The Problem Solving of Bi-objective Hybrid Production with the Possibility of Production Outsourcing through Imperialist Algorithm, NSGA-II, GAPSO Hybrid Algorithms

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Abstract: Purpose: Today, it is important to identify products that must be produced and also the amount of production of each product and the compliance with existing resources. Such a problem is often called the "hybrid production". In addition, today, the advanced industrial units do not focus only on the production of a product and try as much as possible to meet a significant share of their customers' needs. Today, since the number and type of customers' needs are constantly changing, an industrial unit with all of its many developments will fail in producing a huge range of products. Outsourcing is a good strategy for such units so that it monitors the highest quality products while maintaining their customers.

Methods: It is clear that problem solving is more complex when the number of sources and products increases in the area of hybrid products, and using modern innovative and meta-heuristic methods is inevitable to solve such problems. A mathematical model has been presented for the problem of hybrid production considering the outsourcing and two algorithms of Imperialist competitive algorithm; NSGA-II and hybrid algorithm; GA and PSO have resolved the problem in this study. The results indicated that the convergence of Imperialist competitive algorithm to the final value was excellent and had less fluctuation in the answers.

Implications: However, the convergence rate is lower unlike NSGA-II and GAPSO procedures.

Keywords: Multi-purposes hybrid production, Outsourcing, Meta-heuristic algorithms.

1-Introduction

In today's markets, industry units want to increase their throughput by improving their business processes and products. Developed countries, such as the United States and Japan, applied different management philosophies such as just in time (JIT), lean manufacturing, total quality management (TQM), Theory of Constraints (TOC), and so forth in order to increase their productivity. System restrictions are said to introduce the environment of factory as a deterrent in achieving the targets set by the organization. Such limitations may be physical/tangible like machinery, vendors, and so forth or physical/intangible, such as politics, size of performance, and so forth. Another way to overcome some of these limitations and increase the throughput is outsourcing.

Manufacturing outsourcing occurs when a company closes out the contract with one or more contractors to provide a part of the production outside the country rather than doing a full production within the country. In practice, outsourcing is not only a decision to build or "pure" purchase but it includes the transfer of domestic production to foreign purchases (8) (Lie, 2006).

In other words, the issue of production outsourcing is the decision-making on which products or parts must be produced within the country and which products or parts must be produced by one or more external suppliers. Generally, there are five main reasons for outsourcing: improving the company's focus on key products, achieving world-class manufacturing facilities, accelerating interest from reengineering benefits, dividing risks, and the resources releases for other purposes. (14) (Nazari Shirkoohi et al., 2010). It is expected that production costs decreases relative to domestic production by outsourcing because the foreign suppliers enjoy from soft, flexible, and expertise-focused production scheduling (13) (Roudhoft & Varlop 1999). In recent years, outsourcing became an important strategy for many businesses. For a successful outsourcing decision, the benefit of cost savings is an important criterion. Therefore, decision on outsourcing needs careful analysis of the related cost. Several researchers, such as Goldratt and Patterson have used the constraint theory approach in order to solve the problem of hybrid production. Balakrishnan suggested that linear integer programming is a better tool than the constraints theory (10) (Balakrishnan, 1999). Mabin showed that the approach of constraints theory solving has been used improperly by Balakrishnan who argued that the theories of constraints and linear programming could be effectively used in combination (11) (Mabin, 2001).Coman and Ronen formulated a problem of hybrid production outsourcing as a problem of linear programming (4)(Coman & Ronen, 2000) They tested three approaches to solving: the standard evaluation, the theory of standard constraints, and its solution showed that solving the constraints theory was worse than the solution of developed linear programming (4)(Coman and Ronen, 2000). Lai developed a decision model that integrated the problem of increased capacity using an approach of mathematical planning (8) (Lai 2006). In this work, the benefits of expanding the capacity of different types of sources or outsourcing were simultaneously evaluated. She solved the issue of the single bottleneck single product building or buying through the bottleneck capacity for a better decision making and integrated the capacity issues with the financial issues. This was followed by Balakrishnan and colleagues. They presented a linear model that can be solved using spreadsheet software and produced better solutions to the problem of outsourcing (17) (Balakrishnan & Cheng, 2005).

Theory of constraints (TOC) is a management philosophy that defines three important indicators of the performance ;that is ,output, inventory ,and operational costs**) (6)** Goodfree et al **,.** **.(2001**Goldratt and his colleague explain in their book "The goal **"**that TOC is a management philosophy that focuses on the limitations of a system, uses five stages for continuous improvement ,and flattens the performance of an organization in achieving its goal (27**) (**Goldratt **&**Cox **.(2004 ,**The philosophy of management originates from operations management .Among the various applications of TOC, applications for the hybrid generation ,logistics ,scheduling, sizes of the performance, process of problem solving, the project management and market segmentation are known .According to TOC, throughput ,due date ,and other key performance measures of a company can be controlled and optimized by only controlling the bottleneck resources in the company .The problem of hybrid production is one of the TOC applications that maximizes the profits by defining and identifying the quantity of products for production**.**

TOC approach to solving the hybrid production has been used by many researchers, such as Goldratt (1990), Peterson (1992), Plenert (1993), and Lee et al. (1996) (14) (Nazari Shirkoohi et al., 2010). In most previous studies on the hybrid production, this issue has been developed by the increase in the number of limited resources or products. Balakrishnan claimed that when there are multiple constraints, conventional heuristic TOC is not a good solution to the hybrid problem (10) (Balakrishnan, 1999). He claimed that an integer linear programming is a better tool than TOC. Mabin (2001) showed that Balakrishnan improperly uses the approach to TOC. He argued that the TOC and LP could be effectively used in combination (11) (Mobin, 2001). Plenert

provided an example that the heuristic TOC does not produce an optimal or even feasible solution (28) (Plenert, 1993). It is very difficult to identify the dominant bottleneck in cases of more than one bottleneck. According to the TOC approach, a source with the maximum amount of planned utilization is regarded as a dominant bottleneck resource. Heuristic TOC was tested by solving a test problem, its solution consists of utilization of 100% of the bottleneck resource (14) (Nazari Shirkoohi et al., 2010). In some other studies, the TOC approach has been applied and tested in cases that the bottleneck resource in the optimization solution has been not utilized as 100%. In other words, unemployment time is considered for the bottleneck resource within the optimization time. The main objective of TOC is to maximize outputs, which is gained by identifying and taking advantage of the critical constrained resource (CCR). However, the disadvantage of these methods is that in an attempt to establish an explicit TOC approach, only those problems with small sizes can be solved to optimaize within a reasonable computation time (6) (Goodfree et al., 2001). One of the weaknesses of the TOC solving approach is that the method only considers the limitations within the company while both internal and external constraints should be addressed in the area of hybrid production outsourcing. Availability and cost of company are examples of internal constraints availability and cost of outsourcing are instances for external constraints. Thus, the model intended for the issue of integrated hybrid production outsourcing should simultaneously view both internal and external constraints. Moreover, Coman and his colleague formulated a hybrid production outsourcing problem as a LP problem (4) (Coman & Ronen, 2000). They tested three approaches to solving: SCA assessment, standard TOC, and LP. They showed that the TOC solution was worse than the advanced LP solution. Taghipour et al. (2020), studied Assessment and Analysis of Risk Associated with the Implementation of Enterprise Resource Planning (ERP) Project Using FMEA Technique. Taghipour et al. (2015), studied Construction projects risk management by risk allocation approach using PMBOK standard. Taghipour et al. (2015), studied Necessity Analysis and Optimization of Implementing Projects with The Integration Approach of Risk Management and Value Engineering. Taghipour et al. (2020), studied Evaluating Project Planning and Control System in Multi-project Organizations under Fuzzy Data Approach Considering Resource Constraints. Taghipour et al. (2015), studied Risk assessment and analysis of the state DAM construction projects using FMEA technique. Taghipour et al. (2015), studied Analysing the Effects of Phisical Conditions of the Workplace on Employees Productivity. Taghipour et al.(2015), studied Analysing the Effects of Phisical Conditions of the Workplace on Employees Productivity. Taghipour et al.(2015), studied Evaluation of the effective variables of the value engineering in services (Qazvin post center case study).**Khalilpour et al.(2018), studied The Impact of Accountants Ethical Approaches on the Disclosure Quality of Corporate Social Responsibility Information an Islamic in Iran. Sedaghatmanesh & Taghipour (2015), studied Reduction of Losses and Capacity Release of Distribution System by Distributed Production Systems of Combined Heat and Power by Graph Methods.Taghipour et al.(2015), studied A Survey of BPL Technology and Feasibility of Its Application in Iran (Gilan Province). Azarian and Taghipour (2020), studied The Impact of Implementing Inclusive Quality Management on Organizational Trust (Case Study: Education). Changiz Delivand et al.(2021), studied Investigating the effective factors in measuring customers' credibility with a combined approach of data mining and multidisciplinary decision making. Taghipour et al. (2015), studied Implementation of Software-Efficient DES Algorithm. Taghipour et al. (2020), studied Evaluating CCPM method versus CPM in multiple petrochemical projects. Soleymanpour et al. (2020), studied Mathematical modeling for the location-allocation problem allocation of mobile operator subscribers' affairs' agencies under uncertainty conditions. Taghipour et al. (2020), studied Application of Cloud Computing in System Management in Order to Control the Process. Hoseinpour et al. (2020), studied The Problem Solving of Bi-objective Hybrid Production with the Possibility of Production Outsourcing through Meta-

Heuristic Algorithms. Mohammadi et al. (2021), studied Investigating the role and impact of using ICT tools on evaluating the performance of service organizations. Medhat & Taghipour(2021) studied Investigating the Role of Social Capital in Recreating the Historical-Cultural Context of Shiraz. Asadifard et al. (2020), studied A Multi-Objective Mathematical Model for Vehicle Routing Problem Considering the Time Window and Economic and Environmental Objectives Using the Metaheuristic Algorithm Based on Pareto Archive. Khorasani & Taghipour (2020), studied The Location of Industrial Complex Using Combined Model of Fuzzy Multiple Criteria Decision Making (Including Case-Study).

Küttner stated that prediction of demand and the amount of used resources should be addressed in order to solve the problem of hybrid production solving and make decision on the size of production for each crop (19) (Küttner, 2004). Lea and his colleague showed that the option of hybrid production affected the sizes of the company's performance, such as profit, work in progress goods (WIP), customer service, and the capability of factory management (20) (Lea and Fredendall, 2002).

2 - Mathematical Model

The mathematical model of the problem of the is introduced to better understand and solve two-purposes hybrid production problems with the possibility of production outsourcing according to the resource constraints. The presented mathematical model is based on the following hypotheses:

- The amount of each product outsourcing is smaller than the percent of that product GDP.
- Variables of decision-making are integer.
- Satisfying the market demand of all products is maximized.
- The total value of domestic production and amount of any product outsourcing is smaller than or equal to the amount of market demand for the product.
- Available resources for production and amount of the available products are already certain.
- Costs of operation are set to be constant.

2.1. Introduction to the problem parameters

Op_i = the cost of outsourcing product i

RM_i = the amount of demand for product i

D_i = the amount of demand for product i

b_j = total capacity of the resource j within the planned period

a_{ij} = the need to the capacity to produce the product i in resource j

P_i = value of the product market

OE = operation costs

2.2. Introduction to the variables of decision

X_i = the variable of decision-making on the amount of domestic production of product i

Y_i = the variable of decision-making on the amount of outsourcing the product i

$X_i + Y_i$= the satisfied demand of the product i

2.3. A Mathematical Model

The linear programming formulation for the problem of two-purposes hybrid production with the possibility of production outsourcing is adjusted as follows:

$$F_1 = Max \sum_{i=1}^{n} (P_i - RM_i) X_i + (P_i - OP_i)Y_i - OE \tag{1}$$

$$F_2 = Max \sum_{i=1}^{n} (X_i - Y_i)/X_i \tag{2}$$

Subject to:

$$\sum_{i=1}^{n} a_{ij}X_i \le b_j \qquad j \in \{1,2,\dots,m\} \tag{3}$$

$$X_i + Y_i \le D_i \qquad i \in \{1,2,\dots,n\} \tag{4}$$

$$Y_i \le \alpha.X_i \qquad 0 < \alpha < 1 \tag{5}$$

$$X_i, Y_i \in integer \tag{6}$$

Purpose functions are displayed by F1 and F2. F1 is the objective function to maximize the profit and F2 is the objective function to maximize the satisfaction of the demand market. It is assumed in the first objective function that the company meets the market demands by the products produced within the company and by outsourcing. The difference between revenues and expenses, including the cost of raw materials, cost of outsourcing, and operational expenses could be indicative of the company's profits.

2.1. Alternative purpose functions F2

Under the second purpose, it is tried to maximize the maximum amount of market demand that can be replaced with the following two functions.

$$F_2' = Maxmin \left\{ \frac{X_i + Y_i}{D_i} \right\} \tag{7}$$

$$F_2'' = Max \sum_{i=1}^{n} \gamma_i \left\{ \frac{X_i + Y_i}{D_i} \right\}^n \tag{8}$$

Equation (7) maximizes the minimum ratio of supply of the product i demand to the degree of market demand of product i. Equation (8) maximizes total ratio of supply of the product i demand to the degree of market demand of product i, which has been squared. In fact, it increases the degree of the importance of the supplied demand. Inequality (3) is introduced as the capacity constraint, meaning that the total domestic production of products using a source is smaller than or equal to the maximum capacity of the resource. Equation (4) explains the demand restriction as the sum of domestic production and outsourcing of any product that is smaller than the total market demand for the product. The relation (5) is considered as the outsourcing constraint. If the restriction does not add, all demands will be met, and this means that outsourcing is always profitable. However, adding the outsourcing constraint will remove the problem of continuous

profitability of outsourcing and meaninglessness of the second constraint; that is, the demand constraint.

3. The proposed solution

The colonial competitive algorithm, NSGA-II, and a hybrid algorithm, which is a combination of the particles crowding and genetic algorithms, was used to solve the proposed model.

3.1. Colonial competitive algorithm

3.1.1. The production of the initial empires

The solution in the colonial competitive algorithm is in the form of an array. Each array contains the optimized values of the variable. In terms of genetic algorithms, the array is called chromosome. In the above-mentioned algorithm, the title country has been used as the array. In an N-dimensional optimization problem, a country is a $1 \times N$ array and the array is defined as Country = [p1, p2, p3, ..., pN]. Where pi is the optimized variable, each variable in a country is characterized as a sociopolitical feature of the country. From this perspective, the algorithm seeks the best country that is the one with the best combination of sociopolitical characteristics, such as culture, language, and economic policies. After the production of countries, the non-dominance technique and the crowding distance to create fronts and rank the members of each front have been demonstrated. Then, the members of the front are stored in the archive. The emperors are selected as predetermined and from the intended archive.

To calculate the cost or any imperial power, the value of each objective function is obtained for each empire. Then the cost per objective function is computed as equation (9).

$$Cost_{i,n} = \frac{\left|f_{i,n}^{p} - f_{i,n}^{p,best}\right|}{f_{i,total}^{p,max} - f_{i,total}^{p,min}} \qquad (9)$$

where Costi n is the normalized value of the objective function i for the emperor n .Moreover , ,are respectively the best ,maximum ,and minimum values of the objective function i in each iteration. The value of normalized cost of each emperor is obtained as equation (10)

$$Total\ Cost_n = \sum_{i=1}^{r} Cost_{i,n} \qquad (10)$$

so that r is the purpose function value. Each emperor's power is calculated as equation (11) after gaining the normalized cost and the colonies are distributed among them according to the power of any emperor.

$$p_n = \left|\frac{Total\ Cost_n}{\sum_{i=1}^{N_{imp}} Total\ Cost_i}\right| \qquad (11)$$

Then, the initial number of colonies into an empire is determined by equation (12).

$$NC_n = round\{p_n . N_{col}\} \qquad (12)$$

so that the initial number of the colonies of emperor n, N col is the total number of colonies. NC n will be randomly selected from the colonies and given to each emperor. Empire with more power than the weaker emperor would have greater colonies.

3.1.2. The total power of an empire

The total power of an empire is affected by the power of the emperor's country, although the power of the colonies of an empire affects the total power of that empire. Therefore, the total power of an empire is computed as equation (13).

$$TP\ Emp_n = (Total\ Cost(imperialist_n) + \xi mean\{Total\ Cost(colonies\ of\ empire_n)\} \tag{13}$$

TP Emp n is total power of empire n and ξ is a positive number less than one.

3.1.3. The movement of the colonies of an empire towards the emperor

The colonies will be moved toward their empires after the division of colonies .Figure 1 shows the movement .
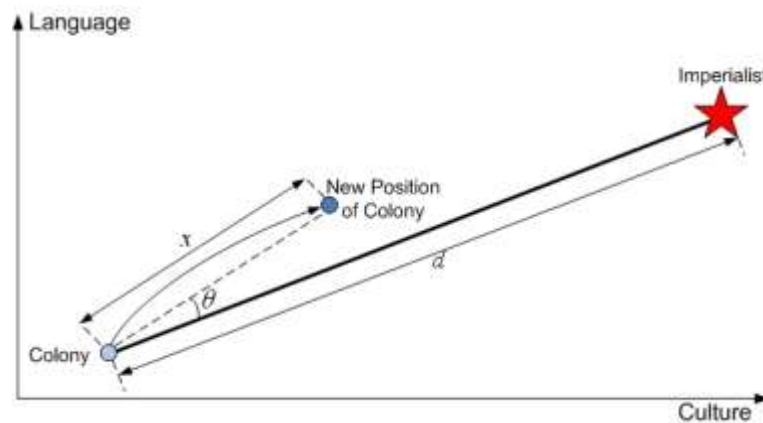

Figure 1. The movement of colonies toward the emperor with a random angle

d is the distance between colony and empire. X is a random variable with uniform distribution between zero and $\beta \times d$ and β is an integer greater than one. The movement direction is shown by angle θ. P_A shows the response rates that are close to the emperor.

3.1.4. Transfer of information between colonies

The crossover of a genetic algorithm was used to transfer data between the colonies. A variety of one-point and two-point crossovers was used to denote the crossover. Tournament selection method was used to select the colonies. Moreover, the percentages of responses that are undergoing crossover are shown with P_c.

3.1.5. Revolution

In each ten years' period, a revolution will happen in some colonies. This method is similar to the mutation method in the genetic algorithm and takes place to escape from local searches.

3.1.6. Archive updated colonies

In each ten years period, initial population of the colonies, assimilated population, the population of data transfer between colonies, population of the revolution, and population of the recovery of the emperor are integrated together for any empire. This is called the integrated population. Then, the archive update is done in accordance with the integrated population. Thus, the best colonies were selected by the size of colonies population for the intended empire NC(i) according to the non-dominated sorting and the crowding distance size of population density and from colony to for the Emperor's chosen.

3.1.7. Changing the location of a colony and an emperor

In some cases, it is possible that a colony has better conditions (in terms of the front of the answer and the crowding distance) than its empire after decades. In this case, the colony is replaced with the emperor.

3.1.8. An imperial competition

Power of the weaker empire will decline and power of the stronger empire will increase in the imperialist competition. The weakest colony of the weakest empire will seize in all competitions of the empires competitions. . On the other hand, the first selection of the weakest colonies of the weakest empires is conducted by the emperor; that is, the winner emperor of the whole empire in the imperial competition. In this competition, the strongest empire certainly shall not take possession of the colony, although these empires are more likely to seize them. The competition just was modeled by choosing one of the weakest colonies of the weakest empires. Then, total normalized cost was obtained as equation (14) to calculate the probability of the appropriation of any empire.

$$NTP\ Emp_n = max\{TP\ Emp_i\} - TP\ Emp_n \tag{14}$$

NTC_n is total normalized power of nth empire and TC_n is total power of the nth empire. With a total normalized power, probability of each empire appropriation is calculated as equation (15).

$$P_{p_n} = \left| \frac{NTP\ Emp_n}{\sum_{i=1}^{N_{imp}} TNTP\ Emp_i} \right| \tag{15}$$

Then, roulette wheel method is used to allocate the selected colony to one of the empires.

3.1.9. Removal of the empires without power

Empires with no power will fall and their colonies will be distributed among other empires in the imperial competition. When an empire loses its other colonies, then it will fall.

The stop condition

The stop condition or the end of the imperial competition occurs when there is only one empire among all countries. Figure 2 illustrates the overall process of colonial competitive algorithm and its details.
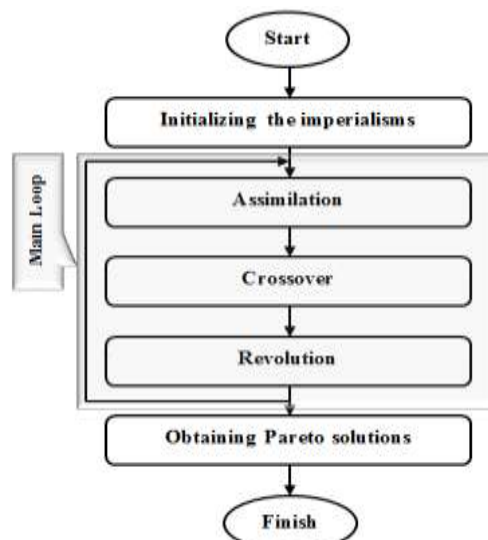
3.2. The developed colonial competitive algorithm

Due to the complexity of the problem of two-purposes hybrid production bonded with the possibility of production outsourcing, an approach was proposed to overcome the bounded and discrete problems. Colonial competitive algorithm must be changed at the stages of initial production of countries, absorption of colonies by the empires, and revolution of the colonies in order to meet the requirement of discreteness and other constraints in the problem.

The capacity constraints, demand, and outsourcing should be applied in the problem of maximization of two-purposes hybrid production with the possibility of outsourcing.

3.3. Results of colonial competitive algorithm

Above examples was coded by colonial competitive algorithm MATLAB software. Table 1 shows the parameters used in the statement of the problem and algorithm parameters. Problem solving took place in three stages using this algorithm. each stage of The algorithm and problem parameters are fixed in each three stages. The weighting method was used to the process of problem optimization.

Table 1. Parameters of the problem and algorithm in colonial competition algorithm

Parameters of the problem	Value	Parameters of the algorithm	Value	
Number of variables	6	Number of countries	100	
Lower bound of variables	0	Number of initial empires	8	
Upper bound of variables	100		500	
		Number of decades)iteration stages(	Revolution rate	0.3
			Absorption coefficient	2
			Absorption angle	0.5
			Zeta) ζ(	0.02

3.3.1. The optimization of the two-purposes function

Purposes should be first scaled up in order to optimize the multi-purposes problems. Therefore, the function of the first purpose is divided into 10,000 to be scaled up with the function of the second purpose. Classical weighting method is used to obtain the optimal values. Thus, the optimization of the target function as $F=w_1F_1+w_2F_2$ was conducted in different values of w1 and w_2 in table 2 provided that $w_2 = 1-w_1$.

Table 2. Results of the optimization of multi-purposes function through ICA

w_1	X_1	X_2	X_3	Y_1	Y_2	Y_3	*Total Cost*
0	69	70	38	26	27	14	2.44
0.02	71	76	35	28	22	9	2.386648
0.04	69	70	39	27	26	14	2.403632
0.06	65	63	48	25	25	16	2.353412

0.08	64	74	40	25	23	16	2.329264
0.1	68	69	42	25	27	14	2.33612
0.12	57	71	48	21	27	17	2.279944
0.14	69	66	43	27	24	15	2.280344
0.16	68	64	43	26	25	17	2.247408
0.18	73	70	37	26	27	14	2.259148
0.2	68	72	40	27	27	11	2.21988
0.24	70	72	38	28	25	15	2.200208
0.26	56	68	50	22	27	20	2.148732
0.28	79	72	32	21	28	12	2.108872
0.3	68	67	42	26	26	16	2.10674
0.34	72	66	41	28	21	16	2.048184
0.38	58	69	48	21	27	17	1.987016
0.4	51	67	54	20	26	21	1.97016
0.42	73	66	40	27	25	15	1.972296
0.44	68	66	41	27	25	16	1.92312
0.48	68	76	37	26	23	12	1.867232
0.5	73	74	35	25	25	12	1.8557
0.52	72	69	38	23	26	15	1.829608
0.54	70	68	40	28	25	15	1.833708
0.56	62	69	46	23	26	14	1.781872
0.6	71	74	36	23	26	14	1.74532
0.62	68	73	39	27	26	15	1.76328
0.66	42	72	55	14	28	22	1.661156
0.68	64	64	48	24	24	18	1.66928
0.7	58	63	52	23	25	17	1.62652
0.74	34	71	62	11	28	24	1.587972
0.76	41	72	58	15	27	23	1.6
0.78	65	70	43	21	27	17	1.558116
0.8	42	67	60	16	25	22	1.52384
0.86	47	69	55	18	27	21	1.481448
0.88	53	69	50	20	27	19	1.438576
0.9	38	73	59	13	27	22	1.4353

0.92	30	59	74	12	23	26	1.406112
0.98	40	67	62	15	25	24	1.368424
1	33	71	64	11	26	25	1.3516

Variables obtained from the optimization have been set in the functions of the first and second purposes and the values of F1 and F2 have been plotted as Pareto chart in figure 3. Since we are looking for the maximum values of F1 and F2, the values obtained in the top right corner of the diagram are a part of the optimal values. These plots are shown in red in the figure.
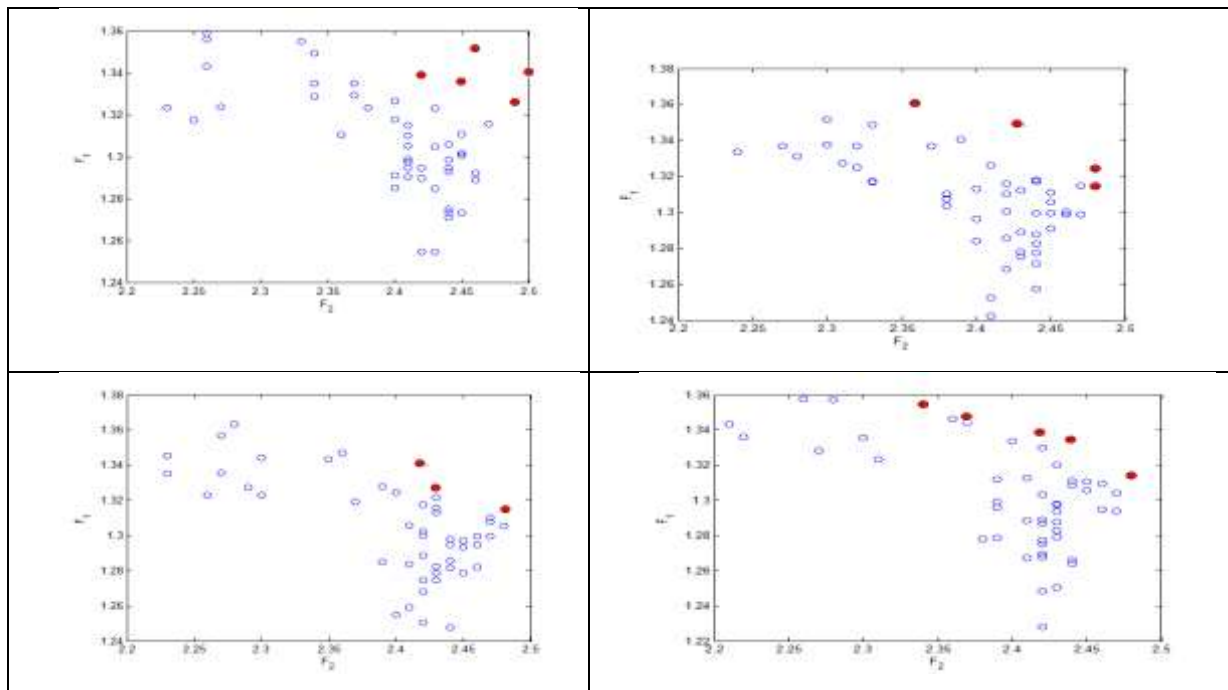
Figure 3: Pareto chart in the optimization of multi-purposes function at different stages of program

Several problems have been solved and compared with the changes in the parameters below.

Table 3. Results of the changes in absorption coefficient (β)

w_1	X_1	X_2	X_3	Y_1	Y_2	Y_3	Total Cost
0	73	73	36	27	26	14	2.49
0.1	72	74	36	28	26	14	2.38232
0.2	66	72	41	25	28	16	2.25156
0.3	73	74	35	27	25	14	2.12678

β = 1	0.4	67	73	40	26	27	15	2.02072
	0.5	67	67	44	25	25	17	1.8857
	0.6	60	74	44	24	26	16	1.7728
	0.7	43	71	57	17	28	22	1.66866
	0.8	70	70	39	28	28	15	1.56544
	0.9	60	68	48	24	25	19	1.45126
	1	20	72	72	8	27	28	1.3822
β = 3	0	66	71	40	26	23	14	2.4
	0.1	75	65	40	23	22	16	2.29548
	0.2	73	71	33	24	28	13	2.18316
	0.3	68	59	48	25	23	16	2.05604
	0.4	67	71	38	26	28	9	1.92712
	0.5	61	62	51	24	23	20	1.8661
	0.6	69	74	38	25	26	11	1.73964
	0.7	69	56	50	21	22	20	1.61406
	0.8	64	73	39	25	27	14	1.50336
	0.9	33	70	64	10	28	20	1.39914
	1	47	67	56	17	25	22	1.3162

Table 4. Results of the changes in the number of countries

	w_1	X_1	X_2	X_3	Y_1	Y_2	Y_3	Total Cost
Countries = 50	0	68	72	37	27	27	12	2.43
	0.1	66	74	40	21	26	13	2.28752
	0.2	73	71	37	27	24	8	2.16724
	0.3	73	76	34	26	24	13	2.10774
	0.4	76	61	42	23	23	15	1.94352
	0.5	73	66	40	19	25	16	1.8234
	0.6	32	69	65	11	27	26	1.7318

	w_1	X_1	X_2	X_3	Y_1	Y_2	Y_3	Total Cost
	0.7	40	71	58	16	26	23	1.63874
	0.8	68	72	40	26	24	15	1.53416
	0.9	40	70	60	16	27	24	1.47414
	1	51	66	55	18	25	18	1.2958
	0	72	78	33	27	21	11	2.42
	0.1	68	65	44	27	19	17	2.2872
	0.2	69	65	43	24	26	16	2.203
	0.3	74	72	34	26	27	10	2.07534
	0.4	71	72	36	25	27	13	1.9728
Countries = 75	0.5	60	71	46	18	27	18	1.8566
	0.6	71	70	38	28	28	12	1.76692
	0.7	53	60	58	20	20	22	1.60998
	0.8	66	68	44	25	25	17	1.54872
	0.9	68	70	41	24	28	16	1.44166
	1	27	69	69	8	26	24	1.3222

Table 5. Results of the changes in revolution rate

	w_1	X_1	X_2	X_3	Y_1	Y_2	Y_3	Total Cost
	0	74	71	32	23	28	11	2.39
	0.1	73	67	40	26	24	15	2.33452
	0.2	74	72	34	25	25	12	2.18468
R.R = 0.1	0.3	77	70	34	23	24	12	2.04822
	0.4	54	71	50	19	26	20	1.97432
	0.5	71	55	48	26	22	18	1.8382
	0.6	61	74	43	19	26	17	1.73952
	0.7	70	71	37	28	28	11	1.6282

	0.8	60	70	46	24	27	14	1.52504
	0.9	55	70	50	20	21	20	1.40528
	1	32	70	65	11	22	25	1.3138
	0	65	75	38	26	24	15	2.43
	0.1	65	67	45	23	24	18	2.30866
	0.2	67	67	42	26	25	16	2.20172
	0.3	72	72	37	26	25	14	2.11056
	0.4	65	73	40	24	26	15	1.97512
R.R = 1	0.5	63	68	44	24	26	17	1.8591
	0.6	62	68	43	24	26	17	1.72452
	0.7	64	63	48	22	24	16	1.60266
	0.8	57	68	50	18	27	17	1.5172
	0.9	37	74	59	9	26	23	1.41096
	1	58	61	54	20	23	20	1.3036

Thus, with regard to the results of the multi-purposes function optimization and the comparison of the obtained results and charts, we can see that the first numerical solved example with the parameters listed in table 1 lead to better results for the functions F1 and F2.

3-4 - Results of GA-PSO

This section provides the above-mentioned example using a hybrid GA-PSO algorithm. Table (6) shows its parameters.

Table 6. Parameters of the problem and algorithm in GAPSO algorithm

Parameters of the problem	Value	Parameters of the algorithm	Value
Number of variables	6	Number of population	50
Lower bound of variables	0	Number of iterations	500
Upper bound of variables	100		20
		The maintained percentage	
			Percentage of crossover 70
			Percentage of mutation 30

3.4.1. The optimization of a two-purpose function

This section deals with the optimization of the problem of two-purpose hybrid production using GA-PSO algorithm. The goals optimization must be scaled up as ICA algorithm. Therefore, the function of the first purpose is divided into 10,000 so that it scales up with the

second purpose. Like the colonial algorithm approach, the classical weighting method is used to deal with two-purpose function. Figure 5-17 shows Pareto chart, which actually is the values of F1 for F2. Since this problem seeks to maximize the value of F1 and F2, values in the upper right corner of the diagram will be the same optimum points. These values are shown in the figure in the black dots. Moreover, the values of the points are in the figure.
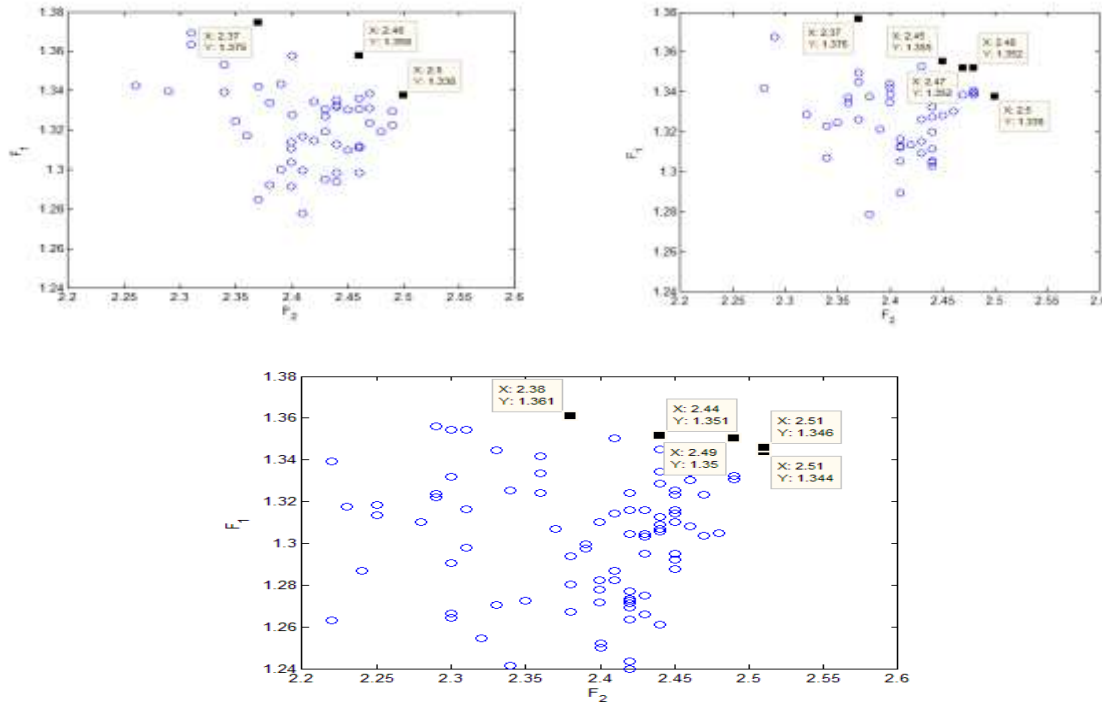

Fihure 4. Pareto diagram of F1 values to F2 values

Table 7. Values of total cost in the optimization of two-purpose function through GAPSO method

w_1	X_1	X_2	X_3	Y_1	Y_2	Y_3	*Total Cost*
0	68	51	54	27	20	20	-2.4
0.02	62	67	47	24	26	18	-2.417848
0.04	70	70	39	28	28	14	-2.443304
0.06	63	70	44	23	28	17	-2.382812
0.08	65	67	45	25	26	18	-2.370064
0.1	69	69	41	23	23	16	-2.2968
0.12	69	65	44	23	26	17	-2.304736
0.14	62	64	49	22	21	19	-2.218072
0.16	66	59	50	21	23	20	-2.2156
0.18	70	65	43	28	26	14	-2.253216
0.2	70	70	40	28	28	14	-2.26752

0.22	70	63	44	28	25	17	-2.217792
0.24	65	70	42	26	28	14	-2.176352
0.26	61	70	46	23	26	18	-2.1527
0.28	65	70	43	19	26	17	-2.089536
0.3	61	70	46	19	28	17	-2.08198
0.32	64	67	46	24	25	15	-2.054672
0.34	63	62	50	25	24	19	-2.056204
0.36	70	60	46	28	23	16	-2.0214
0.38	66	62	48	25	23	19	-2.007972
0.4	70	70	40	28	28	12	-2.01568
0.42	69	68	42	26	26	13	-1.960444
0.44	60	68	48	24	27	19	-1.975032
0.46	68	67	42	27	26	14	-1.912748
0.48	68	66	44	27	26	16	-1.923376
0.5	70	70	40	28	27	14	-1.9097
0.52	59	61	53	22	24	21	-1.842456
0.54	66	53	54	26	20	21	-1.811724
0.56	68	63	46	27	25	18	-1.836304
0.58	60	63	51	23	25	20	-1.790236
0.6	70	58	48	28	23	19	-1.78236
0.62	70	67	40	28	25	16	-1.739808
0.64	65	55	53	26	18	21	-1.683808
0.66	48	66	57	19	24	21	-1.673236
0.68	48	69	55	16	27	22	-1.671096
0.7	43	67	60	17	26	21	-1.63944
0.72	61	67	48	20	25	19	-1.617936
0.74	66	59	50	26	23	20	-1.62008
0.76	40	70	60	16	27	24	-1.613496
0.78	68	70	41	27	26	14	-1.564404
0.8	29	66	70	11	25	25	-1.52592
0.82	61	68	47	23	27	17	-1.52554
0.84	30	70	66	11	28	26	-1.519728
0.86	33	66	67	13	26	26	-1.496096

0.88	46	70	53	18	28	21	-1.44216
0.9	67	56	51	26	22	20	-1.42532
0.92	50	69	54	20	27	19	-1.426944
0.94	52	67	54	19	26	20	-1.396572
0.96	51	65	56	20	26	22	-1.399296
0.98	42	65	62	15	26	24	-1.37274
1	38	58	69	15	22	27	-1.3398

3.5. The results of NSGA-II algorithm

This section optimizes the problem of two-purpose hybrid production problem with the algorithm NSGA-II. This algorithm is limited to a multi-purposes problems and it is not possible to separate the problem into two one-purpose problems. Multi-purposes optimization using the algorithm is simultaneously performed by two functions (F1) and (F2). The dominance method is used to separate the optimum values in this algorithm.

Table 8. Parameters of the problem and algorithm in NSGA-II algorithm

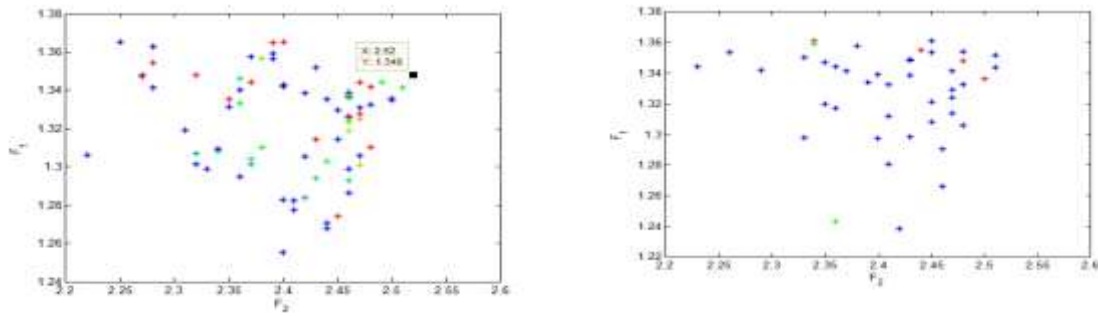| Parameters of the problem	Value	Parameters of the algorithm	Value
The number of variables	6	Number of population	50
Lower bound of variables	0	Number of iterations	100
Upper bound of variables	100	Rate of crossover	0.8
		Rate of mutation	0.3
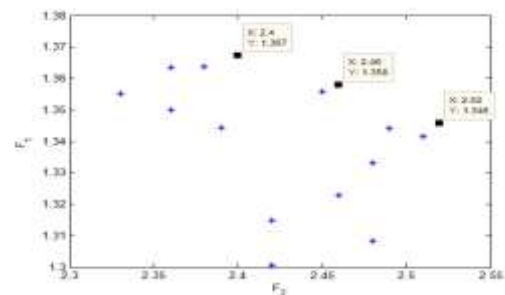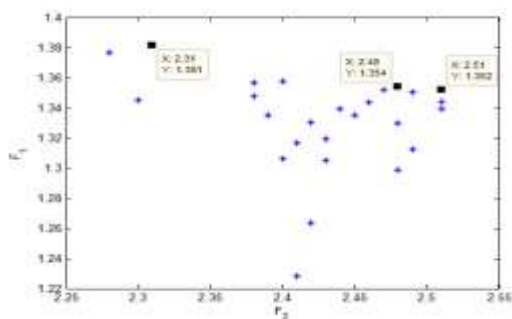
Figure 5. Pareto diagram of NSGA-II method

4. Conclusion and suggestion for further research

A two-purpose model with the possibility of outsourcing was given with regard to a relatively modern view to the problem of hybrid production in this research and algorithms of colonial competition, NSGA-II and a hybrid algorithm of GAPSO was used to solve it. The results were presented with the changes in parameters. The results indicated that the convergence speed of colonial competitive algorithm was lower than GAPSO and NSGA-II methods so that it would be reached the final solution after 500 iterations. It can be also said that GAPSO and NSGA-II algorithms had higher convergence speed and for example, GAPSO algorithm would reach the final response after about 30 iterations and subsequent iterations would not improve the response. It is notable that the convergence of colonial competition algorithm was excellent to the final amount and had fewer fluctuations in the answers. However, its convergence rate is low compared to two other algorithms. In addition, the following items can be interesting areas for future research:

- To consider new constraints, such as a minimum meeting of a certain level of market demand,

- To use uncertainty for modeling the hybrid production,

- To consider a hybrid production model in several steps and at several time intervals, meaning that it is possible to store in each period.

- To present heuristic methods for taking into account the lower bound for the problem,

- To combine the colonial competitive algorithm with other meta-heuristic algorithms to enhance the speed of convergence at a high precision,

- To combine the colonial competitive algorithm with classical optimization algorithms and test it for solving various problems optimization

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