi}{\partial x}=-\sqrt{\frac{bv(n+1)}{2}}x^{\frac{n-1}{2}}\left(f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right). \quad (11)$$

Equations (2) and (3) can be written as

$$\left(1+\frac{1}{\beta}\right)f''(\eta)+f(\eta)f''(\eta)-\frac{2n}{n+1}f'^2(\eta)-M^2f'(\eta)=0, \quad (12)$$

$$\left[1+\frac{4}{3R}\right]\theta''(\eta)+\Pr f(\eta)\theta'(\eta)+\frac{2}{n+1}\left[b-\Pr\lambda f'(\eta)\right]\theta(\eta)+\frac{2}{n+1}ae^{-\eta}=0, \quad (13)$$

with the boundary constraints:

$$f'(\eta)=1, \quad f(\eta)=0, \quad \theta(\eta)=1 \quad at \quad \eta=0,$$
$$f(\eta)\to 0, \quad \theta'(\eta)\to 0 \quad as \quad \eta\to\infty \tag{14}$$

where $M^2=\dfrac{2\sigma B_0^2}{pb(n+1)}$ is the magnetic interaction parameter, $R=\dfrac{kk^*}{4\sigma^* T_\infty^3}$ signifies radiation and $\Pr=\dfrac{v}{a}$ signifies Prandtl numeric.

4. Results and discussion:

A nonlinear equations of motion is defined by the set of NODE (12) and (13) as well as the boundary conditions (14). The solution to these equations is gotten numerically by simplifying the nonlinear equations of motion to system of total differential equations using the R-S method, which is very difficult to obtain analytically.

The aim of this section is to examine the effect of various key parameters on velocity profiles and temperature plots for various key parameters such as velocity and temperature. M, R, Pr, Casson fluid , power-law index is denoted by n, radiation parameter is R, temperature term λ, heat sink/source dependent parameter 'a' and heat sink/source dependent parameter 'b'. In this study, The following are the default values for the different parameters we looked at: =10; M=1 ; Pr=0.71; n=3; R=5; λ =15; a=b=0.1;

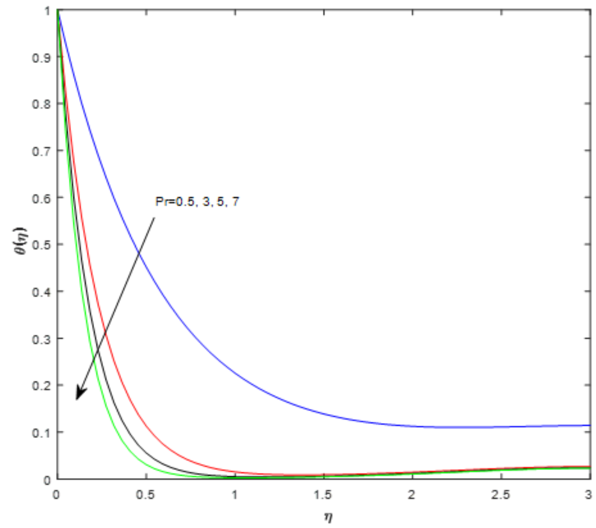

Fig. 1 for distinct values of Pr

The contribution of Pr on ' ' is discussed in Fig. 1. It is evident that a liquid with high Pr values. As a result, we discovered that a rapid rise in Pr induces a drop in temperature, and that the temperature of a motion field is decreasing monotonically function of Pr. The effects of 'n' on have been plotted in Figure 2. The velocity decelerates as 'n' increases, and it is also noted that the effect of 'n' on the velocity is less important. The contribution of 'n' on is displayed by Fig. 3. It has been found that as 'n' increases, the temperature distribution increases. are examine for the distinct values of R is depicted in Fig.4. It has been detected that a drastic elevation in R leads to a reduction in . The effect of ' λ ' on ' ' is examined in Fig.5.

It is detected that the increasing contribution of λ is to lessens the thickness of the thermal layer.

The contribution of 'a' are show in Fig. 6. increases as the value of 'a' rises. As a consequence, the thickness of the thermal boundary layer increases. Fig. 7 illustrates the effects of 'b' on . It can be shown that as the value of 'b' increases, the temperature rises. The thickness of the thermally layer upsurge owing to this fact. The addition of b < 0, energy is absorbed by raising the values of b. hence, the thickness of the thermal layer degenerates. Figure 8 connotes for distinct values of ' '. It is obvious that as ' ' increases, the fluid velocity decreases.

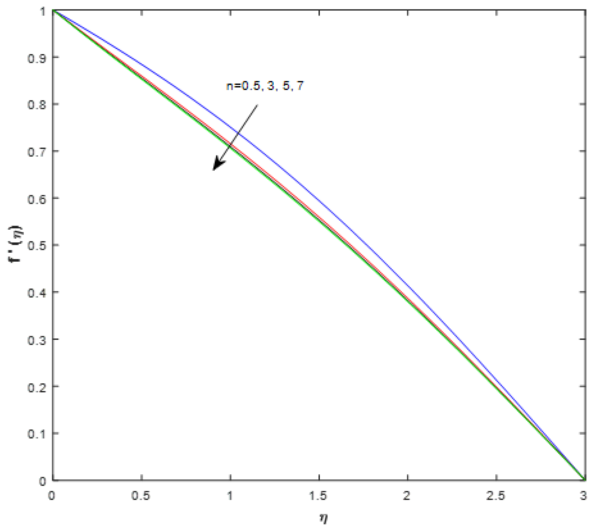

Fig. 2 for distinct values of 'n'

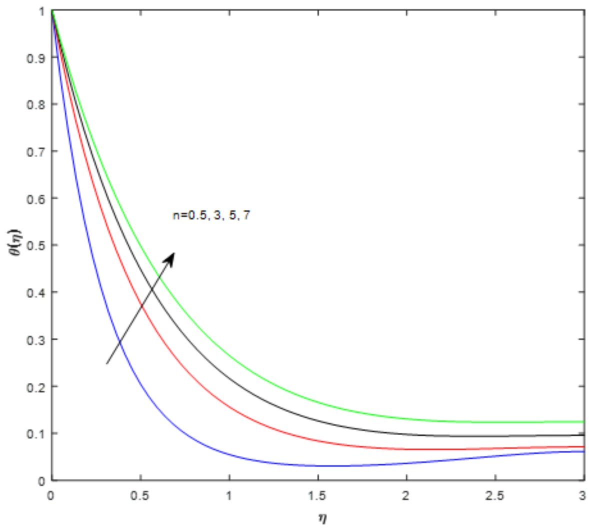

Fig. 3 for distinct values of 'n'

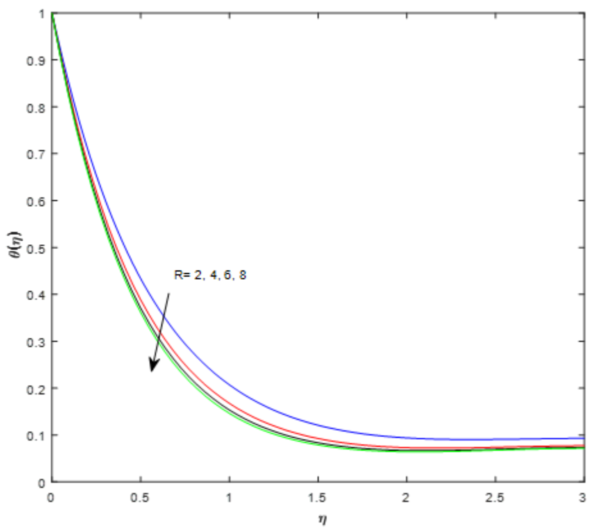

Fig. 4 for distinct values of R

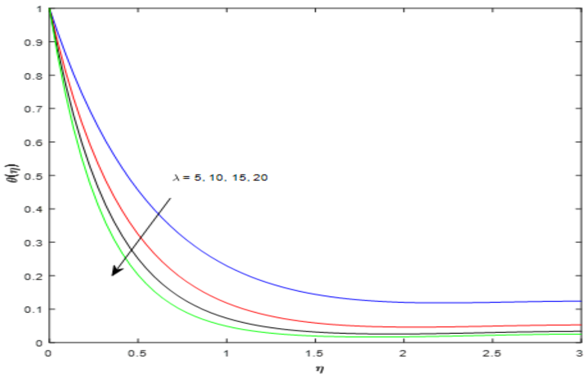

Fig. 5 $\theta(\eta)$ for distinct values of λ

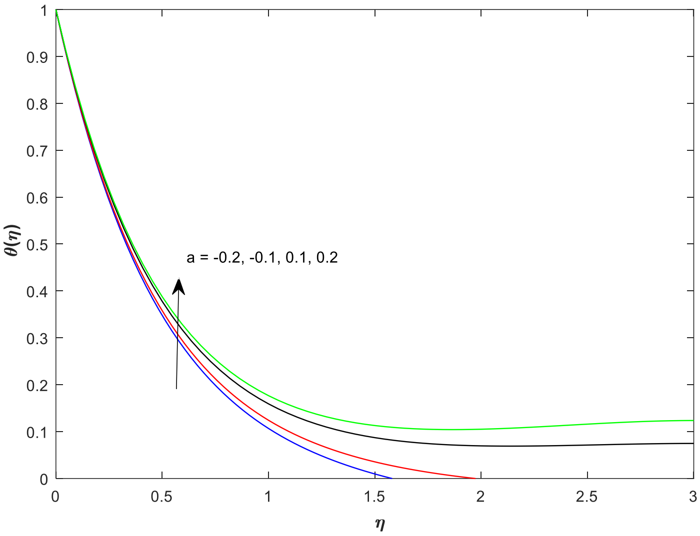

Fig. 6 $\theta(\eta)$ for distinct values of *'a'*

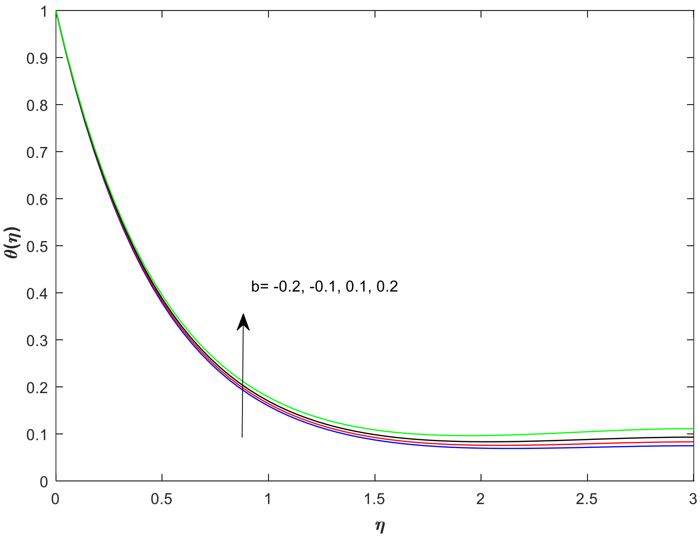

Fig. 7 $\theta(\eta)$ for distinct values of *'b'*

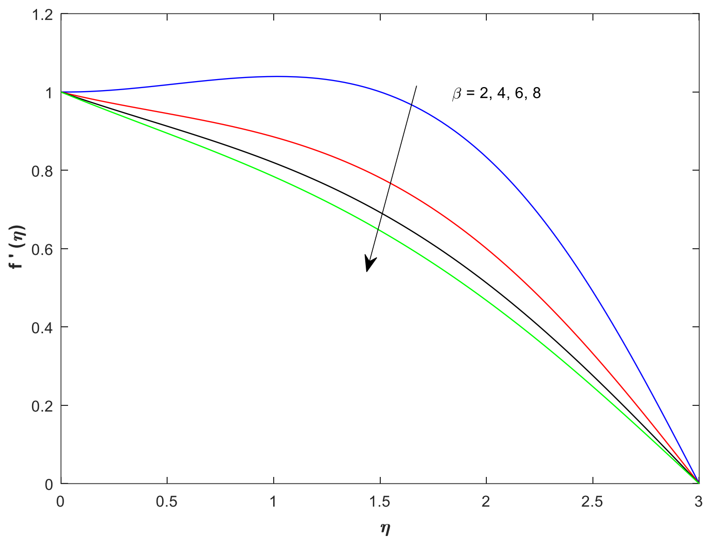

Fig. 8 $f'(\eta)$ for distinct values of β

5 conclusions:

The current research investigates the effect of radiation on Casson fluid MHD flow and heat transfer over a stretched surface. The following are the key outcomes of the current problem:

It is found that increasing n, results in a decrease in the velocity profile. An increase in n, a and b leads to increase in temperature distribution in the thermal boundary layer. A higher Pr, R and λ leads to a decrease in temperature distribution in the thermal boundary layer.

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