4 – Total Mean Cordial Labeling of Union of Graphs with Star and Bistar

R.Ponraj[a], S.Subbulakshmi[b], and S.Somasundaram[c]
[a] Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India.
[b]Research scholar, Department of Mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.
[c]Department of Mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.

Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 20 April 2021

Abstract: In this paper we investigate 4- total mean cordinality of some union of graphs with star and bistar.

1. Introduction

Graphs in this paper are finite, simple and undirected. In this paper, we investigate the 4- total mean cordial behaviour of $K_{1,n} \cup B_{n,n}$, $K_{1,n} \cup J_{n,n}$, $K_{1,n} \cup CH_n$, $K_{1,n} \cup T_n$, $B_{n,n} \cup B_{n,n}$, $B_{n,n} \cup J_{n,n}$, $B_{n,n} \cup CH_n$, $B_{n,n} \cup T_n$. Terms are not defined here follow from Harary[2] and Gallian[1].

1. **K – TOTAL MEAN CORDIAL GRAPH**

Definition 2.1. For a graph *G*, Let *g* be a map from vertex set of *G* to {0, 1, 2,….., *k*-1}, where *k* is an positive integer and *k* >1. For each edge *uv*, assign the label $\frac{g(u)+g(v)}{2}$ if *g(u)+g(v)* is even, $\frac{g(u)+g(v)+1}{2}$ if *g(u)+g(v)* is odd. *g* is called *k* – total mean cordial labeling of *G* if $|tmg(i) - tmg(j)| \leq 1$, for all *i, j* ϵ {0, 1, 2, …….., *k*-1}, where $t_{m\,g}(x)$ denotes the total number of vertices and edges labelled with *x*, *x* ϵ {0, 1, 2, …….., *k*-1}. A graph with admit a *k*-total mean cordial labeling is called *k*-total mean cordial graph (simply *k* – TMC graph).

2. Preliminary

Definition 3.1. The *jellyfish J (m, n)* is obtained from a cycle C_4*: uxvyu* by joining *x* and *y* with an edge and appending *m* pendent edges to *u* and *n* pendent edges to *v*.

Definition 3.2. The *triangular snake* T_n is obtained from the path P_n: $u_1u_2\ldots u_n$ with
$V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$ and edge set $E(T_n) = E(P_n) \cup \{u_i v_i, u_{i+1} v_i : 1 \leq i \leq n-1\}$.

2000 Mathematics Subject Classification. 05C78.
Key words and phrases. Star, bistar, jellyfish, closed helm, triangular snake.

3. Main Result

Theorem 4.1. The graph $K_{1,n} \cup B_{n,n}$ is 4 – TMC.

Proof. Let *u* be an apex vertex and $u_1, u_2, \ldots, u_n$ be the pendent vertices of the star $K_{1,n}$. Let $v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n$ be the pendent vertices and *v*,*w* be the apex vertices of $B_{n,n}$.

Define $\theta : V(K_{1,n} \cup B_{n,n}) \to \{0,1,2,3\}$ by
$\theta(u) = 2$;
$\theta(v) = 3$;
$\theta(w) = 2$;
$\theta(u_1) = \theta(u_2) = \ldots = \theta(u_n) = 0$.

Case 1. n is odd.
Let n = 2r+1, where r ϵ ℕ.
$\theta(v_1) = \theta(v_2) = \ldots = \theta(v_{r+2}) = 2$;
$\theta(v_{r+3}) = \theta(v_{r+4}) = \ldots = \theta(v_{2r+1}) = 3$;
$\theta(w_1) = \theta(w_2) = \ldots = \theta(w_{r+2}) = 0$;
$\theta(w_{r+3}) = \theta(w_{r+4}) = \ldots = \theta(w_{2r+1}) = 2$;

Case 2. n is even.
Let n = 2r, where r ϵ ℕ.
$\theta(v_1) = \theta(v_2) = \ldots = \theta(v_{r+1}) = 2$;
$\theta(v_{r+2}) = \theta(v_{r+3}) = \ldots = \theta(v_{2r}) = 3$;
$\theta(w_1) = \theta(w_2) = \ldots = \theta(w_{r+1}) = 0$;
$\theta(w_{r+2}) = \theta(w_{r+3}) = \ldots = \theta(w_{2r}) = 2$.

Now $t_{m\theta}(0) = t_{m\theta}(1) = \begin{cases} 3r+3 \text{ if } n \text{ is odd} \\ 3r+1 \text{ if } n \text{ is even;} \end{cases}$

$t_{m\theta}(2) = t_{m\theta}(3) = \begin{cases} 3r+2 \text{ if } n \text{ is odd} \\ 3r+1 \text{ if } n \text{ is even.} \end{cases}$

This satisfies the condition of 4 – TMC graph.

Theorem 4.2. The graph $K_{1,n} \cup J_{n,n}$ is 4 – TMC.

Proof. Let w be an apex vertex and $w_1, w_2, \ldots, w_n$ be the pendent vertices of the star $K_{1,n}$. Take the vertex set and edge set of $J_{n,n}$ as in Definition 3.1.

Define $\phi : V(K_{1,n} \cup J_{n,n}) \rightarrow \{0,1,2,3\}$ as follows:

$\phi(w) = 0;$
$\phi(u) = 3;$
$\phi(v) = 2;$
$\phi(x) = 2;$
$\phi(y) = 2.$

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, where $r \in \mathbb{N}$.
$\phi(w_1) = \phi(w_2) = \ldots\ldots\ldots = \phi(w_{2r}) = 0;$
$\phi(w_{2r+1}) = \phi(w_{2r+2}) = \ldots\ldots\ldots = \phi(w_{4r}) = 1;$
$\phi(u_1) = \phi(u_2) = \ldots\ldots\ldots = \phi(u_{2r+1}) = 0;$
$\phi(u_{2r+2}) = \phi(u_{2r+3}) = \ldots\ldots\ldots = \phi(u_{4r}) = 2;$
$\phi(v_1) = \phi(v_2) = \ldots\ldots\ldots = \phi(v_{2r}) = 2;$
$\phi(v_{2r+1}) = \phi(v_{2r+2}) = \ldots\ldots\ldots = \phi(v_{4r}) = 3.$

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r+1$, where $r \in \mathbb{N}$.
$\phi(w_1) = \phi(w_2) = \ldots\ldots\ldots = \phi(w_{2r+1}) = 0;$
$\phi(w_{2r+2}) = \phi(w_{2r+3}) = \ldots\ldots\ldots = \phi(w_{4r+1}) = 2;$
$\phi(u_1) = \phi(u_2) = \ldots\ldots\ldots = \phi(u_{2r+1}) = 0;$
$\phi(u_{2r+2}) = \phi(u_{2r+3}) = \ldots\ldots\ldots = \phi(u_{3r+2}) = 1;$
$\phi(u_{3r+3}) = \phi(u_{3r+4}) = \ldots\ldots\ldots = \phi(u_{4r+1}) = 2;$
$\phi(v_1) = \phi(v_2) = \ldots\ldots\ldots = \phi(v_{2r+1}) = 2;$
$\phi(v_{2r+2}) = \phi(v_{2r+3}) = \ldots\ldots\ldots = \phi(v_{4r+1}) = 3.$

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r+2$, where $r \in \mathbb{N}$.
$\phi(w_1) = \phi(w_2) = \ldots\ldots\ldots = \phi(w_{2r+1}) = 0;$
$\phi(w_{2r+2}) = \phi(w_{2r+3}) = \ldots\ldots\ldots = \phi(w_{4r+2}) = 2;$
$\phi(u_1) = \phi(u_2) = \ldots\ldots\ldots = \phi(u_{2r+2}) = 0;$
$\phi(u_{2r+3}) = \phi(u_{2r+4}) = \ldots\ldots\ldots = \phi(u_{3r+3}) = 1;$
$\phi(u_{3r+4}) = \phi(u_{3r+5}) = \ldots\ldots\ldots = \phi(u_{4r+2}) = 2;$
$\phi(v_1) = \phi(v_2) = \ldots\ldots\ldots = \phi(v_{2r+2}) = 2;$
$\phi(v_{2r+3}) = \phi(v_{2r+4}) = \ldots\ldots\ldots = \phi(v_{4r+2}) = 3.$

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4r+3$, where $r \in \mathbb{N}$.
$\phi(w_1) = \phi(w_2) = \ldots\ldots\ldots = \phi(w_{2r+2}) = 0;$
$\phi(w_{2r+3}) = \phi(w_{2r+4}) = \ldots\ldots\ldots = \phi(w_{4r+3}) = 1;$
$\phi(u_1) = \phi(u_2) = \ldots\ldots\ldots = \phi(u_{2r+2}) = 0;$
$\phi(u_{2r+3}) = 1;$
$\phi(u_{2r+4}) = \phi(u_{2r+5}) = \ldots\ldots\ldots = \phi(u_{4r+3}) = 2;$
$\phi(v_1) = \phi(v_2) = \ldots\ldots\ldots = \phi(v_{2r+2}) = 2;$
$\phi(v_{2r+3}) = \phi(v_{2r+4}) = \ldots\ldots\ldots = \phi(v_{4r+3}) = 3.$

In view of above labeling, we get

$$\text{Now } t_{m\phi}(0) = \begin{cases} 6r+2 \text{ if } n \equiv 0 \pmod 4 \\ 6r+4 \text{ if } n \equiv 1 \pmod 4 \\ 6r+5 \text{ if } n \equiv 2 \pmod 4 \\ 6r+7 \text{ if } n \equiv 3 \pmod 4; \end{cases}$$

$$t_{m\phi}(1) = \begin{cases} 6r+2 \text{ if } n \equiv 0 \pmod 4 \\ 6r+4 \text{ if } n \equiv 1 \pmod 4 \\ 6r+6 \text{ if } n \equiv 2 \pmod 4 \\ 6r+7 \text{ if } n \equiv 3 \pmod 4; \end{cases}$$

$$t_{m\phi}(2) = \begin{cases} 6r+1 \text{ if } n \equiv 0 \pmod 4 \\ 6r+4 \text{ if } n \equiv 1 \pmod 4 \\ 6r+6 \text{ if } n \equiv 2 \pmod 4 \\ 6r+7 \text{ if } n \equiv 3 \pmod 4; \end{cases}$$

$$t_{m\phi}(3) = \begin{cases} 6r+1 \text{ if } n \equiv 0 \pmod 4 \\ 6r+4 \text{ if } n \equiv 1 \pmod 4 \\ 6r+5 \text{ if } n \equiv 2 \pmod 4 \\ 6r+7 \text{ if } n \equiv 3 \pmod 4. \end{cases}$$

This satisfies the condition of 4 – TMC graph.

Theorem 4.3. The graph $K_{1,n} \cup CH_n$ is 4 – TMC.

Proof. Let u be an apex vertex and $u_1, u_2, \ldots, u_n$ be the pendent vertices of the star $K_{1,n}$. Let $v_1 v_2 \ldots v_n v_1$ be the cycle C_n. Now $V(CH_n) = V(C_n) \cup \{v, w_i : 1 \le i \le n\}$ and $E(CH_n) = E(C_n) \cup \{v_i w_i : 1 \le i \le n\} \cup \{vv_i\} \cup \{w_i w_{i+1} : 1 \le i \le n-1\} \cup \{w_n w_1\}$.

Define ψ: $V(K_{1,n} \cup CH_n) \to \{0,1,2,3\}$ by

$\psi(u) = 0$;

$\psi(v) = 2$;

$\psi(u_1) = \psi(u_2) = \ldots = \psi(u_n) = 0$;

$\psi(v_1) = \psi(v_2) = \ldots = \psi(v_n) = 1$;

$\psi(w_1) = \psi(w_2) = \ldots = \psi(w_n) = 3$.

Now $t_{m\psi}(0) = t_{m\psi}(2) = 2n+1$;

$t_{m\psi}(1) = t_{m\psi}(3) = 2n$.

This satisfies the condition of 4 – TMC graph.

Theorem 4.4. The graph $K_{1,n} \cup T_n$ is 4 – TMC.

Proof. Let w be an apex vertex and $w_1, w_2, \ldots, w_n$ be the pendent vertices of the star $K_{1,n}$. Take the vertex set and edge set of T_n as in Definition 3.2.

Define ϕ : $V(K_{1,n} \cup T_n) \to \{0,1,2,3\}$ as follows:

$\phi(w) = 0$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, where $r \in \mathbb{N}$.

$\phi(w_1) = \phi(w_2) = \ldots = \phi(w_r) = 0$;

$\phi(w_{r+1}) = \phi(w_{r+2}) = \ldots = \phi(w_{2r+1}) = 1$;

$\phi(w_{2r+2}) = \phi(w_{2r+3}) = \ldots = \phi(w_{4r}) = 3$;

$\phi(u_1) = \phi(u_2) = \ldots = \phi(u_r) = 0$;

$\phi(u_{r+1}) = \phi(u_{r+2}) = \ldots = \phi(u_{2r}) = 1$;

$\phi(u_{2r+1}) = \phi(u_{2r+2}) = \ldots = \phi(u_{3r}) = 2$;

$\phi(u_{3r+1}) = \phi(u_{3r+2}) = \ldots = \phi(u_{4r}) = 3$;

$\phi(v_1) = \phi(v_2) = \ldots = \phi(v_r) = 0$;

$\phi(v_{r+1}) = \phi(v_{r+2}) = \ldots = \phi(v_{2r-1}) = 1$;

$\phi(v_{2r}) = \phi(v_{2r+1}) = \ldots = \phi(v_{3r-1}) = 2$;

$\phi(v_{3r}) = \phi(v_{3r+1}) = \ldots = \phi(v_{4r-1}) = 3$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, where $r \in \mathbb{N}$.

As in case 1 label the vertices w_i, u_i $(1 \le i \le 4r)$, v_i $(1 \le i \le 4r-1)$.

$\phi(w_{4r+1}) = 0$;

$\phi(u_{4r+1}) = 3$;

$\phi(v_{4r}) = 1$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, where $r \in \mathbb{N}$.

Label the vertices w_i, u_i $(1 \le i \le 4r)$, v_i $(1 \le i \le 4r-1)$ as in case 1.

$\phi(w_{4r+1}) = 0$;

$\phi(w_{4r+2}) = 0$;

$\phi(u_{4r+1}) = 2$;

$\phi(u_{4r+2}) = 1$;

$\phi(v_{4r}) = 3$;

$\phi(v_{4r+1}) = 1$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, where $r \in \mathbb{N}$.

As in case 1 label the vertices w_i, u_i $(1 \le i \le 4r)$, v_i $(1 \le i \le 4r-1)$.

$\phi(w_{4r+1}) = 0$;

$\phi(w_{4r+2}) = 0$;

$\phi(w_{4r+3}) = 3$;

$\phi(u_{4r+1}) = 2$;

$\phi(u_{4r+2}) = 1$;

$\phi(u_{4r+3}) = 2$;

$\phi(v_{4r}) = 3$;

$\phi(v_{4r+1}) = 1;$
$\phi(v_{4r+2}) = 0.$
In view of above labelling, we get

Now $t_{m\phi}(0) = \begin{cases} 7r-1 \text{ if } n \equiv 0 \pmod 4 \\ 7r+1 \text{ if } n \equiv 1 \pmod 4 \\ 7r+3 \text{ if } n \equiv 2 \pmod 4 \\ 7r+4 \text{ if } n \equiv 3 \pmod 4; \end{cases}$

$t_{m\phi}(1) = \begin{cases} 7r \quad \text{ if } n \equiv 0 \pmod 4 \\ 7r+1 \text{ if } n \equiv 1 \pmod 4 \\ 7r+3 \text{ if } n \equiv 2 \pmod 4 \\ 7r+5 \text{ if } n \equiv 3 \pmod 4; \end{cases}$

$t_{m\phi}(2) = \begin{cases} 7r-1 \text{ if } n \equiv 0 \pmod 4 \\ 7r+1 \text{ if } n \equiv 1 \pmod 4 \\ 7r+2 \text{ if } n \equiv 2 \pmod 4 \\ 7r+5 \text{ if } n \equiv 3 \pmod 4; \end{cases}$

$t_{m\phi}(3) = \begin{cases} 7r-1 \text{ if } n \equiv 0 \pmod 4 \\ 7r+1 \text{ if } n \equiv 1 \pmod 4 \\ 7r+3 \text{ if } n \equiv 2 \pmod 4 \\ 7r+4 \text{ if } n \equiv 3 \pmod 4. \end{cases}$

This satisfies the condition of 4 – TMC graph.

Theorem 4.5. The graph $B_{n,n} \cup B_{n,n}$ is 4 – TMC.

Proof. Let $v_1, v_2, \ldots, v_n$, $w_1, w_2, \ldots, w_n$, be the pendent vertices and v, w be the apex vertices of $B_{n,n}$. Let $x_1, x_2, \ldots, x_n$, $y_1, y_2, \ldots, y_n$, be the pendent vertices and x, y be the apex vertices of $B_{n,n}$.

Define θ: V $(B_{n,n} \cup B_{n,n}) \rightarrow \{0,1,2,3\}$ by
$\theta(v) = 0;$
$\theta(w) = 1;$
$\theta(x) = 2;$
$\theta(y) = 3.$
$\theta(v_1) = \theta(v_2) = \ldots = \theta(v_n) = 0;$
$\theta(w_1) = \theta(w_2) = \ldots = \theta(w_n) = 1;$
$\theta(x_1) = \theta(x_2) = \ldots = \theta(x_n) = 2;$
$\theta(y_1) = \theta(y_2) = \ldots = \theta(y_n) = 3.$
Now $t_{m\theta}(0) = t_{m\theta}(2) = 2n+1;$
$t_{m\theta}(1) = t_{m\theta}(3) = 2n+2.$
This satisfies the condition of 4 – TMC graph.

Theorem 4.6. The graph $B_{n,n} \cup J_{n,n}$ is 4 – TMC.

Proof. Let $w_1, w_2, \ldots, w_n$, $z_1, z_2, \ldots, z_n$, be the pendent vertices and w, z be the apex vertices of $B_{n,n}$. Take the vertex set and edge set of $J_{n,n}$ as in Definition 3.1.

Define ϕ: V $(B_{n,n} \cup J_{n,n}) \rightarrow \{0,1,2,3\}$ as follows:
$\phi(w) = 0;$
$\phi(z) = 1;$
$\phi(u) = 3;$
$\phi(v) = 3;$
$\phi(x) = 3;$
$\phi(y) = 1.$
$\phi(w_1) = \phi(w_2) = \ldots = \phi(w_n) = 0;$
$\phi(z_1) = \phi(z_2) = \ldots = \phi(z_n) = 1;$
$\phi(u_1) = \phi(u_2) = 0;$
$\phi(u_3) = \phi(u_4) = \ldots = \phi(u_n) = 2.$
$\phi(v_1) = \phi(v_2) = \ldots = \phi(v_n) = 3.$
Now $t_{m\phi}(0) = t_{m\phi}(1) = 2n+3;$
$t_{m\phi}(2) = t_{m\phi}(3) = 2n+3.$
This satisfies the condition of 4 – TMC graph.

Theorem 4.7. The graph $B_{n,n} \cup CH_n$ is 4 – TMC.

Proof. Let $x_1, x_2, \ldots, x_n$, $y_1, y_2, \ldots, y_n$, be the pendent vertices and x, y be the apex vertices of $B_{n,n}$. Let $v_1 v_2 \ldots v_n v_1$ be the cycle C_n. Now $V(CH_n) = V(C_n) \cup \{v, w_i : 1 \leq i \leq n\}$ and $E(CH_n) = E(C_n) \cup \{v_i w_i : 1 \leq i \leq n\} \cup \{v v_i\} \cup \{w_i w_{i+1} : 1 \leq i \leq n-1\} \cup \{w_n w_1\}$.

Define ψ: V $(B_{n,n} \cup CH_n) \rightarrow \{0,1,2,3\}$ by
$\psi(x) = 2;$

$\psi(y) = 3$;
$\psi(v) = 2$.
Case 1. n is odd.
Let n = 2r+1, where r ϵ ℕ.
$\psi(x_1) = \psi(x_2) = \ldots\ldots\ldots. = \psi(x_{r+2}) = 0$;
$\psi(x_{r+3}) = \psi(x_{r+4}) = \ldots\ldots\ldots. = \psi(x_{2r+1}) = 3$;
$\psi(y_1) = \psi(y_2) = \ldots\ldots\ldots. = \psi(y_{r+2}) = 3$;
$\psi(y_{r+3}) = \psi(y_{r+4}) = \ldots\ldots\ldots. = \psi(y_{2r+1}) = 2$;
$\psi(v_1) = \psi(v_2) = \ldots\ldots\ldots. = \psi(v_{2r+1}) = 0$;
$\psi(w_1) = \psi(w_2) = \ldots\ldots\ldots. = \psi(w_{2r+1}) = 2$.
Case 2. n is even.
Let n = 2r, where r ϵ ℕ.
$\psi(x_1) = \psi(x_2) = \ldots\ldots\ldots. = \psi(x_{r+1}) = 0$;
$\psi(x_{r+2}) = \psi(x_{r+3}) = \ldots\ldots\ldots. = \psi(x_{2r}) = 3$;
$\psi(y_1) = \psi(y_2) = \ldots\ldots\ldots. = \psi(y_{r+1}) = 3$;
$\psi(y_{r+2}) = \psi(y_{r+3}) = \ldots\ldots\ldots. = \psi(y_{2r}) = 2$;
$\psi(v_1) = \psi(v_2) = \ldots\ldots\ldots. = \psi(v_{2r}) = 0$;
$\psi(w_1) = \psi(w_2) = \ldots\ldots\ldots. = \psi(w_{2r}) = 2$.
Now $t_{m\psi}(0) = t_{m\psi}(1) = \begin{cases} 5r+4 & if\ n\ is\ odd \\ 5r+1 & if\ n\ is\ even; \end{cases}$
$t_{m\psi}(2) = t_{m\psi}(3) = \begin{cases} 5r+3 & if\ n\ is\ odd \\ 5r+1 & if\ n\ is\ even. \end{cases}$

This satisfies the condition of 4 – TMC graph.
Theorem 4.8. The graph $B_{n,n} \cup T_n$ is 4 – TMC.
Proof. Let $x_1, x_2, \ldots, x_n$, $y_1 y_2, \ldots, y_n$, be the pendent vertices and x, y be the apex vertices of $B_{n,n}$. Take the vertex set and edge set of T_n as in Definition 3.2.
Define ϕ: V ($B_{n,n} \cup T_n$) →{0,1,2,3} as follows:
$\phi(x) = 0$;
$\phi(y) = 1$.
Case 1. n ≡ 0 (mod 4).
Let n = 4r, where r ϵ ℕ.
$\phi(x_1) = \phi(x_2) = \ldots\ldots\ldots. = \phi(x_{2r}) = 0$;
$\phi(x_{2r+1}) = \phi(x_{2r+2}) = \ldots\ldots\ldots. = \phi(w_{4r}) = 1;\phi$
$\phi(y_1) = \phi(y_2) = \ldots\ldots\ldots. = \phi(y_{4r}) = 3$;
$\phi(u_1) = \phi(u_2) = \ldots\ldots\ldots. = \phi(u_r) = 0$;
$\phi(u_{r+1}) = \phi(u_{r+2}) = \ldots\ldots\ldots. = \phi(u_{2r}) = 1$;
$\phi(u_{2r+1}) = \phi(u_{2r+2}) = \ldots\ldots\ldots. = \phi(u_{3r}) = 2$;
$\phi(u_{3r+1}) = \phi(u_{3r+2}) = \ldots\ldots\ldots. = \phi(u_{4r}) = 3$;
$\phi(v_1) = \phi(v_2) = \ldots\ldots\ldots. = \phi(v_r) = 0$;
$\phi(v_{r+1}) = \phi(v_{r+2}) = \ldots\ldots\ldots. = \phi(v_{2r-1}) = 1$;
$\phi(v_{2r}) = \phi(v_{2r+1}) = \ldots\ldots\ldots. = \phi(v_{3r-1}) = 2$;
$\phi(v_{3r}) = \phi(v_{3r+1}) = \ldots\ldots\ldots. = \phi(v_{4r-1}) = 3$.
Case 2. n ≡ 1 (mod 4).
Let n = 4r+1, where r ϵ ℕ.
As in case 1 label the vertices x_i, y_i, u_i (1 ≤ i ≤ 4r), v_i (1 ≤ i ≤ 4r-1).
$\phi(x_{4r+1}) = 0$;
$\phi(y_{4r+1}) = 1$;
$\phi(u_{4r+1}) = 3$;
$\phi(v_{4r}) = 0$.
Case 3. n ≡ 2 (mod 4).
Let n = 4r+2, where r ϵ ℕ.
Label the vertices x_i, y_i, u_i (1 ≤ i ≤ 4r+1), v_i (1 ≤ i ≤ 4r) as in case 2.
$\phi(x_{4r+2}) = 0$;
$\phi(y_{4r+2}) = 1$;
$\phi(u_{4r+2}) = 3$;
$\phi(v_{4r+1}) = 0$.
Case 4. n ≡ 3 (mod 4).
Let n = 4r+3, where r ϵ ℕ.
As in case 1 label the vertices x_i, y_i, u_i (1 ≤ i ≤ 4r+2), v_i (1 ≤ i ≤ 4r+1).
$\phi(x_{4r+3}) = 1$;

$\phi\ (y_{4r+3}) = 3;$
$\phi\ (u_{4r+3}) = 3;$
$\phi\ (v_{4r+2}) = 0.$
In view of above labelling, we get

Now $t_{m\phi}(0) = \begin{cases} 9r-1\ if\ \text{n} \equiv 0 \pmod 4 \\ 9r+2\ if\ \text{n} \equiv 1 \pmod 4 \\ 9r+5\ if\ \text{n} \equiv 2 \pmod 4 \\ 9r+6\ if\ \text{n} \equiv 3 \pmod 4; \end{cases}$

$t_{m\phi}(1) = \begin{cases} 9r \quad\ if\ \text{n} \equiv 0 \pmod 4 \\ 9r+2\ if\ \text{n} \equiv 1 \pmod 4 \\ 9r+4\ if\ \text{n} \equiv 2 \pmod 4 \\ 9r+6\ if\ \text{n} \equiv 3 \pmod 4; \end{cases}$

$t_{m\phi}(2) = t_{m\phi}(3) = \begin{cases} 9r \quad\ if\ \text{n} \equiv 0 \pmod 4 \\ 9r+2\ if\ \text{n} \equiv 1 \pmod 4 \\ 9r+4\ if\ \text{n} \equiv 2 \pmod 4 \\ 9r+7\ if\ \text{n} \equiv 3 \pmod 4. \end{cases}$

This satisfies the condition of 4 – TMC graph.

REFERENCES

1. J.A.Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **19** (2016) #Ds6.
2. F.Harary, Graph theory, *Addision Wesley*, New Delhi (1969).
3. R.Ponraj, S.Subbulakshmi, S.Somasundaram, *k*-total mean cordial graphs, *J.Math.Comput.Sci.* 10(2020), No.5, 1697-1711.
4. R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-total mean cordial graphs derived from paths, *J.Appl and Pure Math.* Vol 2(2020), 319-329.
5. R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-total mean cordial labeling in subdivision graphs, *Journal of Algorithms and Computation* 52(2020),1-11.
6. R.Ponraj, S.Subbulakshmi, S.Somasundaram, Some 4-total mean cordial graphs derived from wheel, *J. Math. Comput. Sci.* 11(2021), 467-476.