A STUDY OF STRESS ANALYSIS ON GEAR TOOTH USING FINITE ELEMENT METHODS

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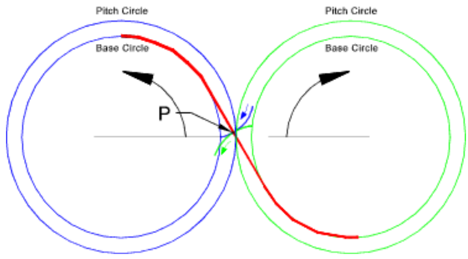
Abstract

In spite of the number of investigations devoted to gear research and analysis there still remains to be developed, a general numerical approach capable of predicting the effects of variations in gear geometry, contact and bending stresses, torsional mesh stiffness and transmission errors. The objectives of this study are to use a numerical approach to develop stress based on the behavior of the pressure angle of the spur gears in mesh, this is to help to predict the effect of gear tooth stresses.

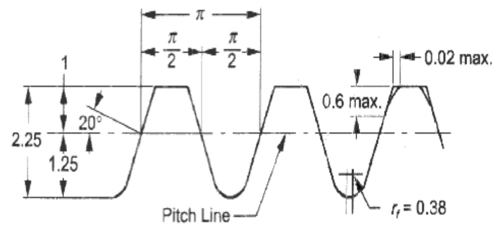
1.Introduction

Gears are machine elements used to transmit rotary motion between two shafts, normally with a constant ratio. The pinion is the smallest gear and the larger gear is called the gear wheel. A rack is a rectangular prism with gear teeth machined along one side- it is in effect a gear wheel with an infinite pitch circle diameter. In practice the action of gears in transmitting motion is a cam action each pair of mating teeth acting as cams. Gear design has evolved to such a level that throughout the motion of each contacting pair of teeth the velocity ratio of the gears is maintained fixed and the velocity ratio is still fixed as each



subsequent pair of teeth come into contact. When the teeth action is such that the driving tooth moving at constant angular velocity produces a proportional constant velocity of the driven tooth the action is termed a conjugate action.

The teeth shape universally selected for the gear teeth is the involute profile. Consider one end of a piece of string is fastened to the OD of one cylinder and the other end of the string is fastened to the OD of another cylinder parallel to the first and



both cylinders are rotated in the opposite directions to tension the string. The point on the string midway between the cylinders P is marked. As the left hand cylinder rotates CCW the point moves towards this cylinder as it wraps on. The point moves away from the right hand cylinder as the string unwraps. The point traces the involute form of the gear teeth.

The lines normal to the point of contact of the gears always intersects the centre line joining the gear centers at one point called the pitch point. For each gear the circle passing through the pitch point is called the pitch circle. The gear ratio is proportional to the diameters of the two pitch circles. For metric gears (as adopted by most of the world's nations) the gear proportions are based on the module.

m = (Pitch Circle Diameter (mm)) / (Number of teeth on gear).

In the USA the module is not used and instead the Diametric Pitch d_p is used

d_p = (Number of Teeth) / Diametrical Pitch (inches)

Profile of a standard 1mm module gear teeth for a gear with infinite radius (Rack).Other module teeth profiles are directly proportion. E.g. 2mm module teeth are 2 x this profile. Many gears trains are very low power applications with an object of transmitting motion with minimum torque e.g. watch and clock mechanisms, instruments, toys, music boxes etc.

2.Spur Gear Forces, Torques, Velocities & Powers

- F = tooth force between contacting teeth (at angle pressure angle α to pitch line tangent. (N)
- F_t = tangential component of tooth force (N)
- F_s = Separating component of tooth force
- α= Pressure angle
- d_1 = Pitch Circle Diameter -driving gear (m)
- d_2 = Pitch Circle Diameter -driven gear (m)
- ω_1 = Angular velocity of driver gear (Rads/s)
- ω_2 = Angular velocity of driven gear (Rads/s)
- z_1 = Number of teeth on driver gear
- z_2 = Number of teeth on driven gear
- P = power transmitted (Watts)
- M = torque (Nm)
- η = efficiency

Tangential force on gears $F_t = F \cos \alpha$
Separating force on gears $F_s = F_t \tan \alpha$
Torque on driver gear $T_1 = F_t d_1 / 2$
Torque on driver gear $T_2 = F_t d_2 / 2$
Speed Ratio $= \omega_1 / \omega_2 = d_2 / d_1 = z_2 / z_1$
Input Power $P_1 = T_1 . \omega_1$

3. Need for at Present Situation

Clear cut analysis of a spur gear drive when a static load is applied to generate the profile of spur gear teeth and to predict the effect of gear bending stress using a two dimensional model. To ensure and estimate the life of the gear Impact of pressure angle on the spur gear drive.

A clean study of stress, deflection, stiffness and load sharing ratio for different pressure angle of a spur gear. To develop and to determine appropriate models of contact elements, to calculate contact stresses for the spur gear of pressure angle 20^0 using ANSYS and compare the results with. To develop a model of spur gear of different pressure angles like 14.5 and 22 and also to find the bending stress. It is a fundamental analysis required for studying the crack and its propagation.

This study is the root cause analysis for every crack and its propagation. To determine the static transmission errors of whole gear bodies in mesh. The objectives in the modeling of gears in the past by other researchers have varied from vibration analysis and noise control, to transmission error during the last five decades. The goals in gear modeling may be summarized as follows:Stress analysis such as prediction of bending stress for different pressure angles.

Evaluating condition monitoring, fault detection, diagnosis, prognosis, reliability and fatigue life by stress, load sharing ratio, stiffness and deflection.Different analysis models will be described. For gears, there are many types of gear failures but they can be classified into two general groups. One is failure of the root of the teeth because the bending strength is inadequate. The other is created on the surfaces of the gears.

There are two theoretical formulas, which deal with these two fatigue failure mechanisms. The other is the Lewis formula and AGMA method, which can be used to calculate the bending stresses. The surface pitting and scoring are the examples of failures which resulted in the fatigue failure of tooth surface

2. Effect of Load Sharing Behavior and Stress

The stiffness, k, of a body is a measure of the resistance offered by an elastic body to deformation. For an elastic body with a single degree of freedom (for example, stretching or compression of a rod), the stiffness is defined as

$$K = F/\delta$$

where

F is the force applied on the body

δ is the displacement produced by the force along the same degree of freedom (for instance, the change in length of a stretched spring)

In the International System of Units, stiffness is typically measured in Newton per metre. In English Units, stiffness is typically measured in pounds (lbs) per inch.

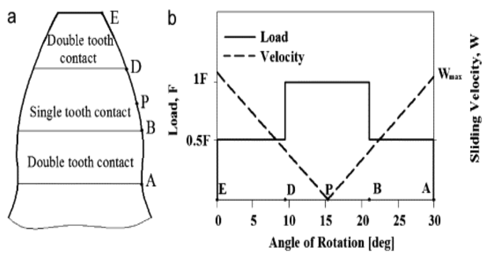
Generally speaking, deflections (or motions) of an infinitesimal element (which is viewed as a point) in an elastic body can occur along multiple degrees of freedom (maximum of six DOF at a point). For example, a point on a horizontal beam can undergo both a vertical displacement and a rotation relative to its undeformed axis. When there are M degrees of freedom a M x M matrix must be used to describe the stiffness at the point. The diagonal terms in the matrix are the direct-related stiffness’s (or simply stiffness’s) along the same degree of freedom and the off-diagonal terms are the coupling stiffness’s between two different degrees of freedom (either at the same or different points) or the same degree of freedom at two different points. In industry, the term influence coefficient is sometimes used to refer to the coupling stiffness.

It is noted that for a body with multiple DOF, the equation above generally does not apply since the applied force generates not only the deflection along its own direction (or degree of freedom), but also those along other directions.

For a body with multiple DOF, in order to calculate a particular direct-related stiffness (the diagonal terms), the corresponding DOF is left free while the remaining should be constrained. Under such a condition, the above equation can be used to obtain the direct-related stiffness for the degree of freedom which is unconstrained. The ratios between the reaction forces (or moments) and the produced deflection are the coupling stiffness’s.

The inverse of stiffness is compliance, typically measured in units of metres per Newton. In rheology it may be defined as the ratio of strain to stress, and so take the units of reciprocal stress, e.g. 1/Pa.

3. Load Sharing



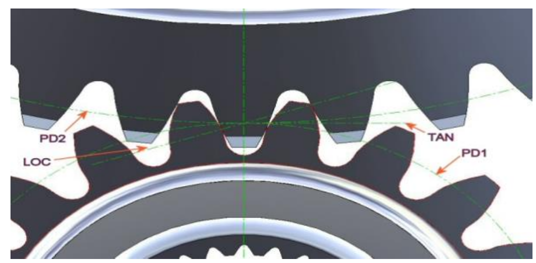
It is clear that the first point of action “A” which is located on the first meshing tooth is associated with point “D” which is located on the second meshing tooth of the same pinion. Therefore the load will be shared between these two points. Then point “E” will goes out of contact, therefore the full load will be applied on point “B”,” until the contact being at point “D”, then a new meshing tooth comes in to contact.

In low contact ratio gearing, and when a single pair of teeth is engaged, this pair transmits the full load or the full load is then applied on the one meshing tooth only. Almost, critical conditions (for maximum generated root stresses) occur in the one pair contact zone. When double pair of teeth are engaged, the transmitted load will be divided between two meshing teeth. Practically the load is not divided fairly; load sharing depends on contact ratio value and stiffness of meshing tooth at point of application of load. The load sharing is drawn against the path of contact for low contact ratio sharing. Failure of gear teeth is expected for lower contact ratio.

4. Meshing Cycle

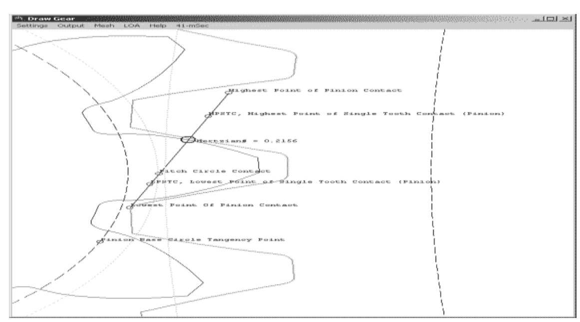
Consider two identical spur gears in mesh. When the first tooth pair is in contact at point A it is between the tooth tip of the output gear and the tooth root of the input gear (pinion). At the same time a second tooth pair is already in contact at point in Figure. As the gear rotates, the point of contact will move along the line of action APE. When the first tooth pair reaches next point shown in Figure, the second tooth pair disengages at point leaving only the first tooth pair in the single contact zone. After this time there is one pair of gear in contact until the third tooth pair achives in contact at point again.

When this tooth pair rotates to point, the another tooth pair begins engagement at point which starts another mesh cycle. After this time there are two pairs of gear in contact until the first tooth pair disengages at point. Finally, one complete tooth meshing cycle is completed when this tooth pair rotates to point.



To simplify the complexity of the problem, the load sharing compatibility condition is based on the assumption that the sum of the torque contributions of each meshing tooth pair must equal the total applied torque.

5. Spur Gear - Tooth Stresses

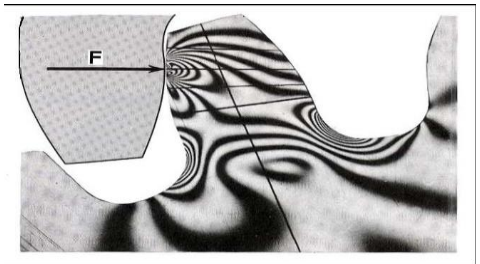


The stress developed by normal force in a photo-elastic model of a gear tooth. The highest stresses exist at regions where the lines are bunched closest together. The highest stress occurs at two locations.

i. At the contact point where the force acts

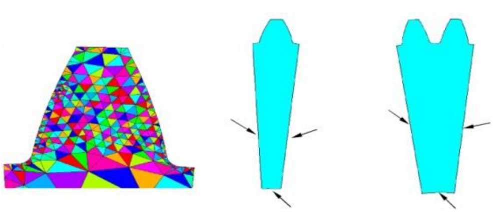
ii. At the fillet region near the base of the tooth Ansys Programming

Ansys package is one of the efficient engineering programming, which has the ability to solve many engineering problems using FEM. This programming



has easiness in use, flexibility in application and reliability for solving problems in engineering fields such as

stress analysis field. The tooth model in this study had been assumed as a plane stress problem due to small width of tooth compared with radius of gear. From the stress distributions on the model, the large concentrated stresses are at the root of the tooth. It shows large Von Mises stresses at the root of the tooth. They are equal to the tensile stresses. The tensile stresses are the main cause of crack failure, if they are large enough. That is why cracks usually start from the tensile side. From the Lewis equation if the diameters of the pinion and gear are always kept the same and the number of teeth was changed, the diametral pitch will be changed or the module of gear will be changed.



a: Tooth meshed with triangular elements
b: One tooth model for C.R<2.0
c: Two teeth model for C.R=2.0

That means that there are different bending strengths between the different teeth numbers. Different Maximum Von Misses with different pressure angle under normal operating conditions, the main source of vibration excitation is from the periodic changes in tooth stiffness due to non-uniform load distributions from the double to single contact zone and then from the single to double contact zone in each meshing cycle of the mating teeth.

This indicates that the variation in mesh stiffness can produce considerable vibration and dynamic loading of gears with teeth, in mesh. For the spur in volute teeth gears, the load was transmitted between just one to two pairs of teeth gears alternately. The torsion stiffness of two spur gears in mesh varied within the meshing cycle as the number of teeth in mesh changed from two to one pair of teeth in contact. Usually the torsional stiffness increased as the meshing of the teeth changed from one pair to two pairs in contact. If the gears were absolutely rigid the tooth load in the zone of the double tooth contacts should be half load of the single tooth contact.

However, in reality the teeth become deformed because of the influence of the teeth bending, shear, and contact stresses. These factors change the load distribution along the path of contact. In addition, every gear contains surface finishing and pitching errors. They alter the distribution of load. Because the teeth are comparatively stiff, even small errors may have a large influence. The elastic deformation of a tooth can result in shock loading, which may cause gear failure.

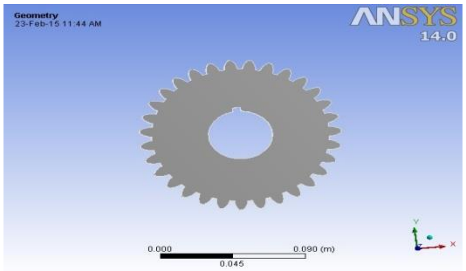
In order to prevent shock loading as the gear teeth move into and out of mesh, the tips of the teeth are often modified so as the tooth passes through the mesh zone the load increases more smoothly. The static transmission error model of gears in mesh can be used to determine the load sharing ratio throughout the mesh cycle. Two

Module (m)	3
Number of teeth (Z)	30
Pressure Angle	14.5
Gear ratio (i)	1
Addendum	1*m
Dedendum	1.25*m
PCD	Z*m
Rim thickness	1*m & 5*m
Material	C45 Steel
Poisson Ratio	0.3
Young's Modulus	2.01e5 N/mm^2

identical spur gears in mesh are considered here.

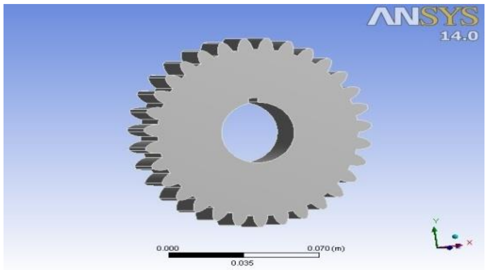
6. Gear Geometrical Modeling

For performing stress analysis of gears, one has to first make 2D model of it. It is a well known fact that preparing the model of the part like a gear is very difficult and time consuming in analysis software like



ANSYS. However, one can make the model in any other third party CAD software and bring the solid model to ANSYS for doing the analysis.

Preparing a 2 D model of a part like gear in CAD software like Pro-E also requires lot of manual input. This would definitely reduce the time required for preparing a 3D model in a significant manner. Few parameters are required to be input to get 2D model of a spur gear.



The involute of spur gear geometrical model is developed in finite element software package ANSYS through APDL (ANSYS Parametric Design Language) program using analytical equations given by Buckingham (1988) [3] then the entire modeling is done in ANSYS package.This model was developed for various pressure angle spur gear drives. These pressure angles are 14.5, 20, and 22. The gear specifications considered for analysis in this work are given in Tables.

6.1 Pressure Angle 14.5

Details of spur gear PA 14.5

Module (m)	3
Number of teeth (Z)	30
Pressure Angle	20
Gear ratio (i)	1
Addendum	1*m
Dedendum	1.25*m
PCD	Z*m
Rim thickness	1*m & 5*m
Material	C45 Steel
Poisson Ratio	0.3
Young's Modulus	2.01e5 N/mm^2

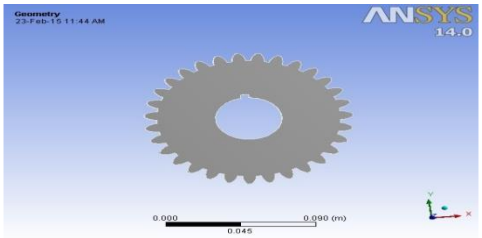
6.2 Pressure Angle 20

Table-4.2 Details of spur gear PA 20

Spur gear with pressure angle 20

6.3 Pressure Angle 22

Details of spur gear PA 22



Spur gear with pressure angle 22

There are various no of parameters taken by changing the pressure angle. All other parameters will be constant only pressure angle will be varied. Material used is carbon steel and value of young modulus & Poisson ratio respectively is 2.01e5 N/mm^2 & 0.292.

7. Conclusion

Following were concluded from this study and the future works in this area are much recommended. Two dimensional spur gear finite element models have been generated and analyzed in this work. The finite element method used to find out the stress of the varying three pressure angle (14.5, 20, and 22) spur gear teeth. Load sharing behaviour based stress analysis of 20^0 pressure angle spur gear teeth by using finite element method is validated with the AGMA Method , Lewis equation method and analytical results of the MiryumB. Sanchez, joseI. Pedrero, Miguel pleguezuelos, “Critical stress and load conditions for bending calculations of involute spur and helical gears” international journal of Fatigue 48 (2013) 28-38.

The average stress value of 20^0 pressure angle spur gear of FEM Method is

Module (m)	3
Number of teeth (Z)	30
Pressure Angle	22
Gear ratio (i)	1
Addendum	1*m
Dedendum	1.25*m
PCD	Z*m
Rim thickness	1*m & 5*m
Material	C45 Steel
Poisson Ratio	0.3
Young’s Modulus	2.01e5 N/mm^2

80919N/mm^{2}and then average value of AGMA Method as 7.991314N/mm^2 and LEWIS Equation as 7.393128 N/mm^2 then comparison of these values in the graph. The graph has been plotted for the Stress and three various pressure Angles ,Using this we can able to get the Linear Equation, $\sigma = -2.949\ \alpha + 18.11$. This clearly shows that by altering any of the one value we can get the other.

As the pressure angle increases, the bending stress decreases and bending load capacity increases. For spur gears, the tooth root bending stress, the total load will be maximum only at the outer part of the single pair tooth contact. These results can be expressed as a recommendation to be considered for standardization purposes and preliminary calculations of spur gear drives.

This is the very first step to be ahead in finding the crack and its propagation which leads to prediction of various designs from failure.

8. References

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