NANO IDEAL GENERALISED CLOSED SETS IN NANO IDEAL TOPOLOGICAL SPACES

Dr. J. Arul Jesti *1 And J. Joycy Renuka *2

*1 Assistant Professor, Department of Mathematics, St. Mary's College (Autonomous), Thoothukudi-628001, Tamil Nadu, India

*2 Research Scholar, St. Mary's College (Autonomous), Thoothukudi-628001, Tamil Nadu, India

ABSTRACT

The purpose of this paper is to define and study a new class of closed sets called $NIgsemi^*$-closed sets in nano ideal topological spaces. Basic properties of $NIgsemi^*$ - closed sets are analyzed and we compared it with some existing closed sets in nano ideal topological spaces.

Key words: $NIgsemi^*$- closed set, closed sets in nano ideal topology, $NIgsemi^*$- open set, nano topology.

1. INTRODUCTION

The concept of ideal topological space was introduced by kuratowski [9]. Also he defined the local functions in ideal topological spaces. In 1990, Jankovic and Hamlett [4] investigated further properties of ideal topological spaces. The notion of I-open sets was introduced by Jankovic et al. [5] and it was investigated by Abd El-Monsef [11]. Later, many authors introduced several open sets and generalized open sets in ideal topological spaces such as $pre\ I$-open sets[2], $semi\ I$-open sets [6], α-I-open sets[6] , αg-I-open sets [23] and gp-I-open sets [23].

In 2013, Lellis Thivagar and Carmel Richard[12] established the field of nano topological spaces which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano-interior and nano-closure. K.Bhuvaneswari et al. [9] introduced and studied the concept of nano generalised closed sets in nano topological spaces. Later Many researchers like [3],[9] obtained several generalizations of nano open sets. In 2012, Robert et. Al [1,2] introduced the class of $semi^*$-open sets and $semi^*$-closed sets in Topological Spaces. In 2015, Paulraj Gnanachandra [19] introduced the notion of $nano\ semi^*$-open sets and $nano\ semi^*$-closed sets in terms of nano generalised closure and nano generalised interior in Nano Topological Spaces. In 2020 [18], further properties of $nano\ semi^*$-open sets were investigated.

M. Parimala et al. [14, 15, 17] introduced the concept of nano ideal topological spaces and investigated some of its basic properties. In 2018, M.Parimala and Jafari [15] introduced the notion of $nano\ I$-open sets and studied several properties. Further she defined nIg - open sets and nIg- closed sets in Nano Ideal Topological Spaces.

In this paper, we introduce a new type of generalized closed and open sets called $NIgsemi^*$-closed set and $NIgsemi^*$-open set in nano ideal topological spaces and investigate the relationships between this set with other sets in nano topological spaces and nano ideal topological spaces. Characterizations and properties of $NIgsemi^*$ -closed sets and $NIgsemi^*$-open sets are studied.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ (or U) represent nano topological spaces on whichno separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, $N\ cl(A)$ and $N\ int(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definitions, which will be used in the sequel.

Definition 2.1 ([12]). Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the ***approximation space***.
Let $X \subseteq U$. Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = \cup \{R(x): R(x) \subseteq X, x \in U\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is, $U_R(X) = \cup \{R(x): R(x) \cap X \neq \varphi, x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 ([12]). If (U, R) is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$
(iii) $L_R(U) = U_R(U) = U$
(iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R Y)$
(vi) $L_R X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(viii) $L_R(X) \subseteq L_R(Y) and U_R(X) \subseteq U_R(Y) whenever X \subseteq Y$
(ix) $U(X^c) = [L_R(X)]^c and L_R(X^c) = [U_R(X)]^c$
(x) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
(xi) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3 ([12]). Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$ and by the property 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\varphi \in \tau_R(X)$.
(ii)   The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii)   The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Therefore, $\tau_R(X)$ is a topology on U called the **nano topology** on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets (briefly n-open sets). The complement of a nano open set is called a ***nano closed set*** (briefly n-closed set)

Definition 2.4 [9]. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be ***ng-closed set*** if $ncl(A) \subseteq B$ whenever $A \subseteq B \subseteq \tau_R(X)$. The complement of an ng-closed set is called a ng-open set.

Definition 2.5 [19] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano generalized interior of A is defined as union of all nano g − open subsets of A and its denoted by $Nint^*(A)$. The nano generalized closure of A is defined as the insertion of all nano g − closed sets containing A and it is denoted by ***$Ncl^*(A)$***.

Lemma 2.6. [19] Let A be a subset of U. Then the following properties hold.

(i) $Nint(A) \subseteq Nint^*(A)$.
(ii) $Ncl^*(A) \subseteq Ncl(A)$.
(iii) $\cup Ncl^*(G_\alpha) \subseteq Ncl^*(\cup G_\alpha)$, where each G_α is nano g-closed.

Definition 2.7.[19] A subset A of a topological space $(U, \tau_R(X)$ is said to be ***nano semi*-open*** [2] (briefly n-$semi^*$ open) if there is a nano open set G in U such that $G \subseteq A \subseteq$

$Ncl^*(G)$ or equivalently if $A \subseteq Ncl^*(Nint(A))$. The complement of nano semi*-open set is nano semi*-closed.

Definition 2.8.[19] Let A be a subset of a nano topological space.

(i)   The *nano semi* $*$-closure of A, $Ns^*cl(A) = \cap\{F: A \subseteq F, F$ is nano semi*-closed

(ii) The nano semi*-interior of A, $Ns^*int(A) = \cup\{G: G \subseteq A, G$ is nano semi*-open$\}$.

Theorem 2.9.[18] Every nano open set is *nano semi* $*$-open.

Definition 2.10[10]. An **ideal I** on a topological space (X, τ) is a nonempty collection of subsets of X, which satisfies the following two conditions:

(i)   If $A \in I$ and $B \subseteq A$ implies $B \in I$

(ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$

Definition 2.11[10] **An ideal topological space** is a topological space (X, τ) with an ideal I on X and it is denoted by (X, τ, I). Given a topological space (X, τ) with an ideal I on X and if $\rho(X)$ is the set of all subsets of X, a set operator $(*) : \rho(X) \to \rho(X)$, called a local function of A with respect to τ and I, is defined as follows: for $A \subseteq X, A^*(I, \tau) = \{x \in X / U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau / x \in U\}$. We simply write A^* instead of $A^*(I, \tau)$.

Definition 2.12[22] For every Ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I, \tau) = \{U - i / U \in \tau \& i \in I\}$. But in general (I, τ) is not always a topology. Additionally $cl^*(A) = A \cup A^*$ defines a **kuratowski closure operator** for $\tau*$ (I). If $A \subseteq X$, $cl(A)$ and $int(A)$ will, respectively, denote the closure and interior of A in (X, τ) and $int^*(A)$ denote the interior of A in (X, τ^*).

Definition 2.13 A subset A of an ideal space (X, τ, I) is ***-closed** (resp. ***-dense** in itself) if $A^* \subseteq A$ (resp. $A \subseteq A^*$).

Definition 2.14 [7] Given a space (X, τ, I) and $A \subseteq X$, A is said to be **I open** if $A \subseteq intA*$. We denoted by $IO(X, \tau) = \{A \subseteq X, A \subseteq int(A^*)\}$ or simply write $I.O$ for $IO(X, T)$ when there is no chance for confusion

Definition 2.15[17] Let $(U, \mathcal{N})$ be a nano topological space, where $\mathcal{N} = \tau_R(X)$. A nano topological spaces $(U, \mathcal{N})$ with an ideal I on U is called a *nano ideal topological space* and is denoted by $(U, \mathcal{N}, I)$. Let $(U, \mathcal{N})$ be a nano topological space and $G_n(x) = \{G_n: x \in G_n, G_n \in \mathcal{N}\}$ be the family of nano open sets which contain x

Definition 2.16 [17]. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space with an ideal I on U, where $\mathcal{N} = \tau_R(X)$ and $(.)_n^*$ be a set operator from $P(U)$ to $P(U)$. ($P(U)$ is the set of all subsets of U). For a subset $A \subset U$, $A_n^*(I, \mathcal{N}) = \{x \in U : G_n \cap A \in I$, for every $G_n \in G_n(x)\}$, where $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$ is called *the nano local function* (briefly, n-local function) of A with respect to I and N. We will simply write A_n^* for $A_n^*(I, \mathcal{N})$.

Theorem 2.17 ([17]). Let $(U, \mathcal{N})$ be a nano topological space with ideals I, I' on U and A, B be subsets of U. Then

(i) $A \subseteq B \Rightarrow A_n^* \subseteq B_n^*$,

(ii) $I \subseteq I' \Rightarrow A_n^*(I') \subseteq A_n^*(I)$,

(iii) $A_n^* = n\text{-}cl(A_n^*) \subseteq n\text{-}cl(A)$ (A_n^* is a nano closed subset of $n\text{-}cl(A)$),

(iv) $(A_n^*)_n^* \subseteq A_n^*$,

(v) $A_n^* \cup B_n^* = (A \cup B)_n^*$

(vi) $A_n^* - B_n^* = (A - B)_n^* - B_n^* \subseteq (A - B)_n^*$,

(vii) $V \in N \Rightarrow V \cap A_n^* = V \cap (V \cap A)_n^* \subseteq (V \cap A)_n^*$ and

(viii) $J \in I \Rightarrow (A \cup J)_n^* = A_n^* = (A - J)n_n^*$.

Theorem 2.18 ([17]). If $(U, \mathcal{N}, I)$ is a nano topological space with an ideal I and $A \subseteq A_n^*$, then $A_n^* = n\text{-}cl(A_n^*) = n\text{-}cl(A)$.

Definition 2.19 ([17]). Let $(U, \mathcal{N})$ be a nano topological space with an ideal I on U. The set operator $n\text{-}cl^*$ is called a **nano∗-closure** and is defined $as\ n\text{-}cl^*(A) = A \cup A_n^*$ for $A \subseteq X$. It can be easily observed that $n\text{-}cl^*(A) \subseteq n\text{-}cl(A)$.

Theorem 2.20 [17]. The set operator $n - cl^*$ satisfies the following conditions:

   (i) $A \subseteq n\text{-}cl^*(A)$,
   (ii) $n\text{-}cl^*(\varphi) = \varphi\ and\ n\text{-}cl^*(U) = U$,
   (iii) $If\ A \subset B, then\ n\text{-}cl^*(A) \subseteq n\text{-}cl^*(B)$,
   (iv) $n\text{-}cl^*(A) \cup n\text{-}cl^*(B) = n\text{-}cl^*(A \cup B)$.
   (v) $n\text{-}cl^*(n\text{-}cl^*(A)) = n\text{-}cl^*(A)$.

Definition 2.21[17]. An ideal I in a space $(U, \mathcal{N}, I)$ is called **$\mathcal{N}$-codense ideal** if $\mathcal{N} \cap I = \{\varphi\}$.

Definition 2.22 [17]. A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is **n ∗-dense** in itself (resp. **n ∗-perfect and n ∗-closed**) if $A \subseteq A_n^*$ (resp. $A = A_n^*$, $A_n^* \subseteq A$).

Lemma 2.23 [17]. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. If A is n ∗-dense in itself, then $A_n^* = n\text{-}cl(A_n^*) = n\text{-}cl(A) = n\text{-}cl^*(A)$.

Definition 2.24.[13,15,20] A subset A of space $(U, \mathcal{N}, I)$ is said to be

1. NI open if $A \subseteq N\ int(A_n^*)$.
2. $NI\alpha$-open if $A \subseteq N\ int(Ncl^*(Nint(A)))$
3. $NI\beta$-open if $A \subseteq Ncl^*(N\ int(Ncl^*(A)))$
4. $NIsemi$-open if $A \subseteq Ncl^*(N\ int(A))$
5. $NI\ pre$-open if $A \subseteq N\ int(Ncl^*(A))$.
6. $NI\ regular$ - open if $A = N\ int(Ncl^*(A))$

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.25[8] A subset A of space $(U, \mathcal{N}, I)$ is said to be

1. a nano ideal generalized closed set ($NI_g -$ closed) if $NIcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
2. a nano ideal generalized - α closed set ($NI_{g\alpha} -$ closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
3. a nano ideal generalized $\beta -$ closed set ($NI_{g\beta} -$ closed) if $NI\beta cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
4. a nano ideal generalized $semi$ - closed set ($NI_{gs} -$ closed) if $NIscl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open
5. a nano ideal generalized pre - closed set ($NI_{gp} -$ closed) if $NIpcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
6. (x) a nano ideal generalized $regular$ - closed set ($NI_{gr} -$ closed) if $NIrcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.

3. NANO IDEAL GENERALISED SEMI*-CLOSED SETS

Definition 3.1. A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be ***nano ideal generalized semi*-closed*** (briefly, $NIgsemi^*$-closed) if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is $nano\ semi^*$-open.

Definition 3.2 A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be ***nano ideal generalized semi*-open*** (briefly, $NIgsemi^*$-open) if $X - A$ is $NIgsemi^*$-closed.

Example 3.3. Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, d\} \subset U$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $\mathcal{N} = \{U, \varphi, \{a, d\}\}$ and the ideal I $= \{\varphi, \{a\}\}$. Then $NIgsemi^*$-closed $=$ $\{U, \varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$

Theorem 3.4. If A and B are $NIgsemi^*$-closed , then $A \cup B$ is $NIgsemi^*$ closed.

Proof. Let A and B are $NIgsemi^*$- closed sets . Then $A^*_n \subseteq V$ where $A \subseteq V$ and V is *nano semi**-open and $B^*_n \subseteq V$ where $B \subseteq V$ and V is *nano semi**- open . Since A and B are subsets of V, $(A^*_n \cup B^*_n) = (A \cup B)^*_n$ is a subset of V and V is n*nano semi**- open. which implies that $(A \cup B)$ is$NIgsemi^*$- closed.

Remark 3.5. The Intersection of two $NIgsemi^*$-closed sets need not be $NIgsemi^*$-closed set which is shown in the following example.

Example 3.6. Let $U = \{a, b, c, d\}$ be the universe, $X = \{c\} \subset U, U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $\mathcal{N} = \{U, \varphi, \{c, d\}\}$ and the ideal $I = \{\varphi, \{c\}\}$. $NIgsemi^*$ -closed $= \{U, \varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$ Let $A = \{a, d\}$ and $B = \{b, d\}$ be $NIgsemi^*$-closed sets. $A \cap B = \{d\}$ is not a $NIgsemi^*$-closed set.

Theorem 3.7. If $(U, \mathcal{N}, I)$ is a nano ideal topological space and $A \subseteq X$, then A is $NIgsemi^*$-closed if and only if $n\text{-}cl^*(A) \subseteq V$ whenever $A \subseteq V$ and V is*nano semi**- open in U.

Proof. Necessity: Since A is $NIgsemi^*$-closed, we have $A^*_n \subseteq V$ whenever $A \subseteq V$ and V is $n\text{-}semi^*$open in U. Now $n\text{-}cl^*(A) = A \cup A^*_n \subseteq V$ whenever $A \subseteq V$ and V is n-open in U.
Sufficiency: Let $A \subseteq V$ and V be *nano semi**- open in U. By hypothesis $n - cl^*(A) \subseteq V$. Since $n\text{-}cl^*(A) = A \cup A^*_n$, we have $A^*_n \subseteq V$.

The following theorem gives characterizations of $NIgsemi^*$-closed sets.

Theorem 3.8. If $(U, \mathcal{N}, I)$ is any nano ideal topological space and $A \subseteq U$, then the following are equivalent.

(1) A is$NIgsemi^*$-closed,

(2) $n\text{-}cl^*(A) \subseteq V$ whenever $A \subseteq V$ and V is *nano semi**-open in U,

(3) For all $x \in n\text{-}cl^*(A), n\text{-}semi^*cl(\{x\}) \cap A \neq \emptyset$.

(4) $n\text{-}cl^*(A) - A$ contains no nonempty *nano semi**-closed set,

(5)$A^*_n - A$ contains no nonempty *nano semi**-closed set.

Proof. (1)$\Rightarrow$(2) If A is $NIgsemi^*$-closed, then $A^*_n \subseteq V$ whenever $A \subseteq V$ and V is *nano semi**-open in U and so $n - cl^*(A) = A \cup A^*_n \subseteq V$ whenever $A \subseteq V$ and V is *nano semi**-open in U. This proves (2).

(2)$\Rightarrow$(3) Suppose $x \in n\text{-}cl^*(A)$. If $n\text{-}semi^*cl(\{x\}) \cap A = \emptyset$, then $A \subseteq U - n\text{-}semi^*cl(\{x\})$. By (2), $n\text{-}cl^*(\{x\}) \subseteq U - n\text{-}semi^*cl(\{x\})$ and hence $n\text{-}cl^*(A) \cap \{x\} = \varphi$, a contradiction, since $x \in n\text{-}cl^*(A)$.

(3)$\Rightarrow$(4) Suppose $F \subseteq n - cl^*(A) - A, F$ is *nano semi**-closed and $x \in F$. Since $F \subseteq U - A, F \cap A = \varphi$.We have $n - semi^*cl(\{x\}) \cap A = \emptyset$ because F is *nano semi**-closed and $x \in F$. From (3), this is a contradiction. Therefore $n\text{-}cl^*(A) - A$ contains no nonempty $n\text{-}semi^*$closed set.

(4)$\Rightarrow$(5) Since $n - cl^*(A) - A = (A \cup A^*_n) - A = (A \cup A^*_n) \cap A^C = (A \cap A^c) \cup (A^*_n \cap A^C) = A^*_n \cap A^C = A^*_n - A$. Therefore $A^*_n - A$ contains no nonempty *nano semi**- closed set.

(5)$\Rightarrow$(1) Let $A \subseteq V$ where V is *nano semi**- open set. Therefore $U - V \subseteq U - A$ and so $A^*_n \cap (U - V) \subseteq A^*_n \cap (U - A) = A^*_n \cap A^C = A^*_n - A$. Therefore $A^*_n \cap (U - V) \subseteq A^*_n - A$. Since A^*_n is always n-closed set, so A^*_n is *nano semi**-closed set and so $A^*_n \cap (U - V)$ is a *nano semi**-closed set contained in $A^*_n - A$. Therefore $A^*_n \cap (U - V) = \emptyset$ and hence$A^*_n \subseteq V$. Therefore$A$ is $NIgsemi^*$-closed

Corollary 3.9. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$ is an $NIgsemi^*$-closed set, then the following are equivalent:

    (i) A is an n^*-closed set.

    (ii) $n\text{-}cl^*(A) - A$ is an $n\text{-}semi^*$closed set.

(iii) $A_n^* - A$ is an $n\text{-}semi^*$closed set.

Proof. (i) $\Rightarrow$ (ii) If A is n^*-closed, then $A_n^* \subseteq A$ and so $n\text{-}cl^*(A) - A = \varphi$. Hence $n\text{-}cl^*(A) - A$ is $n\text{-}semi^*$closed.

(ii) $\Rightarrow$ (iii) Since $n\text{-}cl^*(A) - A = A_n^* - A$, it is clear.

(iii) $\Rightarrow$ (i) If $A_n^* - A$ is $n\text{-}semi^*$closed and A is $n\text{-}semi^*$closed, from Theorem 3.5(v), $A_n^* - A = \varphi$ and so A is n^*-closed.

Theorem 3.10. If A is $NIgsemi^*$-closed and A$\subseteq B \subseteq A_n^*$, then B is $n\text{-}semi^*$closed.

Proof. Let $B \subseteq V$ where V is $n\text{-}semi^*$open in $\mathcal{N}$. Then $A \subseteq B$ implies $A \subseteq V$. Since A is $NIgsemi^*$-closed, $A_n^* \subseteq V$. Also $B \subseteq A_n^*$ implies $B_n^* \subseteq A_n^*$. Thus $B_n^* \subseteq V$ and so B is $NIgsemi^*$-closed.

Theorem 3.11. Let $(U, \mathcal{N}, I))$ be a nano ideal space. Then every subset of U is $NIgsemi^*$-closed if and only if every $n\text{-}semi^*$open set is n^* -closed.

Proof: Suppose every subset of U is $NIgsemi^*$ -closed. If $G \subseteq U$, U is $n\text{-}semi^*$open then G is $NIgsemi^*$-closed and so $(G)_n^* \subseteq G$. Hence G is n^* -closed. Conversely, suppose that every $n\text{-}semi^*$open set is n^*-closed. If G is $n\text{-}semi^*$open set such that $A \subseteq G \subseteq U$ then $(A)_n^* \subseteq (G)_n^* \subseteq G$ and so A is $NIgsemi^*$-closed.

Theorem 3.12. If $(U, \mathcal{N} , I)$ is any nano ideal topological space where $I = \{\varphi\}$, then A is $NIgsemi^*$-closed if and only if A is ng-closed.

Proof. The proof follows from the fact that for $I = \{\varphi\}, A_n^* = n\text{-}cl(A) \supset A$ and so every subset of U is n^*-dense in itself.

Theorem 3.13. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space. Then every subset of U is$NIgsemi^*$-closed if and only if every $n\text{-}semi^*$open set is n^*-closed.

Proof. Suppose every subset of U is $NIgsemi^*$-closed. If V is $n\text{-}semi^*$open, then V is $NIgsemi^*$-closed and so $V_n^* \subseteq V$. Hence V is n^*-closed. Conversely, suppose that every $n\text{-}semi^*$open set is n^*-closed. If $A \subseteq U$ and V is an $n\text{-}semi^*$open set such that $A \subseteq V$, then $A_n^* \subseteq V_n^*$and so A is $NIgsemi^*$-closed.

Theorem 3.14. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space. For every $A \in I, A$ is $NIgsemi^*$-closed.

Proof. Let $A \subseteq U$ where U is $n\text{-}semi^*$open set. Since $A_n^* = \emptyset$ for every $A \in I$, then $n\text{-}cl^*(A) = A \cup A_n^* = A \subseteq U$. Therefore, by Theorem 3.7, A is$NIgsemi^*$-closed.

Theorem 3.15. If$(U, \mathcal{N}, I)$ is an nano ideal topological space, then $A_n^*$is always$NIgsemi^*$-closed for every subset A of U.

Proof. Let $A_n^* \subseteq U$ where U is $n\text{-}semi^*$open. $Since$ $(A_n^*)_n^* \subseteq A_n^*$, we have $(A_n^*)_n^* \subseteq U$ whenever$A_n^* \subseteq U$ and U is $n\text{-}semi^*$open. Hence A_n^* is $NIgsemi^*$-closed

Theorem 3.16. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space and $A \subseteq K$. Then A is $NIgsemi^*$-closed if and only if $A = F - M$ where F is n^*-closed and M contains no nonempty $n\text{-}semi^*$closed set.

Proof. If A is $NIgsemi^*$-closed, then by Theorem 3.8 (5), $M = A_n^* - A$ contains no nonempty $n\text{-}semi^*$closed set. If $F = ncl^*(A)$, then F is n^*- closed such that $F - M = (A \cup A_n^*) - (A_n^* - A) = (A \cup A_n^*) \cap (A_n^* \cap A^c)^c = (A \cup A_n^*) \cap ((A_n^*)^c \cup A) = (A \cup A_n^*) \cap (A \cup (A_n^*)^c) = A \cup ((A_n^* \cap A^c)^c \cap (A_n^*)^c) = A$.

Conversely, suppose $A = F - M$ where F is n^*- closed and M contains no nonempty $n\text{-}semi^*$closed set. Let V be an $n\text{-}semi^*$open set such that $A \subseteq V$. Then $F - M \subseteq V$ which implies that $F \cap (U - V) \subseteq M$. Now $A \subseteq F$ and $F_n^* \subseteq F$ then $A_n^* \subseteq F_n^*$ and so $A_n^* \cap (U - V) \subseteq F_n^* \cap (U - V) \subseteq F \cap (U - V) \subseteq M$. By hypothesis, since $A_n^* \cap (U - V)$ is $n\text{-}semi^*$closed, $A_n^* \cap (U - V) = \emptyset$ and so $A_n^* \subseteq V$. Hence A is $NIgsemi^*$-closed.

Theorem 3.17. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space and $A \subseteq U$. If $A \subseteq B \subseteq A_n^*$, then $A_n^* = B_n^*$ and B is n^*-dense in itself.

Proof. Since $A \subseteq B$, then $A_n^* \subseteq B_n^*$ and since $B \subseteq A_n^*$, then $B_n^* \subseteq (A_n^*)_n^* \subseteq A_n^*$ by Theorem 2.17 (4). Therefore $A_n^* = B_n^*$ and $B \subseteq A_n^* \subseteq B_n^*$. Hence proved.

Theorem 3.18. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space. If A and B are subsets of U such *that* $A \subseteq B \subseteq n\text{-}cl^*(A)$ and A is $NIgsemi^*$-closed, then B is $NIgsemi^*$-closed.

Proof. Since A is $NIgsemi^*$-closed, then by Theorem 3.8 (4), $n\text{-}cl^*(A) - A$ contains no nonempty $n\text{-}semi^*$ closed set. Since $n\text{-}cl^*(B) - B \subseteq n\text{-}cl^*(A) - A$, $n\text{-}cl^*(B) - B$ contains no nonempty $n\text{-}semi^*$ closed set and so by Theorem 3.8 (4), B is $NIgsemi^*$-closed

Theorem 3.19. If $(U, \mathcal{N}, I)$ is any nano ideal topological space, then every nano ideal generalised closed is $NIgsemi^*$-closed set but not conversely.

Proof. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space. Let $A \subseteq U$ and U is $nano\ semi^*$-open. If A is nano ideal generalised closed set then $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is $nano$ open set. We know the fact that every nano open set is $n\text{-}semi^*$ open. Therefore $A_n^* \subseteq V \subseteq U$. Hence every nano ideal generalised closed is $NIgsemi^*$-closed set

Theorem 3.20. If $(U, \mathcal{N}, I)$ is any nano ideal topological space, then every ng-closed set is $NIgsemi^*$-closed but not conversely

Proof. Let V be any nano open set containing A. Since every nano open set is $n\text{-}semi^*$ open, V be any $n\text{-}semi^*$ open set containing A. A Since A is ng-closed, $n\text{-}cl(A) \subseteq V$. By Theorem 2.17(iii), we have $A_n^* \subseteq V$. Then A is $NIgsemi^*$- closed set.
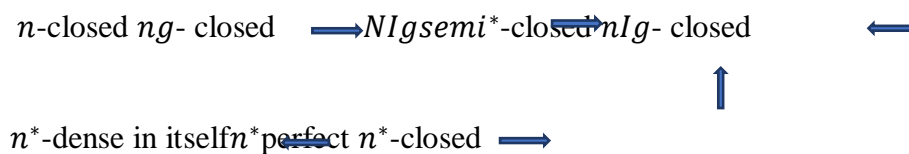
Example 3.21. Let U = {a,b,c,d}, with U/ R= {{b},{d},{a,c}} and X= {a,d},. Then the Nano topology $\mathcal{N} = \{\phi, U, \{d\}, \{a, c\}, \{a. c. d\}\}$ and $I = \{ \phi, \{d\}\}$. Then $NIgsemi^*$-closed sets are {ϕ, U, {b}, {d},{a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}} and ng-closed sets are {ϕ, U, {b}, {a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}. Since {d} is $NIgsemi^*$-closed but it is not ng-closed, every nano generalised closed set is $NIgsemi^*$-closed

Theorem 3.22. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space. Every n∗-closed set is $NIgsemi^*$-closed.

Proof. Let A be a subset of X and A be n∗-closed. Assume that A $\subseteq$ V and V is $n\text{-}semi^*$ open. Since A is n∗-closed, we have $A_n^* \subseteq A$ and so A is $NIgsemi^*$-closed.

Example 3.23. Let U = {a,b,c,d}, with U/ R= {{a},{b},{c,d} and X= {c},. Then the Nano topology $\mathcal{N} = \{\phi, U, \{c, d\}\}$ and $I = \{ \phi, \{c\}\}$ .Then $NIgsemi^*$ -closed sets = {ϕ, U,{a},{b},{c},{a,b},{b,c},{a,c},{a,d},{b.d},{a,b,c},{b,c,d},{a,b,d},{a,c,d}} and n^*- closed sets are {ϕ, U,{a,b},{a,b,c}}. It is clear that every n∗-closed set is $NIgsemi^*$-closed.

For the relationship related to several sets defined in this paper, we have the following diagram:

n-closed ng- closed $\longrightarrow NIgsemi^*$-closed $\rightarrow nIg$- closed $\longleftarrow$

$\uparrow$

n^*-dense in itself n^* perfect n^*-closed $\longrightarrow$

The following theorem gives a characterization of $NIgsemi^*$-open sets.

Theorem 2.31. Let be an nano ideal topological space and $A \subseteq K$. Then A is $NIgsemi^*$-open if a $(U, \mathcal{N}, I)$ nd only if $F \subseteq n\text{-}int^*(A)$ whenever F is $n\text{-}semi^*$ closed and $F \subseteq A$.

Proof. Suppose A is $NIgsemi^*$-open. If F is $n\text{-}semi^*$ closed and $F \subseteq A$, then $U - A \subseteq U - F$ and so $n\text{-}cl^*(U - A) \subseteq U - F$ by Theorem 2.4 (2). Therefore $F \subseteq U - n\text{-}cl^*(U - A) = n\text{-}int^*(A)$. Hence $F \subseteq n\text{-}int^*(A)$.

Conversely, suppose the condition holds. Let V be a $n\text{-}semi^*$open set such that $U - A \subseteq V$. Then $U - V \subseteq A$ and so $U - V \subseteq n\text{-}int^*(A)$. Therefore $n\text{-}cl^*(U - A) \subseteq V$. By Theorem 2.4 (2), $U - A$ is $NIgsemi^*$-closed. Hence A is $NIgsemi^*$-open.

Theorem 3.18. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. Then A is $NIgsemi^*$-open if and only if $F \subseteq n\text{-}int^*(A)$ whenever F is closed and $F \subseteq A$.

Proof. Suppose A is $NIgsemi^*$-open. If F is closed and $F \subseteq A$, then $U - A \subseteq U - F$ and so $n\text{-}cl^*(U - A) \subseteq U - F$. Therefore, $F \subseteq n\text{-}int^*(A)$.

Conversely, suppose the condition holds. Let V be an open set such that $U - A \subseteq V$. Then $U - V \subseteq A$ and so $U - V \subseteq n\text{-}int^*(A)$ which implies that $n\text{-}cl^*(U - A) \subseteq V$. Therefore, $U - A$ is $NIgsemi^*$-closed and so A is $NIgsemi^*$-open.

The following theorem gives a property of $NIgsemi^*$-closed.

Theorem 2.33. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space and $A \subseteq U$. If A is $NIgsemi^*$-open and $n\text{-}int^*(A) \subseteq B \subseteq A$, then B is $NIgsemi^*$-open.

Proof. Since A is $NIgsemi^*$-open, then U−A is $NIgsemi^*$-closed. By Theorem 2.4 (4), $n\text{-}cl^*(U - A) - (U - A)$ contains no nonempty $n\text{-}semi^*$closed set. Since $n\text{-}int^*(A) \subseteq n\text{-}int^*(B)$ which implies that $n\text{-}cl^*(U - B) \subseteq n\text{-}cl^*(U - A)$ and so $n\text{-}cl * (U - B) - (U - B) \subseteq n\text{-}cl^*(U - A) - (U - A)$. Hence B is $NIgsemi^*$-open.

The following theorem gives a characterization of $NIgsemi^*$-closed sets in terms of $NIgsemi^*$-open sets.

Theorem 2.34. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space and $A \subseteq U$. Then the following are equivalent.

(1) A is $NIgsemi^*$-open

(2) $A \cup (U - A_n^*)$ is $NIgsemi^*$-closed,

(3) $A_n^* - A$ is $NIgsemi^*$-open.

Proof. (1)⇒(2) Suppose A is $NIgsemi^*$-open. If V is any $n\text{-}semi^*$open set such that $A \cup (U - A_n^*) \subseteq V$, then $U - V \subseteq U - (A \cup (U - A_n^*)) = U \cap (A \cup (A_n^*)^c)^c = A_n^* \cap A^c = A_n^* - A$. Since A is $NIgsemi^*$-closed, by Theorem 2.4 (5), it follows that $U - V = \emptyset$ and so $U = V$. Therefore $A \cup (U - A_n^*) \subseteq V$ which implies that $A \cup (U - A_n^*) \subseteq U$ and so $(A \cup (U - A_n^*)_n^* \subseteq U_n^* \subseteq U = V$. Hence $A \cup (U - A_n^*)$ is $NIgsemi^*$-closed.

(2)⇒(1) Suppose $A \cup (U - A_n^*)$ is $NIgsemi^*$-closed. If F is any $n\text{-}semi^*$closed set such that $F \subseteq A_n^* - A$, then $F \subseteq A_n^*$ and F is not a subset of A which implies that $U - A_n^* \subseteq U - F$ and $A \subseteq U - F$. Therefore $A \cup (U - A_n^*) \subseteq A \cup (U - F) = U - F$ and $U - F$ is $n\text{-}semi^*$open. Since $(A \cup (U - A_n^*))_n^* \subseteq K - F$ which implies that $A_n^* \cup (U - A_n^*))_n^* \subseteq U - F$ and so $A_n^* \subseteq U - F$ which implies that $F \subseteq U - A_n^*$. Since $F \subseteq A_n^*$, it follows that $F = \emptyset$. Hence A is $NIgsemi^*$-open.

(2)⇔(3) Since $U - (A_n^* - A) = U \cap (A_n^* \cap A^c)^c = U \cap ((A_n^*)^c \cup A) = (U \cap (A_n^*)^c) \cup (U \cap A) = A \cup (U - A_n^*)$ is $NIgsemi^*$-closed. Hence $A_n^* - A$ is $NIgsemi^*$-open

CONCLUSION We defined the concept of $NIgsemi^*$-open sets and $NIgsemi^*$-closed sets in nano ideal topological spaces. We also discussed some of their properties with suitable examples. We hope that this paper is just a beginning of a new structure. It will inspire many to contribute to the cultivation of nano ideal topology in the field of Mathematics.

REFERENCES

[1] A. Robert and S. Pious Missier, "On $Semi^*$-closed sets", AJEM, 1(4) (2012), pp 173 - 176.

[2] A. Robert and S. Pious Missier, "On $Semi^*$-open sets", Int.J.Math. and Soft Comp., 2(2) (2012), pp 95 − 102

[3] C. R. Parvathy and S. Praveena, "On nano generalized pre regular closed sets in nano topological spaces", IOSR Journal of Mathematics, 13 (2)(2017), pp 56-60

[4] D. Jankovic and T. R. Hamlett, "Compatible extensions of ideals", Boll. Un. Mat. Ital., (7)6-B (1992)

[5] D. Jankovic and T. R. Hamlett, "New topologies from old via ideals", Amer. Math. Monthly, 97(4) (1990), pp 295- 310.

[6]E . Hatir and T. Noiri, "On decomposition of continuity via Idealization" Acta Math.Hungar, 96 (4) (2002), pp 341-349

[7] E. Hayashi, "Topologies defined by local properties", Math. Ann., 156 (1964), pp 205–215.

[8] G. Gincy and Dr. C. Janaki, "Nano Ideal α-$Regular$ Closed Sets in Nano Ideal Topological Spaced", International Journal of Mathematical Archive, 11(1) (2020), pp 1-6

[9] K. Bhuvaneswari and K. Mythili Gnanapriya, "Nano generalized closed sets in nano topological spaces", International Journal of Scientific and Research Publications, 4(5), May(2014), pp 1–3.

[10] K. Kuratowski, Topology, Vol. I, Academic Press (New York, 1966).

[11] M. E. Abd El-Monsef, E. F. Lashien and A. A. Nasef, "On I-open sets and I-continuous functions", Kyungpook Math. J., 32 (1992), pp 21–30.

[12] M. LellisThivagar and Carmel Richard, "On Nano forms of weakly open sets" International Journal of Mathematics and Statistics Invention 1(1)(2013), pp 31-37

[13] M.Lellis Thivagar and V.Sutha Devi, "New sort of operators in nano ideal topology", Ultra Scientist Vol.28 (1) A (2016), pp 51-64.

[14] M. Parimala and R. Perumal, "Weaker form of open sets in nano ideal topological spaces", Global Journal of Pure and Applied Mathematics (GJPAM), 12(1) (2016), pp 302–305

[15] M. Parimala, S. Jafari, "On Some New Notions in Nano Ideal Topological Spaces", Eurasian Bulletin Of Mathematics Ebm, vol. 1, no. 3 (2018), pp 85-93

[16] M. Parimala, S. Jafari and S. Murali, "nano ideal generalized closed sets in nano ideal topological spaces", annales univ. sci. Budapest 61 (2018), pp 111–119

[17] M. Parimala, T. Noiri and S. Jafari, "New types of nano topological spacess via nano ideals" (communicated)

[18] Paulraj Gnanachandra, "A New Notion Of Open Sets In Nano Topology",International Journal of Grid and Distributed Computing, vol. 13, no. 2, (2020), pp 2311-2317

[19] P.Gnanachandra and K.Surya, "A study on nano semi*-closed sets and their Applications", Proceedings of UGC Sponsored National Seminar on Recent Trends in Mathematics, GVNC,(2015), pp77-80.

[20] Rajasekaran and O. Nethaji, "Simple forms of nano open sets in an ideal nano topological spaces", Journal of New Theory, 24(2018), pp 35-43.

[21] Rishwanth M Parimala and Saeid Jafari "On some new notions in nano ideal topological spaces" IBJM (2018), vol.1, no. 3, pp 85-92.

[22] R. Vaidyanathaswamy, "The localization theory in set topology", Proc. Indian Acad. Sci., 20 (1945), pp 51–61.

[23] T Noiri , Rajamani M and Inthumathi V, "On decomposition of g-continuity via idealization", Bull.Cal.Math.Soc 99 (4) (2007), pp 415-424.