Research Article

ANALYTICAL STUDY ON THERMOHALINE CONVECTIVE INSTABILITY IN A MICROPOLAR FERROFLUID

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Abstract: The present investigation is on linear analytical study of thermohaline convective instability in micropolar ferrofluid using perturbation technique. The fluid layer is heated from below and salted from above. The theory of linear stability is used to reduce the non-linear effects on governing equations and normal mode analysis is taken to study. The critical magnetic thermal Rayleigh number N_{SC} is obtained for sufficient large value of M_1 and an oscillatory instability is determined. The parameters N_1 and N_5' are analyzed for stabilizing behavior and N_3', τ and M_3 give the destabilizing behavior. The results are depicted graphically.

Key words:
Thermohaline convection, Micropolar ferrofluid, Salinity Rayleigh number, Perturbation technique, linear stability analysis

1. Introduction

Ferrofluids are colloidal suspension of fine magnetic mono domain nano-particles in non - conducting liquids. Such types of ferrofluids have wonderful applications in science and technology. Generally, the ferrofluids are used for cancer treatment in the biomedicine field. An excellent introduction and reviews of this extremely interesting monograph has been given by Rosensweig [1]. In his monograph, the fascinating information is introduced on magnetization. The convection in ferromagnetic fluid is analyzed in various aspects by Chandrasekhar [2]. Finlayson [3] has been investigated the convection in ferrofluid in single component fluid with uniform magnetic fluid. This investigation is extended to porous medium by Vaidyanathan et al. [4]. In non-presence of buoyancy effects, the thermoconvective instability in ferrofluid is given by Lalas and Carmi [5].

The micropolar fluids respond to spin inertia and micro-rotational motions. It can support couple stress and distributed body couples. Eringen [6] introduced the micropolar fluids theory. This theory has been developed by Eringen [7] on thermal effect. An excellent reviews and applications of this fluids theory can be obtained in by Ariman et al. [8] and Eringen [9]. Later, Ahmadi [10] employed firstly the energy method on convective instability of micropolar fluid with use of stability analysis. Pérez-Garcia and Rubi [11] analyzed the micropolar fluids with the effects of overstable motions. Narasimma Murty [12] examined the instability of the Bénard convection in a micropolar fluid using linear stability analysis.

In the effect of porous media, the double-diffusive convection is of greatest interest in mechanical and chemical engineering. In some special case, sodium chloride and temperature field are involving and this is often called as thermohaline convection. Thermohaline convection in a ferrofluid has been analyzed by Vaidyanathan et al. [13] with two-component fluid. The presence of porous medium on ferrothermohaline convection has been given by Vaidyanathan et al. [14].

The theoretical investigation of a micropolar ferromagnetic two component fluid in non-presence of Darcy porous effect has been undertaken by Sunil et al. [15]. The Soret effect is investigated on two component ferrofluid by Vaidyanathan et al. [16] and this is continued to large and small porous effect by Sekar et al. [17, 18]. Reena and Rana [19] have been analyzed the thermosolutal convective instability of micropolar rotating fluids in a porous effect. They used the Darcy model. Chand [20] studied the porous effect on triple-diffusive convective instability in micropolar ferromagnetic fluid.

In present investigation, our intension is to consider salinity gradient on magnetization and magnetic potential equation and thermal convection problem in micropolar fluid of Eringen extend to the thermohaline convection in micropolar ferrofluid. Also, an effect of salinity gradient and how micropolar parameters affect the stability in micropolar ferromagnetic fluid heated from below and salted from above. The stationary and oscillatory instabilities are studied.

2.Mathematical Formulation Of Problem

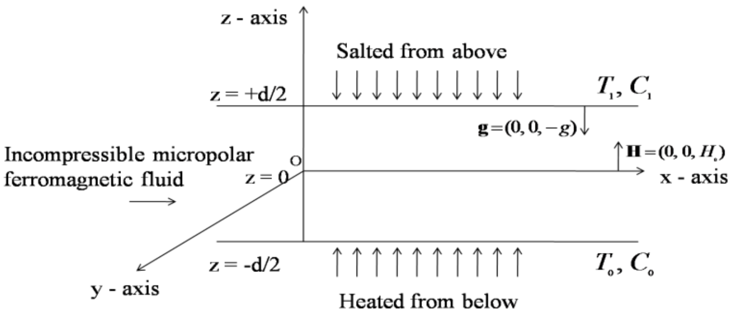

Fig.1 Geometrical configuration

Here we consider, an infinite horizontal micropolar ferrofluid layer heated from below and salted from above. The fluid layer is of thickness *d* and the fluid is considered as an electrically non-conducting incompressible one. The temperature and salinity at the bottom and top surfaces $z=\mp d/2$ are $T_0 \pm (DT)/2$ and $S_0 \mp (DS)/2$, respectively and $\beta_t (=|dT/dz|)$ and $\beta_s (=|dS/dz|)$ are maintained. The governing equations are

The continuity equation is

$$\nabla.\mathbf{q}=0 \tag{1}$$

The momentum and internal angular momentum equations are

$$\rho_0\left(\frac{\partial}{\partial t}+(\mathbf{q}.\nabla)\right)\mathbf{q}=-\nabla p+\rho\mathbf{g}+\nabla.(\mathbf{HB})+2\zeta(\nabla\times\boldsymbol{\omega})+(\zeta+\eta)\nabla^2\mathbf{q} \tag{2}$$

$$\rho_0 I\left(\frac{\partial}{\partial t}+(\mathbf{q}.\nabla)\right)\boldsymbol{\omega}=2\zeta(\nabla\times\mathbf{q}-2\boldsymbol{\omega})+\mu_0(\mathbf{M}\times\mathbf{H})+(\lambda'+\eta')\nabla(\nabla.\boldsymbol{\omega})+\eta'\nabla^2\boldsymbol{\omega} \tag{3}$$

The temperature equation is

$$\left[\rho_0 C_{v,H}-\mu_o\mathbf{H}.\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{v,H}\right]\frac{DT}{Dt}+\rho_s C_s\left(\frac{\partial T}{\partial t}\right)+\mu_0 T\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{v,H}.\frac{D\mathbf{H}}{Dt}=K_1\nabla^2 T+\delta(\nabla\times\boldsymbol{\omega}).\nabla T+\phi \tag{4}$$

The mass flux equation is

$$\rho_0(\partial/\partial t+\mathbf{q}.\nabla)S=K_S\nabla^2 S \tag{5}$$

We can assume the magnetization using Maxwell's equation for non-conducting fluids [16-18] is $\mathbf{M}=\mathbf{H}M(H,T,S)/H$. The linearized magnetic equation in term of H_0, T_a and S_a is

$$M=M_0+\chi(\mathrm{H}-H_0)-K(T-T_a)+K_2(S-S_a) \tag{6}$$

The density equation of state is

$$\rho=\rho_0[1-\alpha_t(T-T_a)+\alpha_S(S-S_a)] \tag{7}$$

where **q**- velocity of fluid, ρ_0 - mean density of the clean fluid, *p*- pressure, ρ - density of the fluid, **g** - gravitational field, **H**-magnetic field, **B**-magnetic induction, ζ -coupling viscosity, ω -microrotation, η -shear viscosity coefficient, *I*-moment of inertia, **M**-magnetization, λ'-bulk spin viscosity, η' -shear spin viscosity, $C_{v,H}$ -effective heat capacity at constant volume, C_s -specific heat solid material, μ_0 -viscosity of the fluid when the applied magnetic field is absent, K_1-thermal diffusivity, *T*-temperature, δ -micropolar heat conduction coefficient, *S*-solute concentration, K_s -concentration diffusivity, H_0 -uniform magnetic field, T_a -average temperature, S_a -average salinity, α_t -thermal expansion coefficient and α_s -analogous solvent coefficient.

The basic state quantities are

$$\mathbf{M}_b(z)=\left[M_0+\frac{K\beta_t\,z}{1+\chi}-\frac{K_2\beta_S\,z}{1+\chi}\right]\mathbf{k},\ \rho(z)=\rho_0[1+\alpha_t\beta_t z-\alpha_S\beta_S z],\ \mathbf{q}=\mathbf{q}_b=\ (0,\ 0,\ 0),$$
$$\mathbf{H}_b(z)=\left[H_0-\frac{K\beta_t\,z}{1+\chi}+\frac{K_2\beta_S\,z}{1+\chi}\right]\mathbf{k},\quad \boldsymbol{\omega}=\boldsymbol{\omega}_b=(0,\ 0,\ 0), \tag{8}$$
$$T_b=T_0-\beta_t z,\ S_b=S_0-\beta_S z,\ p=p_b(z)\quad \text{and}\quad M_0+H_0=H_0^{ext}.$$
where subscript b − the basic state and $\hat{\mathbf{k}}$ − unit vector vertical direction.

A small thermal disturbance is made on the system. Let us take the perturbed components of **M** and **H** be $[M_1^{'},\ M_2^{'},\ M_0(z)+M_3^{'}]$ and $[H_1^{'},\ H_2^{'},\ H_0(z)+H_3^{'}]$, respectively. The perturbed quantities are

$$\mathbf{H}=\mathbf{H}_b(z)+\mathbf{H}',\ \boldsymbol{\omega}=\boldsymbol{\omega}_b+\boldsymbol{\omega}',\ \ p=p_b(z)+p',\ S=S_b(\mathrm{z})+S',$$
$$\mathbf{M}=\mathbf{M}_b(z)+\mathbf{M}',\ \mathbf{q}=\mathbf{q}_b+\mathbf{q}',\ \rho=\rho_b+\rho',T=T_b(\mathrm{z})+\theta, \tag{9}$$

the superscript ' denotes perturbed state.

The perturbed density equation can be calculated as

$$\rho'=\rho_0(-\alpha_t\theta+\alpha_s S') \tag{10}$$

1. normal mode analysis method

We undertake the perturbation quantities by use of normal modes are

$$w(x,y,z,t)=w(z,t)\ \exp[i\,k_x\,x+i\,k_y\,y]$$
$$\theta(x,y,z,t)=\theta(z,t)\ \exp[i\,k_x\,x+i\,k_y\,y]$$
$$\phi(x,y,z,t)=\phi(z,t)\ \exp[i\,k_x\,x+i\,k_y\,y] \tag{11}$$
$$S(x,y,z,t)=S(z,t)\ \exp[i\,k_x\,x+i\,k_y\,y]$$

In Eq. (2), one can get the k[th] component is

$$\left.\begin{aligned}&\left(\rho_0\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\right)w=\mu_0K\beta_t k_0^2\frac{\partial\phi}{\partial z}-\left(\frac{\mu_0K^2\beta_t}{1+\chi}\right)k_0^2\theta+\left(\frac{\mu_0KK_2\beta_t}{1+\chi}\right)k_0^2S'-\mu_0K_2\beta_S k_0^2\frac{\partial\phi}{\partial z}+\left(\frac{\mu_0KK_2\beta_S}{1+\chi}\right)k_0^2\theta\\&\qquad-\left(\frac{\mu_0K_2^2\beta_S}{1+\chi}\right)k_0^2S'-\rho_0g\alpha_t k_0^2\theta+\rho_0g\alpha_S k_0^2S+2\zeta\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\Omega_3^{'}+(\zeta+\eta)\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)^2w\end{aligned}\right\} \tag{12}$$

Internal angular Eq. (3) can be manipulated as

$$\rho_0I\frac{\partial\Omega_3^{'}}{\partial t}=-2\zeta\left[\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)w+2\Omega_3^{'}\right]+\eta'\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\Omega_3^{'} \tag{13}$$

Eq. (4) can be calculated as

$$\rho C_1\frac{\partial\theta}{\partial t}-\mu_0KT_0\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)=K_1\left(\frac{\partial^2}{\partial z}-k_0^2\right)\theta+\left[\rho C_2\beta_t-\left(\frac{\mu_0K^2T_0^2\beta_t}{1+\chi}\right)+\left(\frac{\mu_0\,K\,K_2T_0\beta_S}{1+\chi}\right)\right]w-\delta\beta_t\Omega_3^{'} \tag{14}$$

The Salinity equation is

$$(\partial/\partial t)S+\beta_S w=K_S\left((\partial^2/\partial z^2)-k_0^2\right)S' \tag{15}$$

Using Vaidyanathan et al. [14], one gets

$$(1+\chi)\frac{\partial^2\phi}{\partial z^2}-\left(1+\frac{M_0}{H_0}\right)k_0^2\phi-K\frac{\partial\theta}{\partial z}+K_2\frac{\partial S}{\partial z}=0 \tag{16}$$

The non-dimensional equations can be derived by use of normal mode method as

$$\left.\begin{aligned}\left(\frac{\partial}{\partial t^*}(D^2-a^2)\right)\ w^* &= a\,M_1R^{1/2}\,D\phi^* - a(1+M_1)\,R^{1/2}T^* + a\,R^{1/2}M_1M_5D\phi^*\\ &+ a\,R_S^{1/2}(1+M_4)S^* - a\,R^{1/2}M_1M_5\,T^* + a\,R_S^{1/2}\,\frac{M_4}{M_5}S^*\\ &+ 2N_1(D^2-a^2)\Omega_3^{'*} + (\zeta+\eta)(D^2-a^2)^2\,w^*\end{aligned}\right\} \tag{17}$$

$$I'\frac{\partial\Omega_3^{'*}}{\partial t} = -2\left[\left(D^2-a^2\right)w^* + 2\Omega_3^{'*}\right]N_1 + \left(D^2-a^2\right)N_3^{'}\Omega_3^{'*} \tag{18}$$

$$\left[P_r^{'}\frac{\partial T^*}{\partial t^*} - M_2P_r\frac{\partial}{\partial t^*}(D\phi^*)\right] = (D^2-a^2)T^* + a\ \ (1-M_2-M_2M_5)R^{1/2}w^* - aN_5^{'}R^{1/2}\Omega_3^{'*} \tag{19}$$

$$P_r\frac{\partial S^*}{\partial t^*} = \tau(D^2-a^2)S^* - aR_S^{1/2}\left(\frac{M_5}{M_6}\right)w^*, \tag{20}$$

$$D^2\phi^* - M_3a^2\phi^* - DT^* + \frac{M_5}{\tau}\left(\frac{R}{R_S}\right)^{1/2}DS^* = 0, \tag{21}$$

where the dimensionless quantities are

$$\left.\begin{aligned}&w^* = \frac{wd}{\nu},\ t^* = \frac{\nu t}{d^2},\ T^* = \left(\frac{K_1aR^{1/2}}{\rho_0C_{v,H}\beta_t\nu d}\right)\theta,\ \ \phi^* = \left(\frac{(1+\chi)K_1aR^{1/2}}{\rho_0C_{v,H}\,K\beta_t\nu d^2}\right)\phi,\ \ z^* = \frac{z}{d}, a = k_0d,\ \ D = \frac{\partial}{\partial z^*},\\ &S^* = \left(\frac{K_SaR_S^{1/2}}{\rho_0C_{v,H}\beta_S\nu d}\right)S,\ \ \nu = \frac{\mu}{\rho_0},\ \ \Omega_3^{'*} = \frac{\Omega_3}{\nu}d^3M_1 = \frac{\mu_0K^2\beta_t}{(1+\chi)\rho_0g\alpha_t},\ \ M_2 = \frac{\mu_0K^2T}{(1+\chi)\rho_0C_{v,H}},\ N_1 = \frac{\zeta}{\eta},\\ &M_3 = \frac{1+M_0/H_0}{(1+\chi)},\ M_4 = \ \frac{\mu_0K^2\beta_s}{(1+\chi)\rho_0g\alpha_s}, M_5 = \frac{K_2\beta_s}{K\beta_t},\ \ M_6 = \frac{K_S}{K_1},\ \tau = \rho_0C_{v,H}\left(\frac{K_S}{K_1}\right), N_3^{'} = \frac{\eta'}{\eta d^2},\\ &N_5^{'} = \frac{\delta}{\rho C_2d^2}, I' = \frac{I}{d^2},\ P_r = \frac{\nu}{K_1}\rho C_2,\ P_r^{'} = \frac{\nu}{K_1}\rho C_1,\ R_S = \frac{\rho_0C_{v,H}\beta_s\alpha_sgd^4}{\nu K_S},\ \ R = \frac{\rho_0C_{v,H}\beta_t\alpha_tgd^4}{\nu K_1}\end{aligned}\right\} \tag{22}$$

2. linear stability analysis

The stationary and oscillatory instabilities have been studied using linear theory. The boundary conditions are

$$w^* = D^2w^* = D\phi^* = S^* = \Omega_3^* = T^* = 0 \ \text{ at } \ z^* = \pm 1/2. \tag{23}$$

The exact solutions satisfying above Eq. (23) are

$$\left.\begin{aligned}&w^* = X_1e^{\sigma t^*}\cos\pi z^*,\quad T^* = X_2e^{\sigma t^*}\cos\pi z^*,\quad S^* = X_3e^{\sigma t^*}\cos\pi z^*\\ &D\phi^* = X_4e^{\sigma t^*}\cos\pi z^*,\quad \phi^* = \frac{X_4}{\pi}\sin\pi z^*,\ \Omega_3^* = X_5e^{\sigma t^*}\cos\pi z^*\end{aligned}\right\} \tag{24}$$

where $X_1,\ X_2,\ X_3,\ X_4$ and X_5 are constants. Eqs. (17)–(21) can be mathematical manipulated using Eq. (23) as

$$\left.\begin{aligned}&(\pi^2+a^2)\ \ \left[\sigma + (1+N_1)(\pi^2+a^2)^2\right]X_1 - aR^{1/2}\left[1+(1+M_5)M_1\right]X_2\\ &\quad + a(1+M_4+M_4M_5^{-1})R_S^{1/2}X_3 + a(1+M_5)R^{1/2}\ M_1X_4 - 2(\pi^2+a^2)N_1X_5 = 0,\end{aligned}\right\} \tag{25}$$

$$-2(\pi^2+a^2)N_1X_1 + \left[4N_1 + (\pi^2+a^2)N_3^{'} + I'\sigma\right]X_5 = 0, \tag{26}$$

$$a(1-M_2-M_2M_5)R^{1/2}X_1 - (P_r\sigma + \pi^2 + a^2)X_2 + P_r\varepsilon\sigma M_2X_4 - aN_5^{'}R^{1/2}X_5 = 0, \tag{27}$$

$$aM_6R_S^{1/2}X_1 + \left[(\pi^2+a^2)\tau + \sigma P_r\right]X_3 = 0, \tag{28}$$

$$-R_S^{1/2}\pi^2X_2 + R^{1/2}\,\pi^2(M_5/M_6^{-1})X_3 + R_S^{1/2}(\pi^2+a^2M_3)X_5 = 0, \tag{29}$$

To evaluate the Eigen function, determination of the co-efficient of X_1, X_2, X_3, X_4 and X_5 in Eqs. (25)–(29) is equal to zero. Using the analyses Vaidyanathan et al. [13, 14], Eqs. (25)–(29) have been adopted to get

$$T_1\sigma^4 + T_2\sigma^3 + T_3\sigma^2 + T_4\sigma + T_5 = 0 \tag{30}$$

where

$$T_1 = P_r^{'}(I' - x_1),\ T_2 = P_r^{'}I'x_1^2(1+P_r^{'}) - P_r^{'}x_1(P_r^{'} + I')$$

$$T_3 = a^2x_4P_r^{'}I'R - x_8x_6^2P_r^{'2} + x_1^2(1+P_r^{'})(P_r^{'}+I') + x_8P_r^{'}I'(a^2x_2 - x_1^2) - x_1^2x_6x_7\tau P_r^{'}$$
$$+ a^2M_6P_r^{'}I'R(x_4x_9 + x_3x_8)$$

$$T_4 = -a^2\pi^2\tau x_1x_4I'R - a^2\pi^2P_r^{'}x_4R(x_7 - x_6N_5^{'}) - x_8x_6P_r^{'}(a^2x_2N_5^{'}R + x_1x_6) - x_1x_6^2x_8\tau P_r^{'}$$
$$+ (P_r^{'} + I')(a^2x_2R - x_1^2) - x_1^3x_7x_8\tau(1+P_r^{'}) + (P_r^{'}x_7 - x_1I')(a^2M_6x_3x_8 - a^2M_6x_4x_9R)$$

$$T_5 = x_1\tau(\pi^2(a^2x_4x_7R - a^2x_4x_6N_5^{'}R) + x_8(x_6(a^2x_2R + x_1x_6) - x_7(a^2x_2R - x_1^3)))$$
$$- a^2x_7M_6(x_1x_4x_9R + x_1x_3x_9R_s)$$

$$x_1 = \pi^2 + a^2,\ x_2 = 1 + x_4,\ x_3 = 1 + M_4 + (M_4/M_5),\ x_4 = M_1(1+M_5),$$
$$x_5 = 2N_1,\ x_6 = x_1x_5,\ x_7 = 4N_1 + x_1N_3^{'},\ x_8 = \pi^2 + a^2M_3,\ x_9 = M_5\pi^2/M_6$$

4.1 The case of stationary instability

For steady state, we have σ = 0 at the marginal stability. Then the Eq. (30) leads to get Eigen value R_{sc} for which solution exists. Using the analyses [14]-[15], the critical magnetic Rayleigh number R_{sc} has been obtained using

$$R_{sc} = \frac{Nr}{Dr} \tag{31}$$

where

$$Nr = (\pi^2 + a^2)^3\left(\left(4N_1 + (\pi^2+a^2)N_3^{'}\right)(1+N_1) - 4N_1^2\right) - a^2(1 + M_4 + M_4M_5^{-1})\left(4N_1 + (\pi^2+a^2)N_3^{'}\right)M_6R_s\tau^{-1}$$

$$Dr = a^2\left(1 + M_1(1+M_5)\right)\left(4N_1 + (\pi^2+a^2)\left(N_3^{'} - 2N_1N_5^{'}\right)\right)$$
$$-\frac{a^2\pi^2M_1(1+M_5)}{(\pi^2 + a^2M_3)}\left(\left(4N_1 + (\pi^2+a^2)N_3^{'}\right)\left(1 + M_5\tau^{-1}\right) - 2N_1N_5^{'}(\pi^2+a^2)\right)$$

When M_1 is very large, one can gets N_{sc} (= M_1R_{sc}).

$$N_{sc} = \frac{Nr}{Dr} \tag{32}$$

where

$$Dr = a^2(1+M_5)\left(4N_1 + (\pi^2+a^2)\left(N_3^{'} - 2N_1N_5^{'}\right)\right)$$
$$-\frac{a^2(1+M_5)\pi^2}{(\pi^2 + a^2M_3)}\left(\left(4N_1 + (\pi^2+a^2)N_3^{'}\right)\left(1 + M_5\tau^{-1}\right) - 2N_1N_5^{'}(\pi^2+a^2)\right)$$

Here a is denoted as critical wave number a_c. Analysis of the classical results is given below:

Assuming $\varepsilon = 1,\ N_3^{'} = 1,\ N_1 = 0,$ and $N_5^{'} = 0$ in Eq. (32), one get

$$N_{sc} = \frac{(\pi^2+a^2)^3 - a^2(1 + M_4 + M_4M_5^{-1})M_6\tau^{-1}R_S}{a^2(1+M_5)\left(1 - \pi^2\left(1 + M_5\tau^{-1}\right)/(\pi^2 + a^2M_3)\right)} \tag{33}$$

which is an expression for N_{sc} of Vaidyanathan et al. [13].

Moreover, if $M_4,\ M_6, \tau^{-1},\ R_S = 0$ in Eq. (33), it gives N_{sc} of the Finlayson [3], a single component fluid.

4.2 The case of oscillatory instability

Taking $\sigma = i\sigma_1 \quad (\sigma_1 > 0)$ in Eq. (30), it leads to R_{oc} has been derived using

$$R_{oc} = \left(T_1 Y_2 \sigma_1^6 + (T_1 Y_5 + T_2 Y_3)\sigma_1^4 + (Y_2(Y_2 + Y_6) + Y_3 Y_4)\sigma_1^2 + Y_5(Y_2 + Y_6)\right) / Dr$$

where

$$Y_1 = a^2 M_6 P_r' I'(x_4 x_9 + x_3 x_8) - a^2 x_4 P_r' I' - a^2 x_2 x_8 P_r' I',$$

$$Y_2 = x_1^2 (1 + P_r')(P_r' + I') - x_8 x_6^2 P_r'^2 - x_1^3 x_8 P_r' I' - P_r' \tau x_1^2 x_8 x_7,$$

$$Y_3 = -a^2 \pi^2 I' \tau x_1 x_4 - a^2 \pi^2 P_r' x_4 (x_7 - x_6 N_5') - a^2 x_3 x_6 P_r' x_2 N_5' - x_1 x_6^2 x_8 \tau P_r' + a^2 x_2 (P_r' + I') - a^2 M_6 x_4 x_9 (P_r' x_7 - x_1 I'),$$

$$Y_4 = -x_8 x_6 P_r' \tau - (P_r' + I') x_1^2 + x_1^3 x_7 x_8 \tau (1 + P_r') - (P_r' x_7 - x_1 I') a^2 M_6 x_3 x_8 R_s,$$

$$Y_5 = x_1 \tau \pi^2 (a^2 x_4 x_7 - a^2 x_4 x_6 N_5') + a^2 x_2 x_6 x_8 N_5' + a^2 x_2 x_6 x_7 x_8 + a^2 M_6 x_1 x_4 x_7 x_9,$$

$$Y_6 = \tau x_1^4 x_7 x_8 + a^2 M_6 P_r' x_7 x_8 - \tau x_1^2 x_6^2 x_8,$$

$$Dr = \left(Y_2 \sigma_1^2 + Y_5\right)^2 - \sigma_1^2 Y_3^2, \; \sigma_1^2 = (-A_2 \pm \sqrt{A_2^2 - 4A_1 A_3}) / 2A_1,$$

$$A_1 = T_2 Y_2 + T_1 Y_3, \; A_2 = T_2 Y_5 + Y_4 Y_2, \quad \text{and} \quad A_3 = Y_4 Y_5 - Y_3 (Y_2 + Y_6)$$

3. Discussion of Results

In this investigation, thermohaline convection in micropolar ferrofluid layer is studied. The fluid layer is heated from below and salted from above and the convective system is subjected to a transverse uniform magnetic field. The thermal perturbations and linear stability analysis are used in the study. Here we consider the free boundary conditions. The magnetic numbers M_1 and M_2 are considered the values 1000 and 0, respectively. M_3 is taken from 5 to 25 (Vaidyanthan et al. [14]) and τ ranges from 0.05 to 0.11 (Vaidyanthan et al. [14]) and R_s taken from -500 to 500. The magnetization parameters M_4 and M_6 are taken to be 0.1 and M_5 = 0.5 (Vaidyanthan et al. [16]). Further, N_1, N_3' and N_5' are taken to be non-negative values which is presented by Eringen [21] and he assumed the clausius-Duhem inequality. P_r is taken as 0.01.

The variation of N_{sc} with the coupling parameter N_1 is depicted in Fig. 2 (a) and (b). It is observed from the Fig. 2 (a) that the convective system gives stabilizing behavior, when increasing values of M_3 and R_S. Due to the increasing value of N_1 from 0 to 1 and R_S from -500 to 500 on the system, N_{sc} gets the highest values and the system has more stabilizing effect. But, an increasing of M_3 from 5 to 25, the system shows the stabilizing effect and it is less pronounced. M_3 analyzed for destabilizing behavior always [13-14, 16-17]. But, introducing of N_1 on M_3, the system gets stabilizing effect. Fig. 2 (b) represents the plot of N_{sc} versus N_1 for different τ. This figure shows that N_1 has the stabilizing behavior for increasing value of τ. This is because, the greater the mass and heat transports and more buoyancy energy, which contributes to thermal instability. Also, it is shown from the Fig. 2 (c) that the increase in N_1 stabilizes the system for increasing of τ and R_S. Also, in the presence of R_S = 500, a_c is close to zero. In this moment, the system has an equilibrium state.

Figs. 3 (a) and (b) display the variation of N_{sc} versus spin diffusion parameter N_3' for increasing of R_S and M_3 and τ, respectively. In Fig. 3 (a), we observe that N_{sc} decreases with increasing of N_3', which leads to destabilize the system. Moreover, when R_S = 500, N_{sc} gets zero value. Therefore, the system has a null effect. From Fig. 3 (b), it is seen that as N_3' increases from 2 to 8, there is a decrease in N_{sc} indicating destabilization for different τ. Fig. 3 (c) shows the variation of a_c versus N_3' for various τ, R_S and M_3. When τ increases from 0.05 to 0.1, R_S increases from – 500 to 500 and M_3 increases from 5 to 25, there is a decrease in a_c. It is clear that there is a destabilization on the system which is not much pronounced and when R_S = 500, there is an oscillation in a_c.

Figs. 4 (a) and (b) show the variation of N_{sc} versus micropolar heat conduction parameter N_5' for different M_3, τ and. It is clear from the Fig. 4 (a) that N_5' leads to an increase in N_{sc}. Therefore, N_5' has a stabilizing effect. It is very clear from the Fig. 4 (b) that increase in N_5', it is stabilizing behavior for various R_S. Fig. 4 (c) represents the critical wave number a_c versus the N_5' for various physical parameter τ, R_S and M_3. In this figure N_5' shows a stabilizing behavior. In such situation also the system ha no effect when R_S = 500.

The increase in non-buoyancy magnetization parameter M_3 is obtained to cause large destabilization, because both thermal and magnetic mechanisms favor destabilization. This can be studied from Figs. 5 (a) and (b) in which the increase in M_3 and τ, decrease in N_{sc} and a_c, respectively.

From Fig. 6 (a), it is seen that an increase in R_S, decrease in N_{sc}. An increase of R_S would means that the system is salted from above. Also, when $R_S = 500$ and τ (=0.05, 0.07, 0.09), the N_{sc} gets small values. But for the value τ = 0.11, suddenly N_{sc} gets highest value. In this moment, the convective system gets stabilizing effect. Fig. 6 (b) shows the variation of a_c versus R_S for different τ. When R_S increases from -500 to 500, there is a decrease in a_c promoting instability. When the highest value of R_S (=500) the system tends to the same effect. That is, the system converges to the small values. But, when the highest values of τ (=0.11), the system has an equilibrium position.

4. Conclusion

In the present analysis, the results of a theoretical study on thermohaline convection in a micropolar ferrofluid are considered with free boundary conditions. We conclude that the effect of non-buoyancy magnetization M_3, salinity effect R_S, spin diffusion parameter N_3' have destabilizing behavior and the effect of coupling parameter N_1 and the micropolar heat conduction parameter N_5' have a stabilizing effect due to the microrotation on the onset of convection.

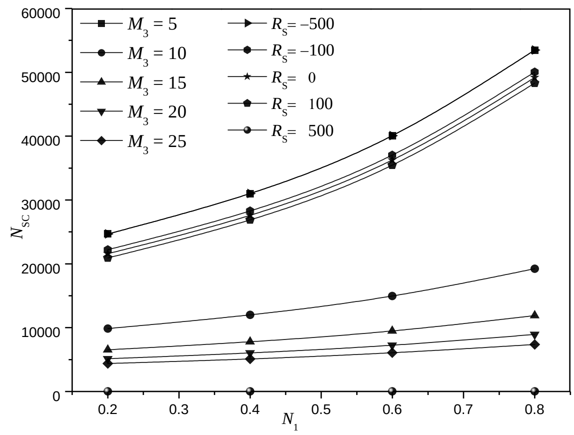

Fig. 2 (a). Variation of N_{sc} versus N_1 for different M_3 and R_S, $N_3'=2, N_5'=0.2$ and $\tau=0.05$

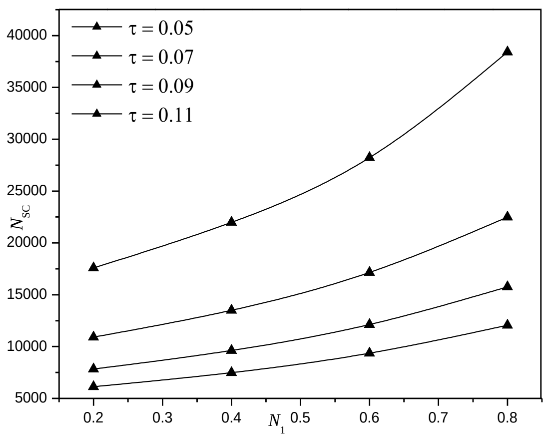

Fig. 2(b). Variation of N_{sc} versus N_1 for different τ, $N_3'=2, N_5'=0.2,$ $M_3 = 5$ and $R_S = -500$.

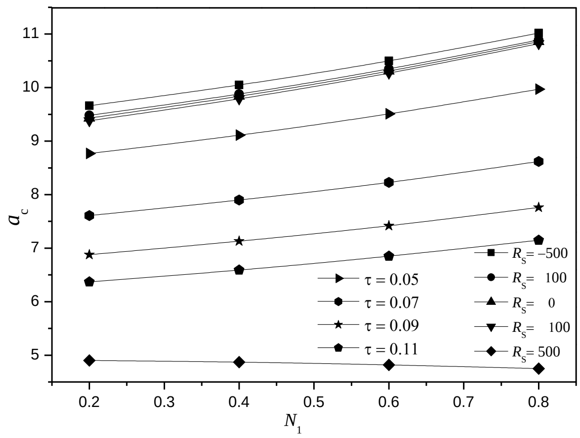

Fig. 2 (c). Variation of a_c versus N_1 for different R_S and τ, $N_3'=2, N_5'=0.2$ and $M_3 = 5$.

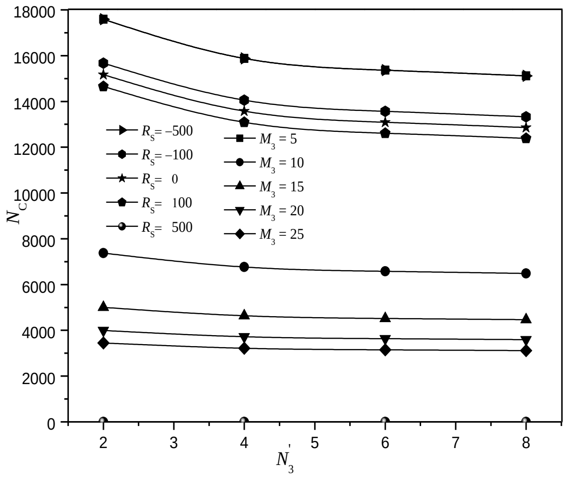

Fig. 3 (a). Variation of N_{sc} versus N_3' for different M_3 and R_S, $N_1 = 0.2,\ N_5' = 0.2$ and $\tau = 0.05$.

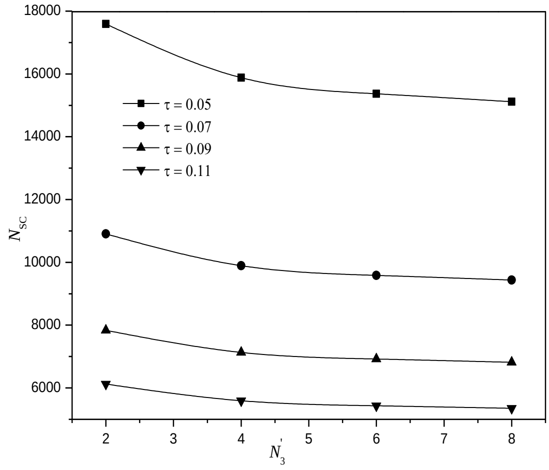

Fig. 3 (b). Variation of N_{sc} versus N_3' for different τ, $N_1 = 0.2,\ N_5' = 0.2,$ M_3 = 5 and R_S = -500.

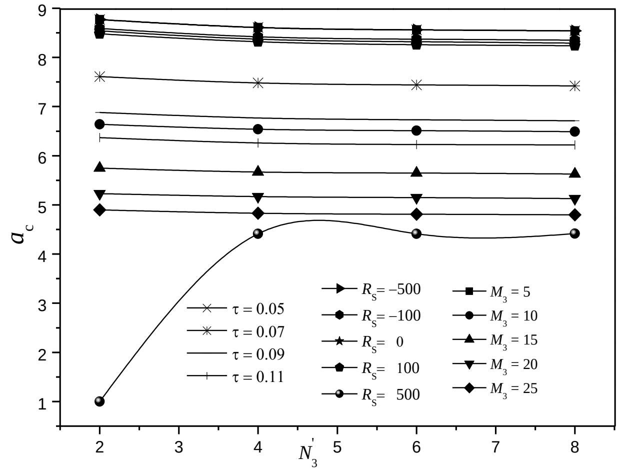

Fig. 3 (c) – Variation of a_c versus N_3' for different R_S, M_3 and τ, $N_1 = 0.2,\ N_5' = 0.2$ and M_3 = 5.

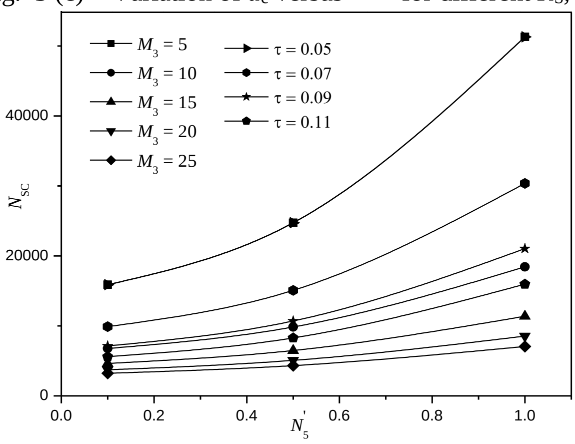

Fig. 4 (a). Variation of N_{sc} versus N_5' for different M_3 and τ, $N_1 = 0.2,\ N_3' = 2$ and R_S = -500.

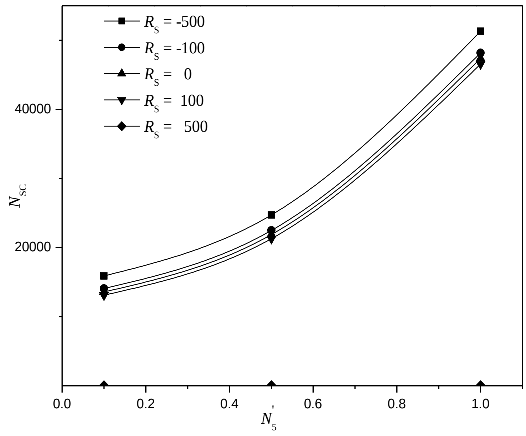

Fig. 4 (b). Variation of N_{sc} versus N_5' for different R_S, $\tau = 0.05,$ $N_1 = 0.2,\ N_3' = 2,$ M_3 = 5 and R_S = -500.

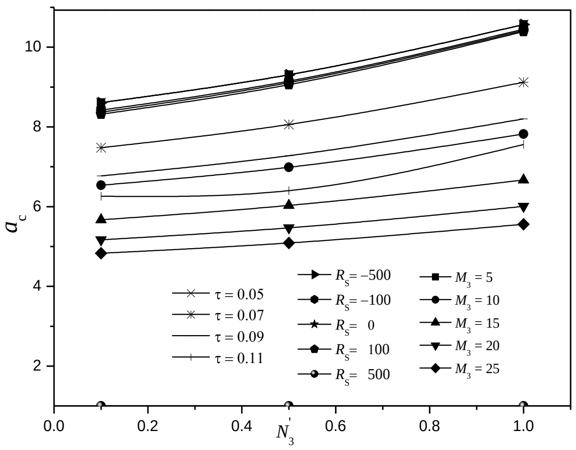

Fig. 4 (c). Variation of a_c versus N_5' for different R_S, M_3 and τ, $N_1 = 0.2$, $N_5' = 0.2$ and M_3 = 5.

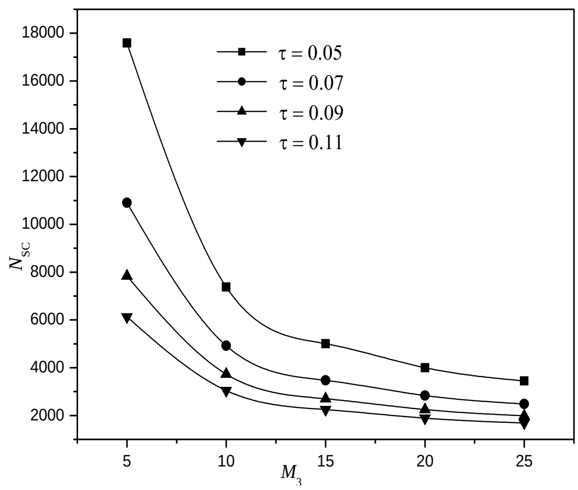

Fig. 5 (a). Variation of N_{sc} versus M_3 for different τ, $N_1 = 0.2$, $N_3' = 2$, $N_5' = 0.2$ and R_S = -500.

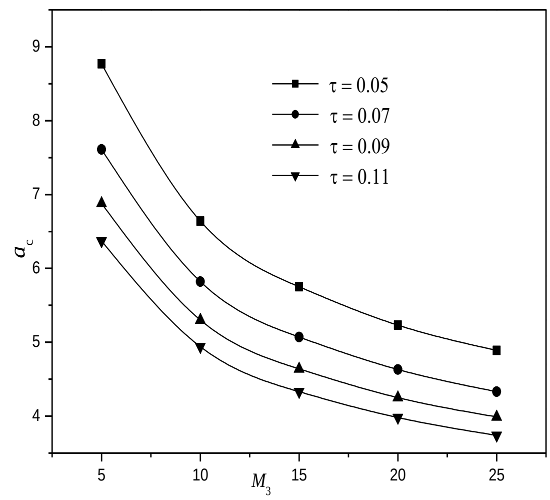

Fig. 5 (b). Variation of a_c versus M_3 for different τ, $N_1 = 0.2$, $N_3' = 2, N_5' = 0.2$ and R_S = -500.

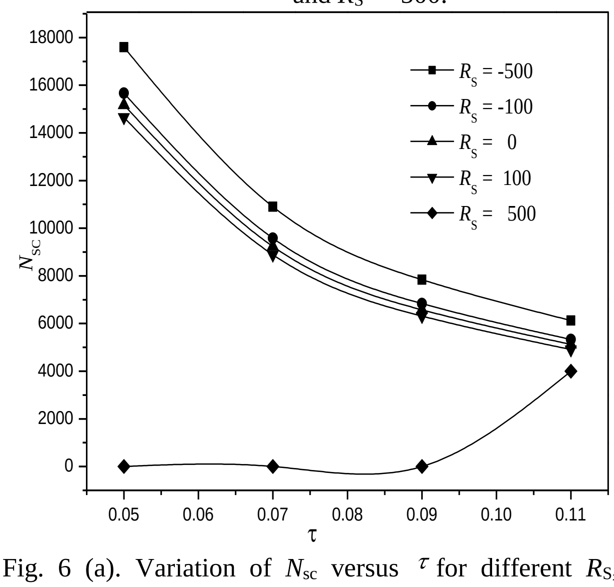

Fig. 6 (a). Variation of N_{sc} versus τ for different R_S, $N_1 = 0.2$, $N_3' = 2$, $N_5' = 0.2$ and M_3 = 5.

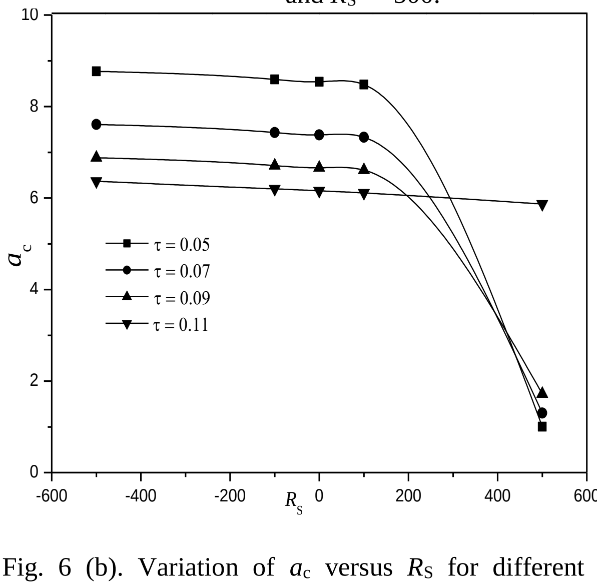

Fig. 6 (b). Variation of a_c versus R_S for different τ, $N_1 = 0.2$, $N_3' = 2, N_5' = 0.2$ and M_3 =5.

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REFERENCES

1. R. E. ROSENSWEIG, *Ferrohydrodynamics*, Cambridge University Press, Cambridge, 1985.
2. S. CHANDRASEKHAR, *Hydrodynamics and Hydromagnetic stability*, Oxford University Press, London, 1961.
3. B. A. FINLAYSON, *Convective instability of ferromagnetic fluids*, International Journal of Fluid Mechanics, **40**, pp.753-767, 1970.
4. G. VAIDYANATHAN, R.SEKAR, R. BALASUBRAMANIAN, *Ferroconvective instability of fluids saturating a porous medium*, International Journal of Engineering Science, **29**, pp.1259-1267, 1991.
5. D. P. LALAS, S. CARMI, *Thermoconvective stability of ferrofluids*, Physics Fluids, **14**, pp.436-437, 1971.

   C. ERINGEN, *Theory of micropolar fluids*, J. Math. Mech., **16**, pp.1–18, 1966.

   C. ERINGEN, *Theory of thermomicrofluids*, Journal of Mathematical Analysis and Applications, **38**, pp.480–496, 1972.
6. T. ARIMAN, M. A. TURK, N. D. SYLVESTER, *Applications of microcontinuum fluid mechanics*, International Journal Engineering Science, **12**, pp.273-293, 1974.

   C. ERINGEN, *Microcontinum fields theories*, II: Fluent Media, Springer, New York, 2001.
7. G. AHMADI, *Stability of micropolar fluid layer heated from below*, International Journal Engineering Science, **14**, pp.81–89, 1976.

   PÉREZ-GARCIA, J. M. RUBI, *On the possibility of overstable motions of micropolar fluids heated from below*, International Journal Engineering Science, **20**, pp.873 – 878, 1982.
8. Y. NARASIMMA MURTY, *Analysis of non-uniform temperature profiles on Bénard convection in micropolar fluids*, Applied Mathematics and Computation, **134**, pp.473 – 486, 2003.
9. G. VAIDYANATHAN, R. SEKAR, A. RAMANATHAN, *Ferro thermohaline convection*, Journal of Magnetism and Magnetic Materials, **176**, pp.321–330, 1997.
10. G. VAIDYANATHAN, R. SEKAR, A. RAMANATHAN, *Ferro thermohaline convection in a porous medium*, Journal of Magnetism and Magnetic Materials, **149**, pp.137–142, 1995.
11. SUNIL, C. PRAGASH, P. K. BHARTI, *Double diffusive convection in a micropolar ferromagnetic fluid*, Applied Mathematics and Computation, **189**, pp.1648–1661, 2007.
12. G. VAIDYANATHAN, R. SEKAR, R. HEMALATHA, R. VASANTHAKUMARI, S. SENTHILNATHAN, *Soret-driven ferro thermohaline convection*, Journal of Magnetism and Magnetic Materials, **288**, pp.460–469, 2005.
13. R. SEKAR, G. VAIDYANATHAN, R. HEMALATHA, S. SENTHILNATHAN, *Effect of sparse distribution pores in a Soret-driven ferro thermohaline convection*, Journal of Magnetism and Magnetic Materials, **302**, pp.20–28, 2006.
14. R. SEKAR, D. MURUGAN, K. RAJU, *Stability analysis of thermohaline convection in ferromagnetic fluid in densely packed porous medium with Soret effect*, World Journal of Engineering **10**(5), 439-447, 2013.
15. REENA, U. S. RANA, *Thermosolutal convection of micropolar rotating fluids saturated porous medium*, IJE Transaction A: Basics **22**, pp. 379–404, 2009.
16. S. CHAND, *Linear stability of triple-diffusive convection in micropolar ferromagnetic fluid saturating porous medium*, Applied Mathematics and Mechanics, **34**, pp. 309–326, 2013.

    C. ERINGEN, *Simple microfluids*, International Journal of Engineering Science, **2**, 205-217, 1964.