System Reliability Estimation with Two Types of Common Cause Shock Failures

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Abstract— This paper discusses the reliability analysis of a three component identical system. The system may affects by two types of failures viz., Lethal Common Cause Shock (LCCS) and Non-Lethal Common Cause Shock (NCCS) failures. By using stochastic process, the set of differential equations of the existing model are derived to attain reliability measures such as system reliability and Mean Time to Failure (MTTF) in both series and parallel cases. In addition, the Maximum Likelihood Estimates (MLE) of the above said measures are derived and shown in numerical illustration by using simulation process. The numerical tables shows the outcomes and recommend that LCCS and NCCS are the most leading causes of failures while studying the performance of the systems in reliability theory.

***Keywords*— Multi-state system, Reliability Analysis, MTTF, LCCS & NCCS Failures, M L Estimation, Simulation**

I. INTRODUCTION

The parameter reliability is used to assess the effectiveness of the system/item and availability under appropriate working conditions for a given period of time. Mostly, the reliability evaluation techniques have been used in aerospace industry and military applications, there after nuclear power plants, electricity supply and continuous process plants were rapidly applied the developments of reliability techniques. Situations where the failures of some or even majority of system units could lead to partial ability or partial system down time to perform required operations are quite common in electrical/mechanical systems. These types of models are also used to designate multi-channel systems.

While assessing the reliability, we need to consider the Common Cause Shock (CCS) failures which can severely degrade the reliability of devices, systems etc. These events are purely external causes which produces multiple failures. As per the reliability literature, in specific two types of CCS failures viz. Lethal common cause shock failures, which is the occurrence of simultaneous outage of all units in the system and the other is non-lethal common cause shock failures, which is the occurrence of random number of units to simultaneous outage of several units in the system. Some attempts have been made in this direction by several authors. Billinton and Allan [1] discussed the role of common cause shock failures in different frame works. Chari et al [2] derived the reliability measures of a two unit system in the presence of common cause shock failures. Dhillon [3], [4] discussed the role of common cause failures as well as human errors in system reliability aspects. Reddy and Verma [5, 6] have discussed reliability measures for 2-component non-identical and identical systems with common cause failures. Sagar et al [7], [9] and Awgichew et al [8] examined the reliability measurements with common cause shock failures for two unit identical system. They derivative M L estimates of two unit system reliability measures such as frequency of failures in the presence of CCS failures. Sreedhar et al [10], [11] analysed 2-unit non identical system with CCS failures. They calculated Maximum Likelihood estimation for estimating reliability indices.

II. MODEL, ASSUMPTIONS AND NOTATIONS

A. System Description

As we discussed in the introduction, none of the authors studies three unit identical systems with LCCS and NCCS failures and maximum likelihood estimation as well. We examined the reliability of the redundant system in series as well as parallel configurations. There are four different possible states for the system operation: perfect state, minor failed state, major failed state, and completely failed states. The failure rates of each unit are constant in nature, but they follow exponential distribution.

B. Assumptions

We considered the following assumptions:

1. The system works until one or more units are functioning.

2. The system units fail individually and also simultaneously due to lethal common cause shock failures or non-lethal common cause shock failures in Poisson manner.
3. Individual, lethal common cause shock and non-lethal common cause shock failures are independent to each other.
4. A repair man is available and ready to restore minor and major faults whether they are failed individually or simultaneously due to common cause shocks.
5. The repair times of failed units depend on the failure mode and are assumed exponentially distributed.

C. Notations:

t	–	Time scale variable
$\lambda / \omega / \beta$	–	*I*ndividual failures / LCCS failures / NCCS failures
μ_0 / μ_1	–	Repair rates
T	–	Time to failure of a unit
$p(q)$	–	The probability of simultaneous failures of units due to NCCS / LCCS
$P_i(t)$	–	Probability that the system is in state (i = 0, 1, 2, 3) at time t
$R_{LNS}(t) / R_{LNP}(t)$	–	Reliability when units are in series / parallel
$\hat{R}_{LNS}(t) / \hat{R}_{LNP}(t)$	–	M L estimate of reliability function for series mode / parallel mode
$E_{LNS}(T) / E_{LNP}(T)$	–	Mean time to failure for series / parallel

III. State Transition Diagram and Description

In view of the stated assumptions, we formulate state transition diagram of the model in Fig.1. The state description of the current model highlights that initially all the units are functioning perfectly and it in a state of s_0. After any one of the three units is down and others are functioning, it switches to state s_1 which is regarded as minor partially down state. If two units have failed, it will be passed to s_2 that is the major partially down state. In both cases, to restore the system we use general repair. State s_3 indicates completely down state due to failure of all the three units. The quantities that appear in Fig.1 are defined as:

$$\left.\begin{aligned} \lambda_0 &= 3(\lambda + \beta p q^2) \\ \lambda_1 &= (\beta p^3 + \omega) \\ \lambda_2 &= (\beta p^2 + \omega) \\ \lambda_c &= \lambda + \beta p \\ \mu_0 &= \mu \\ \mu_1 &= 2\mu \end{aligned}\right\} \quad (1)$$

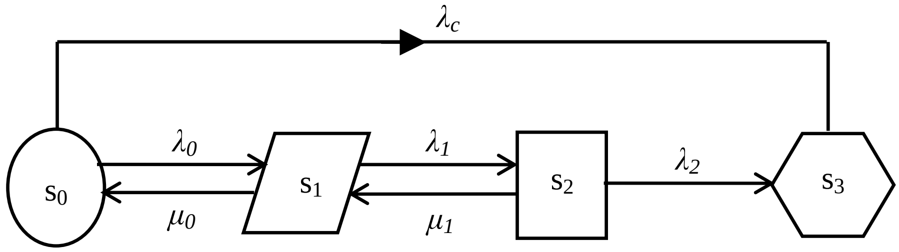

Fig.1 State transition diagram

IV. MATHEMATICAL MODEL

The set of differential equations associated with the current mathematical model for the above state transition diagram are:

$$P_0^{'}(t) = -(\lambda_0 + \lambda_c)P_0(t) + \mu_0 P_1(t) \quad (2)$$

$$P_1^{'}(t) = \lambda_0 P_0(t) - (\lambda_1 + \mu_0)P_1(t) + \mu_1 P_2(t) \quad (3)$$

$$P_2^{'}(t) = \lambda_1 P_1(t) - (\lambda_2 + \mu_1)P_2(t) \quad (4)$$

$$P_3^{'}(t) = \lambda_c P_0(t) + \lambda_2 P_2(t) \quad (5)$$

Initial conditions: $P_0(0) = 1$, and other state probabilities are zero at $t = 0$

Taking Laplace transformation of equations (2) to (5) and using initial conditions, we obtain

$$P_0(t) = \frac{r_1^2 + r_1 K + L}{(r_1 - r_3)(r_1 - r_2)}\exp(r_1 t) - \frac{r_2^2 + r_2 K + L}{(r_1 - r_2)(r_2 - r_3)}\exp(r_2 t) + \frac{r_3^2 + r_3 K + L}{(r_1 - r_3)(r_2 - r_3)}\exp(r_3 t) \quad (6)$$

$$P_1(t) = \frac{\lambda_0(r_1 + \lambda_2 + \mu_1)}{(r_1 - r_3)(r_1 - r_2)}\exp(r_1 t) - \frac{\lambda_0(r_2 + \lambda_2 + \mu_1)}{(r_1 - r_2)(r_2 - r_3)}\exp(r_2 t) + \frac{\lambda_0(r_3 + \lambda_2 + \mu_1)}{(r_1 - r_3)(r_2 - r_3)}\exp(r_3 t) \quad (7)$$

$$P_2(t) = \frac{\lambda_0 \lambda_1}{(r_1 - r_3)(r_1 - r_2)}\exp(r_1 t) - \frac{\lambda_0 \lambda_1}{(r_1 - r_2)(r_2 - r_3)}\exp(r_2 t) + \frac{\lambda_0 \lambda_1}{(r_1 - r_3)(r_2 - r_3)}\exp(r_3 t) \quad (8)$$

$$P_3(t) = 1 - [P_0(t) + P_1(t) + P_2(t)] \quad (9)$$

$$\left.\begin{aligned} r_1 &= -r\sin(\alpha) - \frac{A_1}{3} \\ r_2 &= r\sin\left(\frac{\pi}{3} + \alpha\right) - \frac{A_1}{3} \\ r_3 &= r\sin\left(-\frac{\pi}{3} + \alpha\right) - \frac{A_1}{3} \end{aligned}\right\} \quad (10)$$

Here

$$\left.\begin{aligned} q &= A_3 - \frac{A_1 A_2}{3} + 2\frac{A_1^3}{27} \\ r &= \frac{2}{3}\left(A_1^2 - 3A_2\right)^{1/2} \\ \alpha &= \frac{\sin^{-1}\left(\frac{-4q}{r^3}\right)}{3} \end{aligned}\right\}$$

where

$$\left.\begin{aligned} K &= (\lambda_1 + \lambda_2 + \mu_0 + \mu_1) \\ L &= (\lambda_1 \lambda_2 + \mu_0 \lambda_2 + \mu_0 \mu_1 \end{aligned}\right\} \quad (11)$$

$$A_1 = (\lambda_0 + \lambda_1 + \lambda_2 + \mu_0 + \mu_1 + \lambda_c)$$
$$A_2 = (\mu_0\mu_1 + \mu_0\lambda_c + \mu_1\lambda_c + \mu_1\lambda_0 + \mu_0\lambda_2 + \lambda_1\lambda_c + \lambda_2\lambda_c + \lambda_0\lambda_1 + \lambda_0\lambda_2 + \lambda_1\lambda_2)$$
$$A_3 = (\mu_0\mu_1\lambda_c + \mu_0\lambda_c\lambda_2 + \lambda_1\lambda_2\lambda_c + \lambda_0\lambda_1\lambda_2)$$

V. RELIABILITY CHARACTERISTICS

In this section, we derived some performance measures when three units of the system are in series and in parallel modes.

A. Series System

In this case, all units of the system are in good working condition. The states s_1 to s_2 and s_2 to s_3 are absorbing states and hence no transition is allowed. Therefore, the reliability function is given by:

$$R_{LNS}(t) = P_0(t)$$
$$= \exp(-(4\lambda + \beta p(1+3q^2))t) \tag{12}$$

And the mean time to failure is:

$$E_{LNS}(T) = \int_0^\infty R_{LNS}(t).dt$$
$$= \frac{1}{4\lambda + \beta p(1+3q^2)} \tag{13}$$

B. Parallel System

The reliability function for parallel system is:

$$R_{LNP}(t) = P_0(t) + P_1(t) + P_2(t)$$
$$= M_1 \exp(r_1 t) - M_2 \exp(r_2 t) + M_3 \exp(r_3 t) \tag{14}$$

Where

$$M_1 = \left((r_1^2 + r_1 K + L) + \lambda_0(r_1 + \lambda_2 + \mu_1) + \lambda_0\lambda_1\right) \big/ (r_1 - r_3)(r_1 - r_2)$$
$$M_2 = \left((r_2^2 + r_2 K + L) + \lambda_0(r_2 + \lambda_2 + \mu_1) + \lambda_0\lambda_1\right) \big/ (r_2 - r_3)(r_1 - r_2)$$
$$M_3 = \left((r_3^2 + r_3 K + L) + \lambda_0(r_3 + \lambda_2 + \mu_1) + \lambda_0\lambda_1\right) \big/ (r_1 - r_3)(r_2 - r_3)$$

also K, L, r_1, r_2, r_3 are defined in equations (11) and (10)
and MTTF of parallel system is:

$$E_{LNP}(T) = \int_0^\infty R_{LNP}(t).dt$$
$$= \frac{-(\lambda_0\lambda_1 + \lambda_0\lambda_2 + \mu_1\lambda_0 + \lambda_1\lambda_2 + \mu_0\lambda_2 + \mu_0\mu_1)}{r_1 r_2 r_3} \tag{15}$$

Where $\lambda_0, \lambda_1, \lambda_2, \mu_0, \mu_1$ and r_1, r_2, r_3 are defined in (1) and (10)

C. Numerical Results

For illustration purpose by fixing $\lambda = 0.01, \mu_0 = 1, \mu_1 = 1.5, p = 0.3$ and for different values of time-variable *t* = 0, 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 units of time, we get different values of reliability for series and parallel cases as shown in table 1.

TABLE I

RELIABILITY FOR SERIES AND PARALLEL SYSTEMS

Time (t)	Series System			Parallel System		
	β=0.2, ω=0.2	β=0.3, ω=0.3	β=0.4, ω=0.5	β=0.2, ω=0.2	β=0.3, ω=0.3	β=0.4, ω=0.5
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.6863	0.5918	0.5103	0.8761	0.8293	0.7804
4	0.4710	0.3502	0.2604	0.7703	0.6908	0.6074
6	0.3233	0.2073	0.1329	0.6776	0.5756	0.4722
8	0.2219	0.1227	0.0678	0.5960	0.4796	0.3670
10	0.1523	0.0726	0.0346	0.5243	0.3997	0.2852
12	0.1045	0.0430	0.0277	0.4612	0.3330	0.2217
14	0.0717	0.0254	0.0090	0.4057	0.2775	0.1723
16	0.0492	0.0150	0.0046	0.3569	0.2313	0.1339
18	0.0338	0.0089	0.0023	0.3139	0.1927	0.1041
20	0.0232	0.0053	0.0012	0.2761	0.1605	0.0809

TABLE II

MTTF FOR SERIES AND PARALLEL SYSTEMS

$\mu = 1, p = 0.2$

	Series System (β, ω)	Parallel System

λ				(β, ω)		
	(0.1, 0.1)	(0.2, 0.2)	(0.3, 0.4)	(0.1, 0.1)	(0.2, 0.2)	(0.3, 0.4)
0.01	10.163	6.378	4.647	35.702	21.605	14.702
0.02	7.225	5.081	3.918	27.459	18.452	13.131
0.03	5.605	4.223	3.387	22.542	16.215	11.925
0.04	4.579	3.613	2.983	19.275	14.544	10.970
0.05	3.869	3.157	2.665	16.947	13.250	10.195
0.06	3.351	2.803	2.408	15.205	12.218	9.553
0.07	2.955	2.520	2.197	13.851	11.375	9.014
0.08	2.643	2.289	2.019	12.769	10.674	8.553
0.09	2.390	2.097	1.868	11.885	10.082	8.155
0.10	2.182	1.935	1.739	11.149	9.575	7.809

VI. Estimation and Simulation

A. Estimation

In this, we have attempted Maximum likelihood estimation to estimate the system reliability and MTTF of the present model. However, the system is under the influence of NCCS and LCCS failures in addition to individual failures.

Let the samples $x_1, x_2, ..., x_n$; $y_1, y_2, ..., y_n$ and $w_1, w_2, ..., w_n$ with size '*n*' representing times between individual, NCCS and LCCS failures which will obey exponential law.

Let the samples $z_{11}, z_{12}, ..., z_{1n}$; $z_{21}, z_{22}, ..., z_{2n}$ with size '*n*' number of times between repairs of the units with exponential population law.

$\hat{\hat{x}}, \hat{\hat{y}}, \hat{\hat{w}}, \hat{\hat{z}}_1, \hat{\hat{z}}_2$ are the maximum likelihood estimates of $\lambda, \beta, \omega, \mu_0, \mu_1$ respectively.

Where, $\hat{\hat{x}} = \frac{1}{\overline{x}}; \hat{\hat{y}} = \frac{1}{\overline{y}}; \hat{\hat{w}} = \frac{1}{\overline{w}}; \hat{\hat{z}}_1 = \frac{1}{\overline{z}_1}; \hat{\hat{z}}_2 = \frac{1}{\overline{z}_2}$; $\overline{x} = \frac{\sum x_i}{n}; \overline{y} = \frac{\sum y_i}{n}$ $\overline{w} = \frac{\sum w_i}{n}; \overline{z}_1 = \frac{\sum z_{1i}}{n}; \overline{z}_2 = \frac{\sum z_{2i}}{n}$

B. Simulation

We compute M L estimates such as $\hat{R}_{LNS}(t), \hat{R}_{LNP}(t)$ of the present model by using Monte-Carlo simulation. For a range of specified values of the rates of $\lambda, \beta, \omega, \mu_0, \mu_1$ and for the sample size n=5(5)15 were simulated in each case with N=20000(30000)100000 in order to evolve mean square error (MSE) in each case by using C++ (software).

TABLE III

RELIABILITY ESTIMATION FOR SERIES SYSTEM

$\lambda = 0.1,\ \beta = 0.2,\ \omega = 0.3, p = 0.3, t = 1$
SAMPLE SIZE (n = 5)

N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
20000	0.577989	0.492346	0.025525
50000	0.577989	0.491844	0.025762
80000	0.577989	0.492411	0.025832

SAMPLE SIZE (n = 10)

N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
20000	0.577989	0.522508	0.011552
50000	0.577989	0.523343	0.011446
80000	0.577989	0.523294	0.011507

SAMPLE SIZE (n = 15)

N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
20000	0.577989	0.532862	0.007407
50000	0.577989	0.533398	0.007394
80000	0.577989	0.533402	0.007395

TABLE IV

RELIABILITY ESTIMATION FOR PARALLEL SYSTEM

$\lambda = 0.1, \beta = 0.2, \omega = 0.3, \mu_0 = 1, \mu_1 = 1.5, p = 0.3, t = 1$
SAMPLE SIZE (n = 5)

N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
20000	0.868255	0.830932	0.003984
50000	0.868255	0.831099	0.004021
80000	0.868255	0.831404	0.003957

SAMPLE SIZE (n = 10)

N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
20000	0.868255	0.846471	0.001455
50000	0.868255	0.846431	0.001455
80000	0.868255	0.846591	0.001446

SAMPLE SIZE (n = 15)

N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
20000	0.868255	0.851125	0.000879
50000	0.868255	0.850872	0.000895
80000	0.868255	0.851039	0.000893

VII. RESULT DISCUSSION AND CONCLUSIONS

This paper discusses the reliability measures of a 3-unit system in series and parallel under the lethal and non-lethal common cause shock failures. A study of the model with the support of maximum likelihood estimation were presented and established empirically. The significance of LCCS and NCCS failures in these types of models were conferred through numerical drawing and simulation validity in this article.

Table I shows the evidence for the reliability of the system at various time values. The reliability is decreasing in both series and parallel cases when LCCS and NCCS failure rates are increasing. Table II contains the difference in the MTTF corresponding to different failure rates in series and parallel system. It is observed that MTTF decreases as the failure rate increases and also there is a great advance from series to parallel system. Table III and Table IV show the simulation study in order to establish the validity of the proposed M L estimates. It is perceived that the point estimates become more precise when the sample size is large and mean square error decreases with increasing the sample size. The model conferred in this paper was found to be great significance in proper maintenance analysis, and assessment of the system.

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