MULTIPLICATIVE GEOMETRIC ARITHMETIC INDEX FOR VARIOUS GRAPHS

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Abstract: A topological index, also known as connectivity index, is a molecular structure descriptor calculated from a molecular graph of a chemical compound which characterizes its topology. Various topological indices are categorized based on their degree, distance and spectrum. In this study, the degree-based topological indices such as multiplicative geometric - arithmetic index ((MGA) index)isderived for various graphs.

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1. INTRODUCTION

A graph is an ordered pair $G=(V,E)$, where V is a non-empty finite set, called the set of vertices of G, and E is a set of unordered pairs of V, called the edges of G. If $xy \in E$ then x and y are called adjacent and they are incident with the edge xy.

For a graph $G=(V,E)$, the order is $|V|$, the number of its vertices. And the size is $|E|$, the number of its edges. The degree of a vertex $x \in V$, denoted by $d(x)$, is the number of edges incident with it. [1]

The complete graph on n vertices, denoted by K_n, is a graph on n vertices such that every pair of vertices is connected by an edge. The empty graph on n vertices, denoted by E_n is a graph on n vertices with no edges. Thecompletebipartitegraph $K_{m,n}$ on $n+m$ verticesasthe(unlabelled) graph, isomorphic to $(A \cup B = \{xy : x \in A,\ y \in B\})$, where $|A| = m$ and $|B| = n,\ A \cap B = \varnothing$.The order of a graph $G=(V,E)$ is $|V|$, the number of its vertices. The size of G is $|E|$, the number of its edges. [1,2]

The degree of a vertex v of G, denoted by $d(v)$ or $\deg(v)$, is the number of edges incident to v. A vertex of degree one in G is called a leaf or pendant vertex, and a vertex of degree 0 in G is called an isolated vertex. The minimum degree of G, denoted by $\delta(G)$, is the smallest vertex degree in G. The maximum degree of G, denoted by $\Delta(G)$, is the largest vertex degree in G. The graph G is called k-regular for a natural number k if all vertices have degree k. [2,6]

Let G_1 and G_2 be two graphs with disjoint vertex sets V_1 and V_2, and edge sets E_1 and E_2, respectively. Thenthe join$G_1 + G_2$is the graph consisting of$G_1 \cup G_2$with all edges joining$V_1$with V_2.[6]

The degree-based topological indices is the most investigated categories of topological indices, which is used in mathematical chemistry.

The topological index for a graph is defined in [5],

$$TI(G) = \sum_{pq \in G} F\left(d(p), d(q)\right)$$

Multiplicative geometric - arithmetic index is defined as follows [7]

$$MGA(G) = \prod_{pq \in E(G)} \left(\frac{2\sqrt{\left(d_p \cdot d_q\right)}}{\left(d_p + d_q\right)} \right)$$

In this paper, we calculated and analysed the degree-based topological indices such as multiplicative geometric - arithmetic index (MGA) index. Further investigated the (MGA) index in regular graph, complete graph, complete bipartite graph, union graphs and join graphs are derived. Further explain the theorem by examples.

2. THE MULTIPLICATIVE GEOMETRIC - ARITHMETIC (MGA) INDEX OF VARIOUS GRAPHS

In this section, the Multiplicative geometric - arithmetic index (MGA) index of regular graph, complete graph, complete bipartite graph and join of graphs are investigated.

Theorem 2.1: For a K regular graph, the Multiplicative geometric - arithmetic index (MGA) index) is unity.

Proof: Let G be a K regular graph of ordern. This implies the degree of every vertex in G is K and n number of vertices in the graph G. In a K regular graph there are $\left(\frac{nK}{2}\right)$ edges in regular graph. Therefore (MGA) index for K regular graph is,

$$MGA(G) = \prod_{pq \in E(G)} \left(\frac{2\sqrt{\left(d_p \cdot d_q\right)}}{\left(d_p + d_q\right)} \right)$$

$$= \prod_{pq \in E(G)} \left(\frac{2\sqrt{(K \cdot K)}}{(K + K)} \right)$$

$$MGA(G) = \prod_{pq \in E(G)} \left(\frac{2\sqrt{\left(K^2\right)}}{(K + K)} \right)$$

$$= \prod_{pq\in E(G)} \left(\frac{2K}{(K+K)} \right) =$$

$$= \prod_{pq\in E(G)} (1) = (1)^{\left(nk/_2\right)}$$

$$MGA(G) = 1$$

Hence the (MGA) index for K regular graph is equal to unity.

Example 2.1:

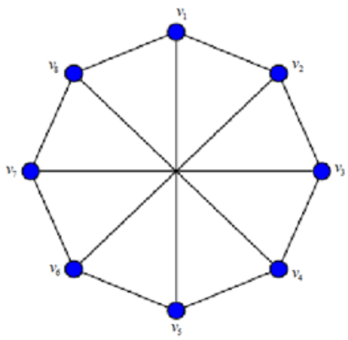

Figure 2.1: 3-Regular graph

The graph G is a 3- regular graph having 8 vertices. Therefore $d(u_i) = 3, \forall u_i \in V$ $O(G) = n = 8, S(G) = 12$. Then $MGA(G) = 1^{12} = 1$.

Theorem 2.2: For a complete graph of n vertices, the (AG) index is unity.

Proof: Let G be a complete graph of order n. This implies the degree of every vertex in G is (n-1). In a (n-1) regular graph there are $\frac{n(n-1)}{2}$ edges. Therefore, (MGA) index

$$MGA(G) = \prod_{pq\in E(G)} \left(\frac{2\sqrt{(d_p \cdot d_q)}}{(d_p + d_q)} \right)$$

$$MGA(K_n) = \prod_{pq\in E(G)} \left(\frac{2\sqrt{(n-1)(n-1)}}{((n-1)+(n-1))} \right)$$

$$= \prod_{pq\in E(G)} \left(\frac{2\sqrt{(n-1)^2}}{((n-1)+(n-1))} \right)$$

$$= \prod_{pq\in E(G)} \left(\frac{2(n-1)}{2(n-1)} \right) = (1)^{\left(n(n-1/_2\right)}$$

$$MGA(K_n) = 1$$

Example 2.3:

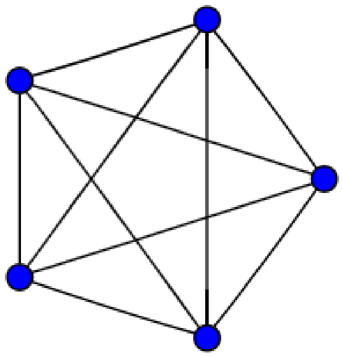

Figure 2.3: Complete graph K_5

The graph G is a complete graph K_5 having 5 vertices. Therefore $d(u_i)=4, \forall u_i \in V$ and $O(G)=n=5$. Then, $MGA(K_5)=1^{10}=1$.

Theorem 2.3: For a complete bipartite graph $K_{m,n}$ the $MGA(G)=\left(\frac{2\sqrt{(m\cdot n)}}{(m+n)}\right)^{mn}$.

Proof: Let G be a complete bipartite graph $K_{m,n}$, This implies the graph contains two disjoint vertex set V_m and V_n there is an edge between the vertex set V_m into vertex set V_n. Therefore degree of every vertex in V_m and V_n is n and m respectively, $d(v_i)=n, \forall v_i \in V_m$ and $d(v_j)=m, \forall v_j \in V_n$,there is mn edges in a complete bipartite graph $K_{m,n}$ of (m,n) vertices. Therefore (MGA) index

$$MGA(G)=\prod_{pq\in E(G)}\left(\frac{2\sqrt{(d_p\cdot d_q)}}{(d_p+d_q)}\right)$$

$$MGA(K_{m.n})=\prod_{pq\in E(G)}\left(\frac{2\sqrt{(m\cdot n)}}{(m+n)}\right)$$

$$=\left(\frac{2\sqrt{(m\cdot n)}}{(m+n)}\right).\left(\frac{2\sqrt{(m\cdot n)}}{(m+n)}\right).\left(\frac{2\sqrt{(m\cdot n)}}{(m+n)}\right).....mn\ times$$

$$MGA(K_{m.n})=\left(\frac{2\sqrt{(m\cdot n)}}{(m+n)}\right)^{mn}$$

Hence the $MGA(K_{m.n})=\left(\frac{2\sqrt{(m\cdot n)}}{(m+n)}\right)^{mn}$.

Example 2.3:

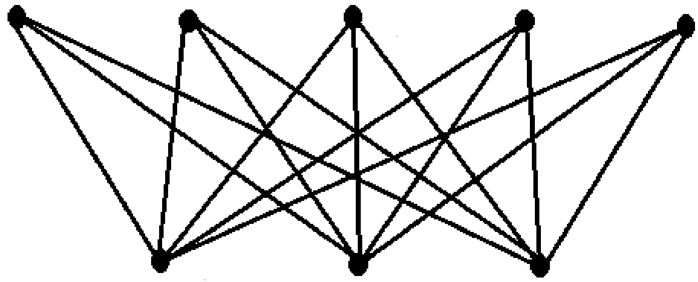

Figure 2.3: Complete bipartite graph $K_{5,3}$

The graph G is a complete bipartite graph $K_{5,3}$ having vertex sets V_5 and V_3. Therefore $d(u_i) = 3, \forall u_i \in V_5$ and $d(u_j) = 5, \forall u_j \in V_3$. The (MGA) index $MGA(G) = \left(\frac{2\sqrt{(m \cdot n)}}{(m+n)}\right)^{mn}$ $MGA(K_{5,3}) = \left(\frac{2\sqrt{(15)}}{(8)}\right)^{15} = \left(\frac{\sqrt{(15)}}{(4)}\right)^{15}$.

Theorem 2.4: For a join of two graphs G_1 & G_2, then the (MGA) index

$$MGA(G_1 + G_2) = \prod_{pq \in E(G_1)} \left(\frac{2\sqrt{\left((d_p + n)\cdot(d_q + n)\right)}}{\left((d_p + d_q + 2n)\right)} \right) \prod_{pq \in E(G_2} \left(\frac{2\sqrt{\left((d_p + m)\cdot(d_q + m)\right)}}{\left((d_p + d_q + 2m)\right)} \right)$$
$$\prod_{pq \in E(G_1+G_2)} \left(\frac{2\sqrt{\left((d_p + n)\cdot(d_q + m)\right)}}{\left((d_p + d_q + m + n)\right)} \right)$$

Proof: Let a join of two graphs G_1 and G_2 be of order m and n respectively. By the definition of join of two graphs G_1 and G_2 there is an edge between every vertex in G_1 and G_2. This implies the degree of vertices in $(G_1 + G_2)$ are $\left(d(v_i) + n\right), \forall v_i \in V_1 \; and \left(d(v_j) + m\right), \forall v_j \in V_2$. Therefore (MGA) index for join of graphs is

$$MGA(G) = \prod_{pq \in E(G)} \left(\frac{2\sqrt{\left(d_p \cdot d_q\right)}}{\left(d_p + d_q\right)} \right)$$

$$MGA(G_1 + G_2) = \prod_{pq \in E(G_1)} \left(\frac{2\sqrt{\left((d_p + n)\cdot(d_q + n)\right)}}{\left((d_p + n) + (d_q + n)\right)} \right) \prod_{pq \in E(G_2)} \left(\frac{2\sqrt{\left((d_p + m)\cdot(d_q + m)\right)}}{\left((d_p + m) + (d_q + m)\right)} \right)$$
$$\prod_{pq \in E(G_2+G_2)} \left(\frac{2\sqrt{\left((m+n)\cdot(m+n)\right)}}{\left((m+n) + (m+n)\right)} \right)$$

$$MGA(G_1+G_2)=\prod_{pq\in E(G_1)}\left(\frac{2\sqrt{\left((d_p+n)\cdot(d_q+n)\right)}}{\left((d_p+d_q+2n)\right)}\right)\prod_{pq\in E(G_2}\left(\frac{2\sqrt{\left((d_p+m)\cdot(d_q+m)\right)}}{\left((d_p+d_q+2m)\right)}\right)\prod_{pq\in E(G_2+G_2)}\left(\frac{2\sqrt{\left((m+n)^2\right)}}{\left(2(m+n)\right)}\right)$$

$$MGA(G_1+G_2)=\prod_{pq\in E(G_1)}\left(\frac{2\sqrt{\left((d_p+n)\cdot(d_q+n)\right)}}{\left((d_p+d_q+2n)\right)}\right)\prod_{pq\in E(G_2}\left(\frac{2\sqrt{\left((d_p+m)\cdot(d_q+m)\right)}}{\left((d_p+d_q+2m)\right)}\right)\prod_{pq\in E(G_2+G_2)}(1)$$

$$MGA(G_1+G_2)=\prod_{pq\in E(G_1)}\left(\frac{2\sqrt{\left((d_p+n)\cdot(d_q+n)\right)}}{\left((d_p+d_q+2n)\right)}\right)\prod_{pq\in E(G_2}\left(\frac{2\sqrt{\left((d_p+m)\cdot(d_q+m)\right)}}{\left((d_p+d_q+2m)\right)}\right)(1)^{mn}$$

$$MGA(G_1+G_2)=\prod_{pq\in E(G_1)}\left(\frac{2\sqrt{\left((d_p+n)\cdot(d_q+n)\right)}}{\left((d_p+d_q+2n)\right)}\right)\prod_{pq\in E(G_2}\left(\frac{2\sqrt{\left((d_p+m)\cdot(d_q+m)\right)}}{\left((d_p+d_q+2m)\right)}\right)\prod_{pq\in E(G_1+G_2)}\left(\frac{2\sqrt{\left((d_p+n)\cdot(d_q+m)\right)}}{\left((d_p+d_q+m+n)\right)}\right)$$

Hence the (MGA) index of the join graph is

$$MGA(G_1+G_2)=\prod_{pq\in E(G_1)}\left(\frac{2\sqrt{\left((d_p+n)\cdot(d_q+n)\right)}}{\left((d_p+d_q+2n)\right)}\right)\prod_{pq\in E(G_2}\left(\frac{2\sqrt{\left((d_p+m)\cdot(d_q+m)\right)}}{\left((d_p+d_q+2m)\right)}\right)\prod_{pq\in E(G_1+G_2)}\left(\frac{2\sqrt{\left((d_p+n)\cdot(d_q+m)\right)}}{\left((d_p+d_q+m+n)\right)}\right)$$

Conclusion:

In this study, the expression for multiplicative geometric - arithmetic index (MGA) index is derived forregular graph, complete graph, complete bipartite graph and join of graphs. Further suitable examples are considered to explain the theorem.

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