Fuzzy Inverse Barrier Method to Find Primal and Dual Optimality Solutions of Fuzzy Linear Programming Problems

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Abstract: This article highlights a fuzzy inverse barrier method with a ranking of R>0 parameters to solve problems of fuzzy linear programming. In the method, a fuzzy inverse barrier function algorithm establishes the existence of a limit to arrive at an optimal solution to a given problem. A current algorithmic method, some fuzzy inverse barrier method examples are included.

Keywords: fuzzy Inverse ranking barrier function, fuzzy optimal solution, fuzzy basic feasible solution
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1. Introduction

Tanaka and Asai [20] were the pioneers in the field of fuzzy linear programming problems. Dubois, D., and Prade, H. [4] provided some fuzzy numbers. In the treatment of fuzzy arithmetic operations by numbers, Hsieh and Chen [10] are familiar with the concept of fuzzy function. Mahdavi-Amiri and Nasseri [13] identified the fuzzy linear programming problems of duality in the fuzzy, suitable solution for the Fuzzy variable linear programming problem. The principle, the inverse barrier approach, was discussed by Carroll [3] and followed by Fiacco[6] and McCormick[12].

The other discussion of algorithms for fuzzy constrained optimization is the fuzzy Inverse Barrier process with rank. In this way, the fuzzy linear programming problem is solved using a new approach to the fuzzy inverse barrier function. The fuzzy inverse barrier approach is a method for approximating fuzzy restricted issues by using a term in the fuzzy objective function that reflects the high cost of violating the fuzzy constraints. The continuous fuzzy inverse barrier function opinion value expands to infinity as the boundary of the viable region of the fuzzy optimization problem is increased using point methodologies. A positive decreasing parameter ξ or η that defines the fuzzy inverse barrier is associated with this method; thus, the degree to which the fuzzy unconstrained problem addresses the restricted problems of the original fuzzy. We easily demonstrate the fuzzy logarithmic barrier method in the case of fuzzy inverse rank R = 0 to the simple one. A non-empty of the FIBM's fuzzy primal and fuzzy dual is restricted. Methods of the Fuzzy inverse barrier are sometimes called fuzzy interior methods.

2. Preliminaries

2.1.”Fuzzy Linear Programming Problem (FLPP)

Consider the following FLPP

Maximum (or Minimum) $\tilde{V} = \tilde{f} q_l$

Constraints of the form

$M\tilde{q_l}(\leq, =, \geq)\tilde{N_s}$, s=1,2,…m ,

and the nonnegative conditions of the fuzzy variables $\tilde{q} \geq$ (0,0,0) where $\tilde{f}^T$ =($\tilde{f}_1$,...,$\tilde{f}_n$) is an j-dimensional constant vector, $M \in R^{i\times j}$, $\tilde{q}$ =($\tilde{q_l}$) , l=1,2,..,n and $\tilde{N_l}$are nonnegative fuzzy variable vectors such that $\tilde{q}_l$ and$\tilde{N_s}$ $\in \mathcal{F}(R)$ for all $1 \leq l \leq n, 1 \leq s \leq m$, is denoted by an FLPP.

2.2. Feasible solution: If $\tilde{q} \in \mathcal{F}(R)^n$is a feasible solution, it must satisfy all of the constraints of the problem.

2.3. Optimal Solution: If we get $\tilde{f}\tilde{q}^* \geq \tilde{f}\tilde{q}$ for all feasible solutions $\tilde{q}$, then we have an optimal solution $\tilde{q}$*.

2.4. Fuzzy basic feasible solution:

Let $M\tilde{q_l} = \tilde{N_s}$ and $\tilde{q_l} \geq \tilde{0}, M = [m]_{s\times l}\ rank(M) = s\ N \neq 0,\ rank\ (\tilde{N}) = l$. Let y_l be the solution to N_y= m_l. The basic solution $\tilde{q}_L = \left(\tilde{q}_{L_1}, \tilde{q}_{L_2}, \ldots, \tilde{q}_{L_m}\right)^T = L^{-1}\tilde{N_s}, \tilde{q}_N = 0$ is a solution of $M\tilde{q_l} = \tilde{N_s}$, $\tilde{q}$ is thus partitioned into $(\tilde{q}_L^T\tilde{q}_N^T)^T$, a fuzzy basic solution corresponding to the basis $\tilde{N_s}$. If $\tilde{q}_L \geq \tilde{0}$, then the fuzzy basic solution is feasible, and $\tilde{v} = \tilde{f}q_l$ is the corresponding fuzzy objective value, where$\tilde{f}_L = (\tilde{f}_{L_1}, \ldots, \tilde{f}_{L_m})$. define $V_l = f_L y_l = f_L L^{-1} m_l$ in response to each fuzzy non-basic variable $\tilde{q}_l, 1 \leq l \leq n, l \neq L_i$, and s=1,…,m. when $\tilde{q}_L > \tilde{0}$, $\tilde{q}$ is referred to as a non-degenerate fuzzy basic feasible solution.

3. Fuzzy Inverse barrier method (FIBM)

Assume that the primal Fuzzy linear programming problem (PFLPP)

Minimize $\tilde{V} = (\tilde{f_1}, \tilde{f_2}, \tilde{f_3})\tilde{p}_1 + (\tilde{f_1}, \tilde{f_2}, \tilde{f_3})^t\tilde{p}_2$

Subject to $M_{11}\tilde{p}_1 + M_{12}\tilde{p}_2 \geq (\tilde{n_1}, \tilde{n_2}, \tilde{n_3})$,

$M_{21}\tilde{p}_1 + M_{22}\tilde{p}_2 \geq (\widetilde{n_1}, \widetilde{n_2}, \widetilde{n_3})^t$.

Where $M_l \in R^{m\times n}, \tilde{f}, \tilde{f}^t, \tilde{p}_i \in R^n, \tilde{N}, \tilde{N}^t \in R^m, \tilde{f} = (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})$,
$\widetilde{N_l} = (\widetilde{n_1}, \widetilde{n_2}, \widetilde{n_3}), i = 1,2$ (1)

Assume that M has the maximum m rank without loss of generality. Assume that a minimum of one possible solution appears to the primary fuzzy linear programming problem.

We specify the fuzzy inverse barrier method"$\tilde{I}(\tilde{p}, \xi)$for any scalar, $\xi > 0$ for the problem.

Define $\tilde{I}(\tilde{p}, \xi): R^n \to R$ by the fuzzy inverse barrier function (FIBF)

$$\tilde{I}(\tilde{p}_1, \tilde{p}_2, \xi) = (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})\tilde{p}_1 + (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})^t\tilde{p}_2 + \frac{1}{\xi}.\frac{1}{\Re}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \quad (2)$$

The fuzzy inverse barrier function (2) of the problem is convex. $\tilde{I}(\tilde{p}, \xi)$is, therefore, a global minimum and ξ is a positive decreasing value of the parameter.

Define $\tilde{I}: R^n \to (-\infty, \infty)$ by

$$\tilde{I}(\tilde{p}_1, \tilde{p}_2) = \begin{cases} M_l\tilde{p} - \widetilde{N_l} \leq 0, if\ M_l\tilde{p} - \widetilde{N_l} = 0, \\ M_l\tilde{p} - \widetilde{N_l} > 0, if\ M_l\tilde{p} - \widetilde{N_l} \neq 0 \text{ for all } l \end{cases}$$

Convert the fuzzy inverse of the rank barrier equation into the two weak fuzzy inequalities.

The fuzzy measure of the problem of fuzzy linear programming is the metric for feasible solutions to a fuzzy interior; we have

$$\vartheta(\tilde{p}, \xi) = \min_{\tilde{q}, s}\left\|(\dot{\wp})^{-\Re/2}\left(\frac{(\dot{\wp})^{\Re+1}s(\tilde{p},\xi)}{\xi}\right) - I\right\|, M^T\tilde{q} + s = \tilde{N} \quad (3)$$

The fuzzy function is the first order and the second-order derivatives of the FIBM:

$$\nabla\tilde{I}(\tilde{p}, \xi) = (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3}) + (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})^t - \frac{1}{\xi}(\dot{\wp})^{-\Re-1}, \quad (4)$$

$$\nabla^2\tilde{I}(\tilde{p}, \xi) = (\Re + 1)(\dot{\wp})^{-\Re-2} \quad (5)$$

The interior of its boundary region was characterized by a fuzzy inverse barrier function, so that

(i) The fuzzy inverse barrier method is a fuzzy continuous function.

(ii) $\tilde{I}(\tilde{p}_1, \tilde{p}_2) \geq 0$.

(iii) $\tilde{I}(\tilde{p}) \to \infty$ as $\widetilde{p_1}, \widetilde{p_2}$. It reaches the set's boundary. The method of the Fuzzy inverse barrier is also called the method of the fuzzy interior.

3.1: Fuzzy Inverse Barrier Lemma

Let $\{\xi^s\}$ is a fuzzy increasing sequence, $\tilde{I}(\tilde{p}^s)$is the function of the fuzzy inverse barrier and $\tilde{f}(\tilde{p})$ is the function of the fuzzy objective, then we get,

(i).A fuzzy inverse barrier function with including parameter $\tilde{I}(\tilde{p}^s, \xi^s)$ included is smaller than are equal to $\tilde{I}(\tilde{p}^{s+1}, \xi^{s+1})$

(ii).$\tilde{I}(\tilde{p}^s)$ will include a fuzzy inverse barrier function objective function. It is more than are equal to $\tilde{I}(\tilde{p}^{s+1})$.

(iii). The fuzzy inverse barrier technique is an increasing sequence of the fuzzy objective function $\tilde{f}(\tilde{p}^s) \geq \tilde{f}(\tilde{p}^{s+1})$.

Proof:

(i). $\tilde{I}(\tilde{p}^s, \xi^s) = (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^s) + \frac{1}{\Re}\frac{1}{\xi^s}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^s) + \frac{1}{\Re}\frac{1}{\xi^{s+1}}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^{s+1}) + \frac{1}{\Re}\frac{1}{\xi^{s+1}}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq \tilde{I}(\tilde{p}^{s+1}, \xi^{s+1})$.

$\tilde{I}(\tilde{p}^s, \xi^s) \leq \tilde{I}(\tilde{p}^{s+1}, \xi^{s+1})$.

(ii). $(\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^s) + \frac{1}{\Re}\frac{1}{\xi^s}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^{s+1}) + \frac{1}{\Re}\frac{1}{\xi^s}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}}$ (6)

$(\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^{s+1}) + \frac{1}{\Re}\frac{1}{\xi^{s+1}}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^s) + \frac{1}{\Re}\frac{1}{\xi^{s+1}}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}}$ (7)

The fuzzy inverse barrier function that we get from fuzzy inequalities (6)&(7),

$(\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^s) + \frac{1}{\Re}.\frac{1}{\xi^s}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} + (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^{s+1}) + \frac{1}{\xi^{s+1}}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^{s+1}) + \frac{1}{\Re}\frac{1}{\xi^s}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}}\widetilde{+f}(\tilde{p}^s) + \frac{1}{\Re}\frac{1}{\xi^{s+1}}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}}$

Then $(\frac{1}{\xi^s} - \frac{1}{\xi^{s+1}})\frac{1}{\Re}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq (\frac{1}{\xi^s} - \frac{1}{\xi^{s+1}})\frac{1}{\Re}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}}$

$\tilde{I}(\tilde{p}^s) \geq \tilde{I}(\tilde{p}^{s+1})$.

(iii). from the proof of (i) $(\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^s) + \frac{1}{\Re}\frac{1}{\xi^{s+1}}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}} \geq (\widetilde{f_1}, \widetilde{f_2}, \widetilde{f_3})(\tilde{p}^{s+1}) + \frac{1}{\xi^{s+1}}\frac{1}{\Re}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\widetilde{N_l})^{\Re}}$.

$\tilde{I}(\tilde{p}^s) \leq \tilde{I}(\tilde{p}^{s+1})$. Then$\tilde{f}(\tilde{p}^s) \geq \tilde{f}(\tilde{p}^{s+1})$.

Lemma (iv): If $\vartheta(\tilde{p}_1, \tilde{p}_2, \xi) \leq 1$, then fuzzy dual of the FIBF with rank is Dual fuzzy feasible solutions.

Lemma (v): If $\vartheta(\tilde{p}_1, \tilde{p}_2, \xi) \leq {}^2/_3$ $, then\ \check{p}$ is a strictly fuzzy feasible solution.

Lemma (vi): If $\vartheta(\tilde{p}_{1,}\tilde{p}_2,\xi) \le 1$ then $\tilde{I}(\tilde{p}^k,\xi^k) = \|(\dot{\wp})^{-\Re/2}I\|^2 \le p\left[\frac{1}{\acute{\alpha}(1-\vartheta(\tilde{p}_{1,}\tilde{p}_2,\xi))}\right]^{\frac{\Re}{\Re+1}}$.

Lemma (vii): If $\vartheta(\tilde{p}_1,\tilde{p}_2,\xi) \le 1$ then $\|\tilde{V}^T\tilde{p} - M^T\tilde{q}(\tilde{p},\xi)\|^2 \le \xi(\tilde{I}(\tilde{p}^k,\xi^k) + \vartheta(\tilde{p}_1,\tilde{p}_2,\xi)\sqrt{\tilde{I}(\tilde{p}^k,\xi^k)}$.

3.2: Fuzzy Inverse Barrier convergence theorem

Theorem:3.2.1

The fuzzy linear programming problem is defined as an increasing sequence of positive fuzzy Inverse barrier parameters $\{\xi^s\}$such that $\xi^s \ge 1, \xi^s \to \infty, s \to \infty$.

Suppose $\tilde{f}(p), M_l\tilde{p} - \tilde{N}_l$& $\tilde{I}(\tilde{p})$is a continuous fuzzy function and there is an optimal solution p^* of $\tilde{V}$. Then every $\tilde{p}$ limit point of $\{\tilde{p}^s\}$

Proof:

Let us assume that $\tilde{p}$ is any boundary point of $\{\tilde{p}^s\}$

$$\tilde{I}(\tilde{p}^S,\xi^S) = \tilde{f}(\tilde{p}^S) + \frac{1}{\Re}\frac{1}{\xi^k}\sum_{\Re=1}^{m}\frac{1}{(M_l\tilde{p}-\tilde{N}_l)^{\Re}} \ge \tilde{f}(p^*) \qquad (8)$$

From the continuity of $\tilde{f}(\tilde{p})$, we get,

$$\lim_{S\to\infty}\tilde{f}(\tilde{p}^S) = \tilde{f}(\tilde{p}), \lim_{s\to\infty}\frac{1}{(M_l\tilde{p}^S-\tilde{N}_l)^{\Re}} \le 0.$$

From the fuzzy Inverse barrier we get,

$$\lim_{s\to\infty}\tilde{I}(\tilde{p}^s,\xi^s) = \tilde{f}(p^*)$$

$\tilde{p}$ is feasible**.**

Theorem: 3.2.2

Let us consider$\vartheta(\tilde{p}_1\tilde{p}_2,\xi) \le \frac{1}{2}, \xi = (1-\acute{\alpha})\xi, where\ \acute{\alpha} = 1/10\sqrt{\tilde{I}(\tilde{p},\xi)}$, the fuzzy strictly feasible $\check{p}$ then $\vartheta(\check{p}_1\check{p}_2,\check{\xi}) \le \frac{1}{2}$.

Proof:

From the definition of the fuzzy measure, $\vartheta(\tilde{p}_1\tilde{p}_2,\xi) = \left\|\frac{(\dot{\wp})^{(\Re/2)+1}s(\tilde{p},\xi)}{\acute{\alpha}} - (\dot{\wp})^{-(\Re/2)}I\right\|$

$$\le \frac{1}{1-\acute{\alpha}}\left(\vartheta(\tilde{p}_1\tilde{p}_2,\xi) + \acute{\alpha}\sqrt{\tilde{I}(\tilde{p},\xi)}\right) = \frac{12}{18}$$

Fuzzy quadratic convergence properties to be applied we get,

$$\vartheta(\tilde{p}_1\tilde{p}_2,\xi) \le \frac{1}{2}.$$

3.3. Fuzzy Inverse Barrier Function Algorithm:

1. Construct a stand-form for the fuzzy linear programming problems.

$\tilde{V} = \tilde{f}(\tilde{p})$. Subject to $(M_l\tilde{p}^k - \tilde{N}_l) \le 0$.

2. Convert fuzzy linear programming issue to the included rank's fuzzy inverse barrier approach.

$$\tilde{I}(\tilde{p},\eta) = \tilde{f}(\tilde{p}) + \frac{1}{\xi}\frac{1}{\Re}\sum_{l=1}^{m}\frac{1}{(M_l\tilde{p} - \tilde{N}_l)^{\Re}}$$

3. Given a problem with fuzzy inequality constraints to Minimize fuzzy inverse barrier function $Min\tilde{I}(\tilde{p},\xi) = \tilde{f}(\tilde{p}) + \frac{1}{\xi}\frac{1}{\Re}\sum_{l=1}^{m}\frac{1}{(M_l\tilde{p}-\tilde{N}_l)^{\Re}}$ as $\xi \to \infty$, the approximation becomes closer to the indicator function.

4. By adding the first-order necessary condition for optimality, we can find the optimum value of the given fuzzy linear programming problem, using the limit $\xi \to \infty$.

5. Calculate $\tilde{I}(\tilde{p}^s_{1,}\tilde{p}^s_{2,}\xi^k) = \min_{\tilde{p}\ge 0}\tilde{I}(\tilde{p}^s_{1,}\tilde{p}^s_{2,}\xi^s)$, then minimize $\tilde{p}^s_{1,}\ \tilde{p}^s_{2,}$& $\xi = 10, s = 1,2, \dots s = I$ then stop. otherwise, start by making a move to step 5.

In a fuzzy inverse barrier approach with problem ranking, the Dual Fuzzy linear programming problem should be applied to the same idea.

4. Numerical Example

Problem:1

Consider the problem of primal fuzzy linear programming

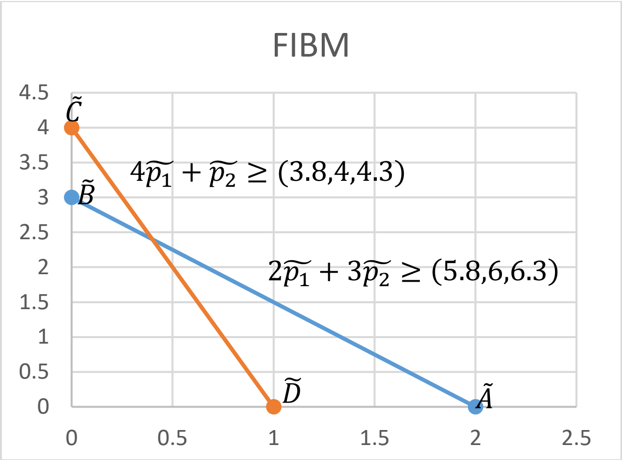
$Min\ \tilde{V} = (3.8,4,4.3)\widetilde{p_1} + (2.8,3,3.3)\widetilde{p_2}$

$2\widetilde{p_1} + 3\widetilde{p_2} \ge (5.8,6,6.3),$

$4\widetilde{p_1} + \widetilde{p_2} \ge (3.8,4,4.3).$

Solution:

The given problem indicates that the corresponding FIBM graph is as follows:



We obtain the fuzzy optimal value of the given primal and dual fuzzy linear programming problem calculation table (i) and (ii) using the FIBF algorithm as follows:

Table: (i)

No.	ξ^k	$\widetilde{p_1}$	$\widetilde{p_2}$
1	10	(0.187,0.505,0.705)	(1.613,1.632,1.863)
2	10^2	(0.230,0.570,0.770)	(1.604,1.610,1.820)
3	10^3	(0.244,0.591,0.791)	(1.601,1.603,1.806)
4	10^4	(0.248,0.597,0.797)	(1.600,1.601,1.802)
5	10^5	(0.249,0.599,0.799)	(1.600,1.600,1.801)
6	10^6	(0.250,0.600,0.800)	(1.600,1.601,1.800)

Table: (ii)

No.	ξ^k	$\widetilde{q_1}$	$\widetilde{q_2}$
1	10	(0.950,1.059,1.222)	(0.625,0.774,0.936)
2	10^2	(0.772,0.882,1.029)	(0.526,0.655,0.795)
3	10^3	(0.716,0.826,0.968)	(0.494,0.617,0.751)
4	10^4	(0.698,0.808,0.949)	(0.485,0.605,0.737)
5	10^5	(0.693,0.803,0.943)	(0.481,0.602,0.732
6	10^6	(0.691,0.801,0.941)	(0.480,0.601,0.731)
7	10^7	(0.690,0.800,0.940)	(0.480,0.600,0.730)

The fuzzy inverse barrier functions have the best solution to the given problem's primal-dual.
(p_1)̃=(0.25,0.6,0.8),(p_2)̃=(1.6,1.6,1.8),
Min Ṽ=(5.5,7.2,9.4)
(q_1)̃=(0.69,0.8,0.94),
(q_2)̃=(0.48,0.6,0.73),
Max Ṽ=(5.8,7.2,9.1)

5. Conclusion:

The FIBF, including rank for solving the primal-dual partial fuzzy linear programming problem using the proposed algorithm to achieve an improved optimal solution, is included in this paper. The optimal solution graph should also be attached to the fuzzy inverse barrier process graph. The table for the problems discussed above indicates that the computational method for the primary-dual FIBF algorithm that we built when the fuzzy Inverse Barrier parameter ξ is the optimal solution provides an increased convergence rate

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