Find A Correction Factor Between The Analytical And Numerical Solution For Reynolds Equation (With And Without) Considering THL Effect

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Abstract
In this paper ,an analytical study of a conventional hydrodynamics bearing was carried out. An integrated program was built using a computer program and through the Matlab facility in order to solve Reynolds equation numerically using finite difference method and analytically through the short bearing theory.The effects of the thermal distortion on the bearing performance arestudied. The thermal distortion is a result of the temperature riseeffects. This study was carried out for journal speed 3000 rpm. The oil film pressure distribution is obtained by solving Reynolds equationnumerically and analytically.Pinkus's approach has been used in computing the temperature distribution along the circumferential direction of the bearing surface. Analytical solution for Reynolds equation overestimates the bearing characteristics and Capacity compared with numerical solution results. It was found that THL effect the bearing performance positively (in the low range of temperature) by increasing stiffness and load capacity also it was found that using correction factor is useful to get more accurate assessment for the bearing performance by solving Reynolds equation analytically.
Key words: Reynolds, Hydrodynamic Bearing ,Short Theory Bearing , Finite difference method , Correction Factor , THL,Pinkus Equation ,Vogel Equation.

Introduction

Journal bearings are some of the oldest and most relevant machine elements. When it comes to the design, the concept of these bearings did not significantly change over the course of history, retaining its original simplicity and functionality. However, as the needs of mankind grew, machines were becoming more and more complex, and they were working in increasingly difficult conditions. To keep the bearing operation reliable even in such conditions, the materials of the bearings and their lubricants were improved over time. The bearing materials were changed in terms of improving their slip properties and embed ability, and the lubricants were changed in terms of improving their rheological properties[1].Journal Bearings consists of two cylinders rotating relative to each other. The outer cylinder (bearing) is stationary and the inner called the Journal (shaft) rotating with an angular velocity. The main purpose of the journal bearing is to support the rotating machinery by providing sufficient lubrication to separate the moving parts and to minimize the friction due to rotation. The amount of pressure generated depends on the viscosity and density of the fluid. Viscosity is the property that defines the resistance of the fluid to motion. The viscosity is due to the molecular attraction between the adjacent layers of fluid. . The high-pressure fluid film in the clearance between the journal and the bearing due to rotation of the journal provides the hydrodynamic film lubrication, and the load capacity to the bearing[2].In the hydrodynamic lubrication regime of a journal bearing, the journal and its housing are separated by a thin layer of oil, and the friction and wear are minimized. Viscous friction of the oil causes heat generation, which in certain conditions can significantly worsen the performance of the bearing. A higher temperature of the oil film reduces the load carrying capacity of the bearing due to decrease of the oil film stiffness. In extreme cases, this can lead to a seizure of the bearing due to the rupture of the oil film, when "welding" takes place at the contact points of micro-irregularities. For this reason, the need for knowing the distribution of temperature in the lubricant is imposed as an imperative, in order to prevent the aforementioned negative phenomena. Furthermore, knowing the temperature of the oil film can also be helpful when selecting the journal and bearing materials[1]. The fluid-dynamic motion of oil film within the lubrication gap is described by the well-known Reynolds equation . Explicit analytical solutions for the pressure distribution can be obtained only for asymptotic configurations (e.g. infinitely short and long journal bearings), while for other journal bearing configurations numerical solution of Reynolds equation is required[3].

Thermal effect on the hydrodynamic bearings is considered a very important matter, since it is the first work in this field. In the recent years, with the development of the high speed machinery, the thermal characteristics of bearings have been held of much account by many researchers. The clarification of thermal characteristics in high-speed journal bearings and their effect on the bearing operation is very important from the point of reliable and durable operation of bearing and bearing systems. This is called the THL theory which considers the oil film temperature as a key of this theory. In most literature surveys, this theory deals with the effects of temperature on the lubricants viscosity and with the change of oil film thickness due to thermal distortion of the bearing surface.It should be noted that the excessive temperature rise makes the thermal effect as a problem in bearings, which leads to degradation of lubricant and could cause bearing jamming[4].In this paper, the Reynolds equation will be solved (with and without) considering THL effect using the numerical solution and the analytical solution, and a comparison between the two solutions was made in order to find the correction factor.

Bearing Geometry

The bearinggeometry that studied in this research is shown schematically in Figure (1). The journal rotates with an angular velocity (ω) about its center (O_j), and the distance between (O_j) and (O_b) is eccentricity (e). The angle between the load line on the bearing center (O_b) and the line that connecting centers (O_j,O_b) is called attitude angle (φ). The oil film thickness *(h)* is distributed along the circumference of the bearing from h_{max} at (θ=0) to h_{min}at (θ=π).

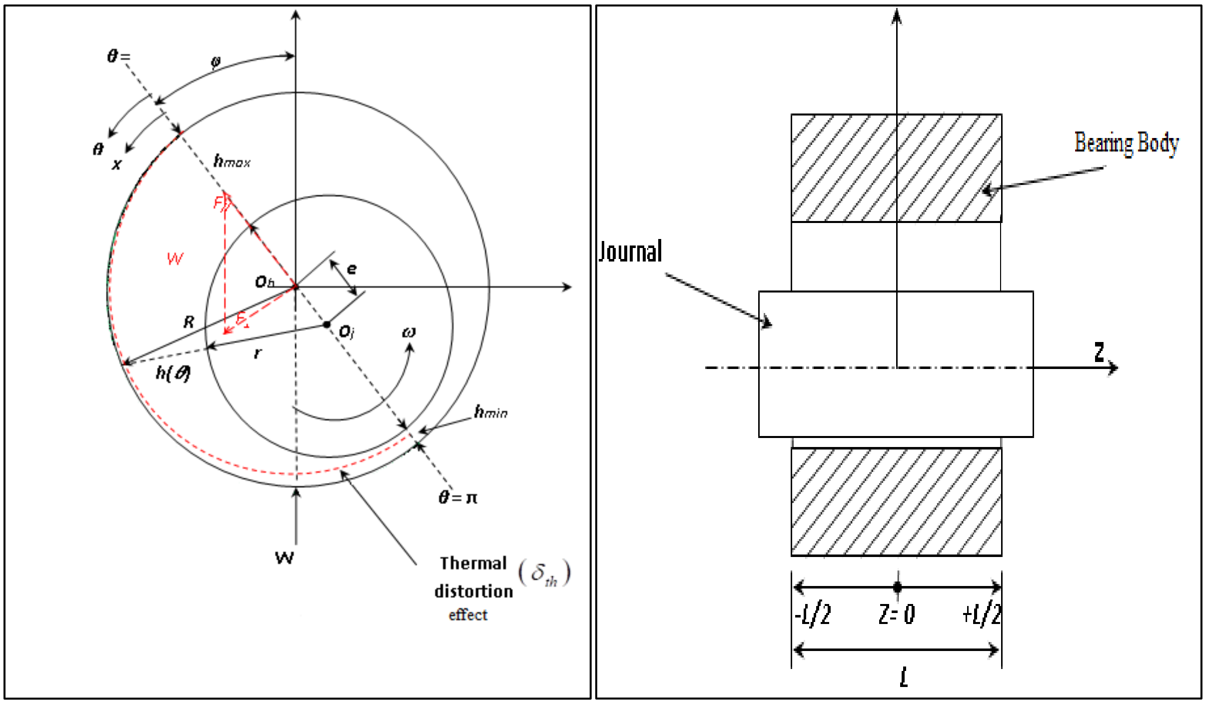

Figure (1)Schematic drawing of the journal bearing

Theoretical analysis

Studying any hydrodynamic bearing, whether it is of a traditional circular or non-circular type, requires solving Reynolds equation analytically or numerically, knowing that the numerical solution gives more accurate results, on the one hand, because the analytical solution cannot be implemented until after Deleting more than one term from the equation limits, in addition to the fact that the analytical solution cannot be applied to non-conventional bearings because the bearing surface is not continuous but is divided into pads[5]

In this paper, the focus is on the pressure outputs that generated under the influence of the load, shaft rotation speed and specification of the oil, where Reynolds equation was solved numericallyusing of finite differencesmethod, then it was solved analyticallyusing the theory of short bearings and a comparison between the two methods was made In order to find a correction factor between them. The form below of Reynolds equation is one of more than ten forms of it. This form is described as a second-order differential equation with two dimensions.

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Z}\left(\frac{h^3}{\mu}\frac{\partial P}{\partial Z}\right)=6U\frac{\partial h}{\partial x}+12\frac{\partial h}{\partial t} \text{-------(1)}$$

Since there is no change in the thickness of the slice with time, that is, $(\frac{\partial h}{\partial t}=0)$Since there is no dynamic loading and the load is in static and steady .

Numerical Solution

Reynolds equation is solved using finite difference method(FDM) to get the generated pressure distribution of the oil film at theinner surface of the bearing. The solution of the finite deference model of Reynolds equation wasachieved by dividing the bearing surface into circumferential and axialgrids. Then, the oil film pressure value at any node ($P_{i,j}$) is obtained bysolving Reynolds equation. The mesh number is the same in bothdirections, circumferential (m) and axial (k). The finite difference grids areshown schematically in Figure (2)**,** in this figure a five nodes scheme of finite difference is used. It was found that the typical number of grids thatachieve a reasonable accuracy and do not consume long running time, in thisnumber is (50x50):The element size in the circumferential direction ($\triangle x$) can be determined asfollow:

$$\triangle \mathrm{x} = \frac{\pi \mathrm{R}}{\mathrm{m}-1}$$

And also the element size in the axial direction (Δz) can be determined as follow:

$$\triangle \mathrm{z} = \frac{\mathrm{L}}{\mathrm{k}-1}$$

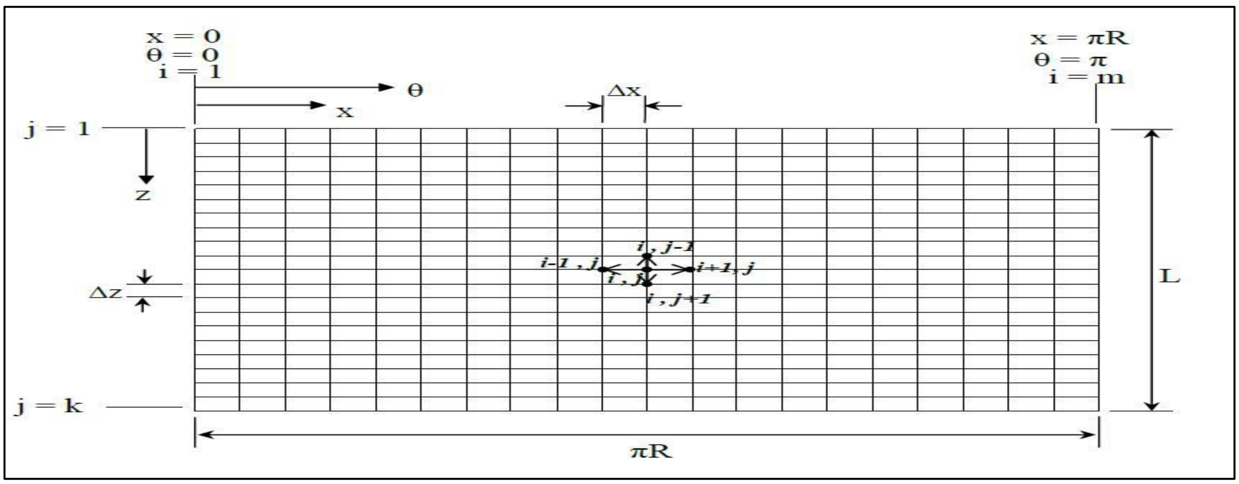

Figure (2)Bearing surface , Divided into Axial and Circumferential grids

$$\frac{\partial}{\partial \mathrm{x}}\left(\frac{\mathrm{h}^3}{\mu}\frac{\partial \mathrm{P}}{\partial \mathrm{X}}\right) + \frac{\partial}{\partial \mathrm{Z}}\left(\frac{\mathrm{h}^3}{\mu}\frac{\partial \mathrm{P}}{\partial \mathrm{Z}}\right) = 6\mathrm{U}\frac{\partial \mathrm{h}}{\partial \mathrm{x}} + 12\frac{\partial \mathrm{h}}{\partial \mathrm{t}} \text{-------(1)}$$

$$\frac{\mathrm{h}^3}{\mu}\frac{\partial^2 \mathrm{P}}{\partial \mathrm{X}^2} + \frac{3\mathrm{h}^2}{\mu}\frac{\partial \mathrm{h}}{\partial \mathrm{X}}\frac{\partial \mathrm{P}}{\partial \mathrm{X}} - \frac{\mathrm{h}^3}{\mu^2}\frac{\partial \mu}{\partial \mathrm{X}}\frac{\partial \mathrm{P}}{\partial \mathrm{X}} + \frac{\mathrm{h}^3}{\mu}\frac{\partial^2 \mathrm{P}}{\partial \mathrm{Z}^2} + \frac{3\mathrm{h}^2}{\mu}\frac{\partial \mathrm{h}}{\partial \mathrm{Z}}\frac{\partial \mathrm{P}}{\partial \mathrm{Z}} - \frac{\mathrm{h}^3}{\mu^2}\frac{\partial \mu}{\partial \mathrm{Z}}\frac{\partial \mathrm{P}}{\partial \mathrm{Z}} = 6\mathrm{U}\frac{\partial \mathrm{h}}{\partial \mathrm{X}}$$

Assuming that there is no misalignment and the viscosity is constant in the z-direction:

$$\frac{\partial h}{\partial Z} = 0 \; and \; \frac{\partial \mu}{\partial Z} = 0$$

Therefore equation (1) becomes:

$$\frac{\mathrm{h}^3}{\mu}\frac{\partial^2 \mathrm{P}}{\partial \mathrm{X}^2} + \frac{3\mathrm{h}^2}{\mu}\frac{\partial \mathrm{h}}{\partial \mathrm{X}}\frac{\partial \mathrm{P}}{\partial \mathrm{X}} - \frac{\mathrm{h}^3}{\mu^2}\frac{\partial \mu}{\partial \mathrm{X}}\frac{\partial \mathrm{P}}{\partial \mathrm{X}} + \frac{\mathrm{h}^3}{\mu}\frac{\partial^2 \mathrm{P}}{\partial \mathrm{Z}^2} = 6\mathrm{U}\frac{\partial \mathrm{h}}{\partial \mathrm{X}}$$

The first and second derivatives of the pressure ($P_{i,j}$) in the circumferential direction are

$$\left.\frac{\partial \mathrm{P}}{\partial \mathrm{X}}\right)_{\mathrm{i,j}} = \frac{\mathrm{P}_{\mathrm{i+1,j}} - \mathrm{P}_{\mathrm{i-1,j}}}{2\left(\frac{\pi \mathrm{R}}{\mathrm{m}-1}\right)}$$

$$\left.\frac{\partial^2 \mathrm{P}}{\partial^2 \mathrm{X}}\right)_{\mathrm{i,j}} = \frac{\mathrm{P}_{\mathrm{i+1,j}} - 2\mathrm{P}_{\mathrm{i,j}} + \mathrm{P}_{\mathrm{i-1,j}}}{\left(\frac{\pi \mathrm{R}}{\mathrm{m}-1}\right)^2}$$

Similarly, the pressure gradient in the z-direction, the first and second derivatives of ($P_{i,j}$)

$(\frac{\partial P}{\partial Z})_{i,j}$‘ $(\frac{\partial^2 P}{\partial^2 Z})_{i,j}$‘$(\frac{\partial \mu}{\partial X})_{i,j}$‘ $(\frac{\partial u}{\partial X})_{i,j}$

Then by substituting the above values into Reynolds equation, gives

$$P_{i,j} = A_1 P_{i+1,j} + A_2 P_{i-1,j} + A_3 P_{i,j+1} + A_4 P_{i,j-1} - C_5$$

Solving the above equation using the iteration technique gives the oil film pressure at all the points in the circumferential and axial directions. This was achieved using the (the computer program _MATLAB) for solving the simultaneous equations, which represent all the nodes

Analytical Solution

The approximate analytical solutions of Reynolds equation is based on the assumptions that one of the two terms in the left side of Eq. (1) can be neglected. The first term can be neglected when the journal bearing is considered as a bearing with high length to diameter ratio (long bearing, L/D>>1) and the second term when the journal bearing is considered as a bearing with low length to diameter ratio (short bearing L/D<1). Such solutions for pressure distribution give considerably simplified mathematical expressions[6]. They are exact solutions in the sense that they satisfy Eq. (1) for the case of infinite axial length or of infinite short bearing. However they are regarded as approximate solutions when they are used to determine the pressure distribution in bearings of finite length .In this paper, the theory of short bearings isused.

$$\frac{\partial}{\partial Z}\left(\frac{h^3}{\mu}\frac{\partial P}{\partial Z}\right) = 6U\frac{\partial h}{\partial x}$$

$$\frac{\partial^2 P}{\partial Z^2} = \frac{6U\mu}{h^3}\frac{\partial h}{\partial X}$$

We integrate both sides twice

$$P = \frac{6U\mu}{h^3}\frac{\partial h}{\partial X}\frac{Z^2}{2} + C_1 Z + C_2$$

$$h = C_r\left(1 + n\cos\left(\frac{X}{r}\right)\right)$$

$$P = \frac{3U\mu}{C_r(1+n\cos\theta)^3} * \frac{n}{r}\sin\theta\left(\frac{L^2}{4} - Z^2\right)$$

Temperature Profile

The representation of the temperature change is so difficult to estimate the exact performance of the bearing. High temperature effects bearing performance by reducing oil viscosity and thermal deformations. The temperature rises due to shear in lubricant layers, and the value of the shear stress is effected by many factors, the most important of which are velocity the and external load. Thermal effects are calculated through the energy equation, and for the purpose of estimating the temperature, Pinkus equation was relied on from the aforementioned energy equation within the half Summerfield boundary[7].

$$T_i = \left[T_\circ + \frac{1}{\beta_\circ}\ln(1 + \lambda I_{\theta i})\right]$$

$\boldsymbol{\lambda}$: is a constant given by:

$$\boldsymbol{\lambda} = 2\omega\left[\left(\frac{r}{C_r}\right)^2\left(\frac{\beta_\circ \mu_\circ}{C_P \rho}\right)\right]$$

And:

$I_{\theta i}$: is the integral of the angle between the inlet oil at ($T_\circ$) to the section at (T_i)

$$I_{\theta i} = \int [\frac{C_r}{C_r + ecos(\theta_i)}]^2 d\theta_i$$

Viscosity – Temperature Relationship

There are several viscosity-temperature equations, some of which are purely empirical whereas others are derived from theoretical models. In this work, Vogel's equation is used to calculate viscosity of the oil film at any point in the bearing surface[8].

$$\boldsymbol{\mu_i = a e^{b/[(T_i + 273.15) - c]}}$$

a = 0.0000736317

b = 797.7122

c = 177.3562

where:

(a , b , c) are constants and wear found for ISO VG32 oil from a table in [8].

Thermal Distortion Effect (THL)

The thermal distortion occurs due to the rise of the bearing surface temperature. In this work, the effect of temperature on the bearing surface is calculated and is neglected for the shaft. This is because the thermal distortion of the shaft is very little compared with that of the bearing surface.The thermal distortion of the bearing surface is calculated using the thermal expansion equation[9]. as follow:

$$\delta_{th,i} = \alpha_{AL} t \, \Delta T_i$$

Where:

$$\Delta T_i = T_i - T_\circ$$

Hence :

$$h_{th\ i,j} = h_{i,j} - \delta_{th\ ,i}$$

Results and Discussion

Reynolds equation has been solved numerically and analytically using a computer program with the aid of Matlab. After running the program and obtaining the results, in order to make a comparison between the numerical and analytical solution to the of Reynolds equation (with and without) the effect of thermal distortion THL, A set of relationships are drawn represented by the following.Figures (3) and (4) show the distribution of the circumferential pressure (numerically and analytically), respectively, at a speed of 3000 rpm. These curves,show the effect of the oil layer change on the pressure values. This is duethermal deformations which leads to an increase in the values of Pressure.These results is in agreement with the results of refs [7][10].Figure (5) shows the distribution of peripheral pressure for the case (HD) result from numerical and analytical solution using 3000 rpm speed and n = 0.7. it was found that the value of the pressure using the theory of short bearings is greater than that obtained from using finite differences method, and these results are consistent with the results of ref[6].It showed a slight difference as a result of the different design and operational values of the current survey from the traditional supports of other researchers.Figure (6) shows the relationship between numerical pressure and analytical pressure (HD. The correction coefficient of the analytical solution was calculated. The accuracy of the correction equation was 0.989 when using a first-degree (linear) equation and the accuracy was 0.992 when using a second-degree (quadratic) equation, so the second-order correction equation was applied in calculating the pressure because its accuracy is greater.Figure (7) shows the change in oil film pressure (P) circumferentially (HD - using correction factor). It was found that the numerical solution and the analytical solution after using the correction factor are almost equal and there is no longer a significant difference between the two solutions. Figure (8) shows the distribution of peripheral pressure for the case (THL) numerically and analytically at a speed of 3000 rpm and n = 0.7. Also it was found that the value of the pressure using the theory of short bearings is greater than the value of the pressure using finite differences method.Figure (9) shows the relationship between the numerical pressure and the analytical pressure (THL).Through this curve. The correction factor of the analytical solution was calculated. The accuracy of the correction equation was 0.99 when using an equation of the first degree (linear). The equation of the first degree was sufficient because of its high accuracy.Figure (10) shows the change in oil membrane pressure (P) circumferentially (THL - using correction factor).Also it was found that the numerical solution and the analytical solution (after using the correction factor)are almost equal and there is no longer a significant difference between the two solutions.

Conclusions

Based on the current results, the following can be concluded

1- The obtained values from the analytical solution of short bearing theory method are higher than that of numerical solution using finite differences.
2- It is not possible to rely heavily on the results of the analytical solution, because it is based on a set of hypotheses, so we resort to the correction factor.
3- Using a correction factor to deal with the analytical solution is very difficult because of; first, the complication of the parameters and; second their effect varies with the variation of the other parameters
4- Hydrodynamic pressure and high temperatures cause thermal deformations that change the thickness of the oil film.
5- THL significantly affect the performance of the bearing, especially at high speeds and a large Eccentricity ratio.

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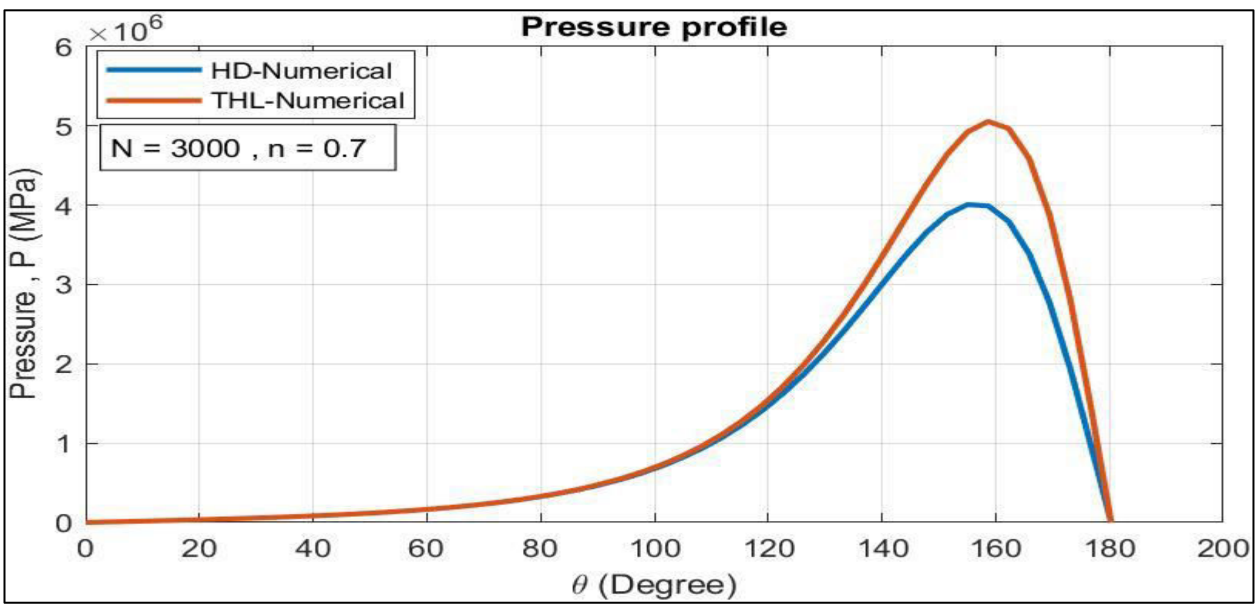

Figure (3)Pressure Profile (numerically).

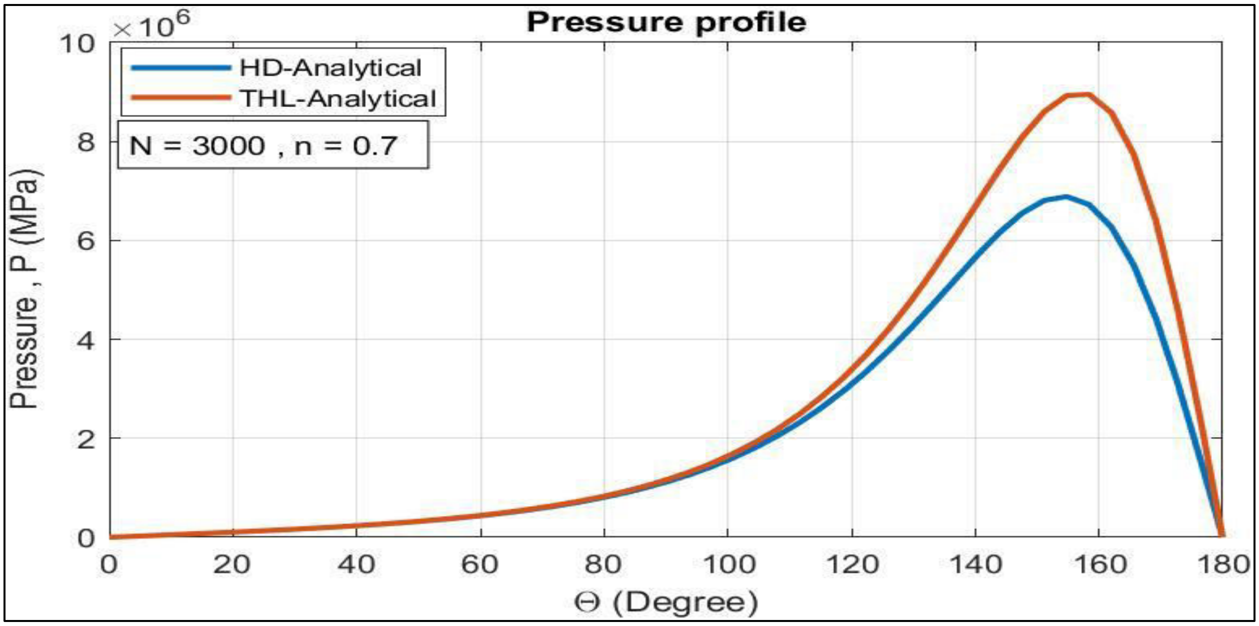

Figure (4) Pressure Profile (Analytically).

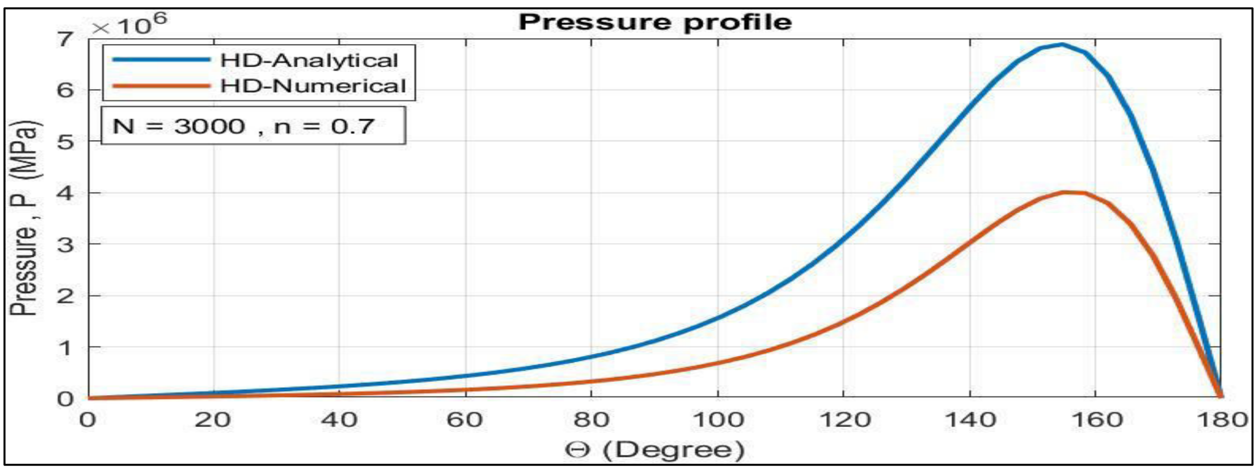

Figure (5) Pressure Profile (HD).

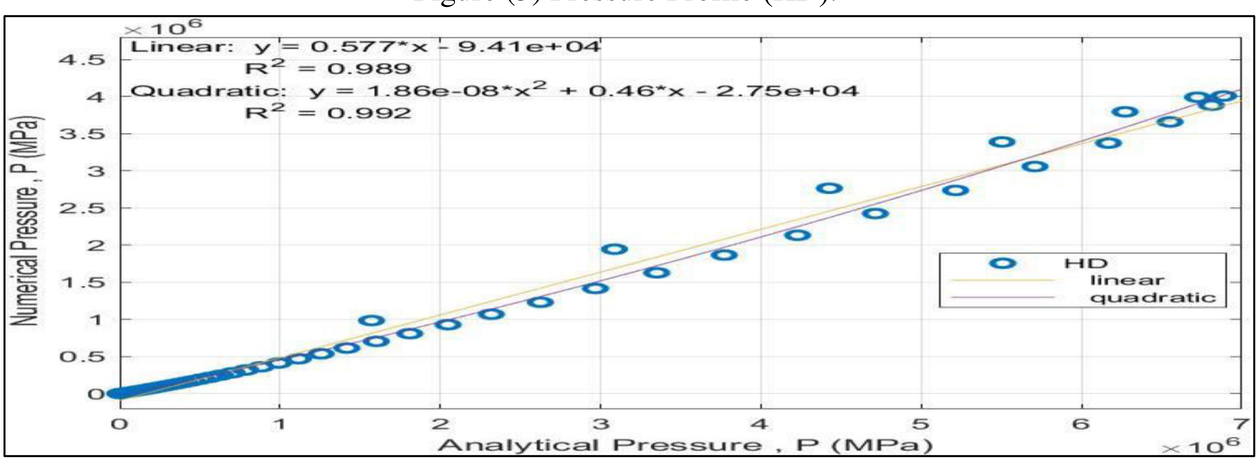

Figure (6) the relation between Numerical Pressure and Analytical Pressure (HD).

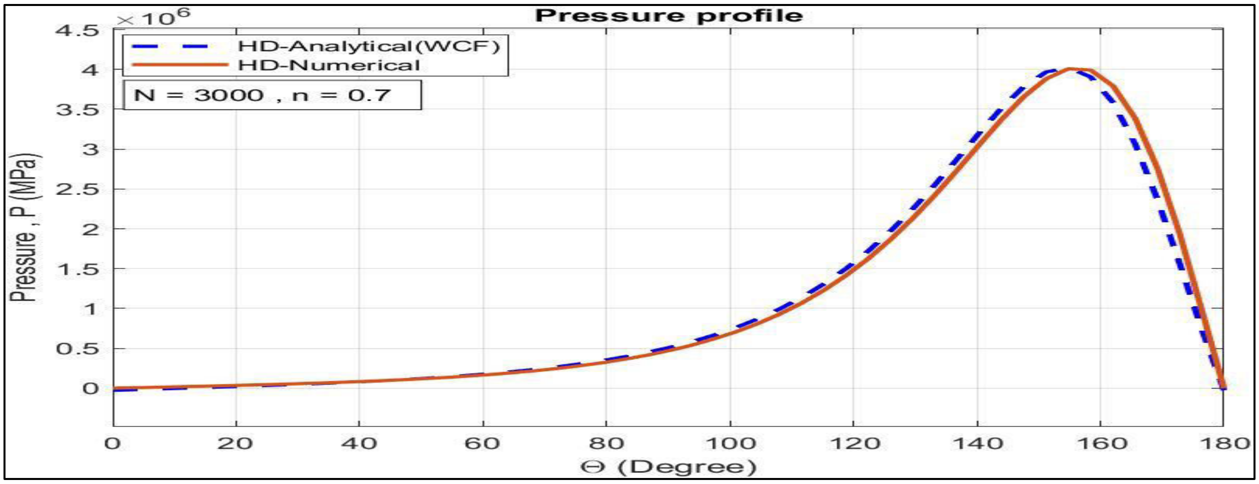

Figure (7) Pressure Profile (HD – using correction factor).

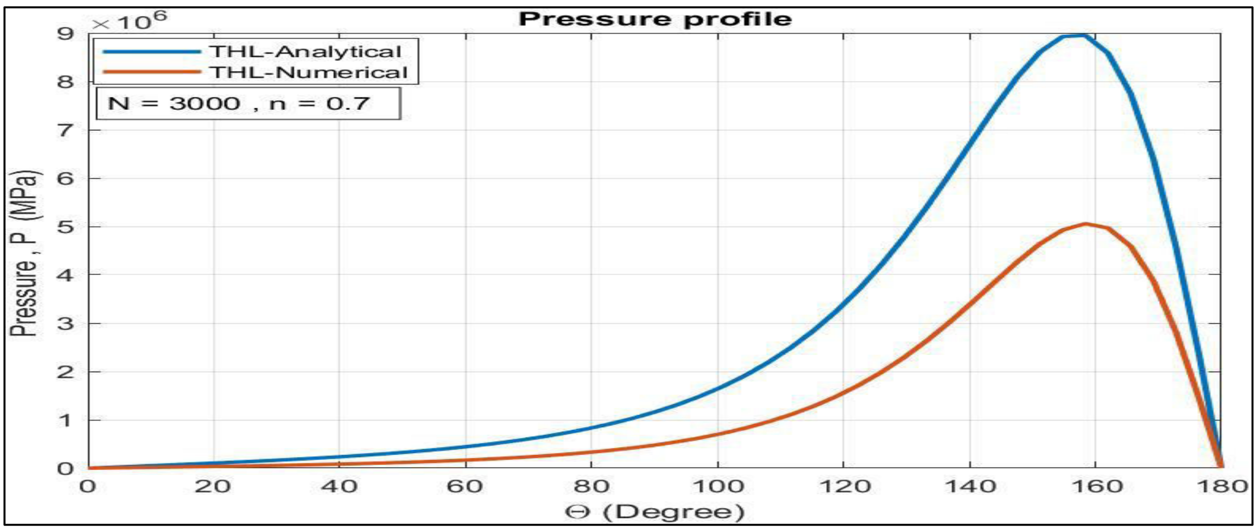

Figure (8) Pressure Profile (THL).

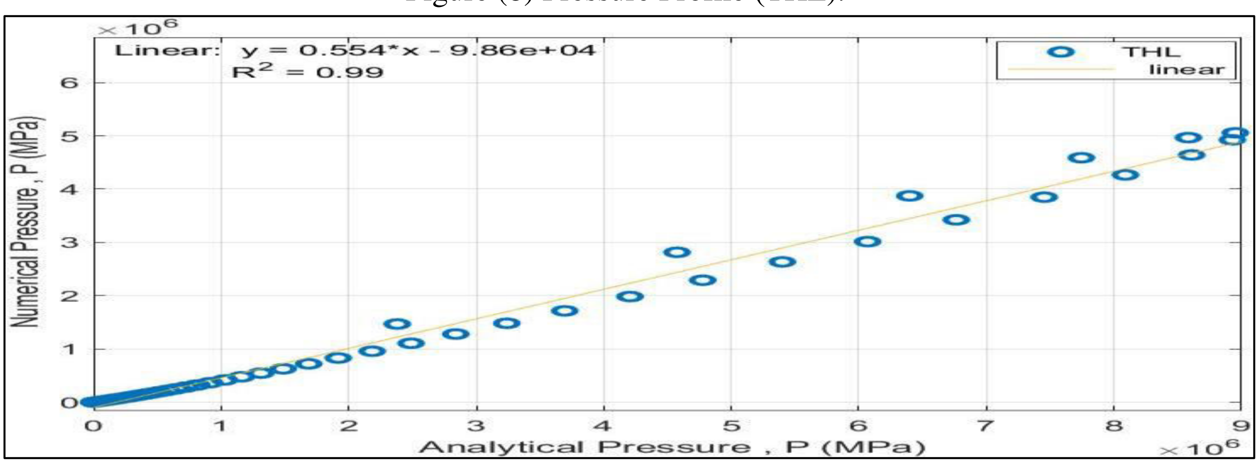

Figure (9) the relation between Numerical Pressure and Analytical Pressure (THL).

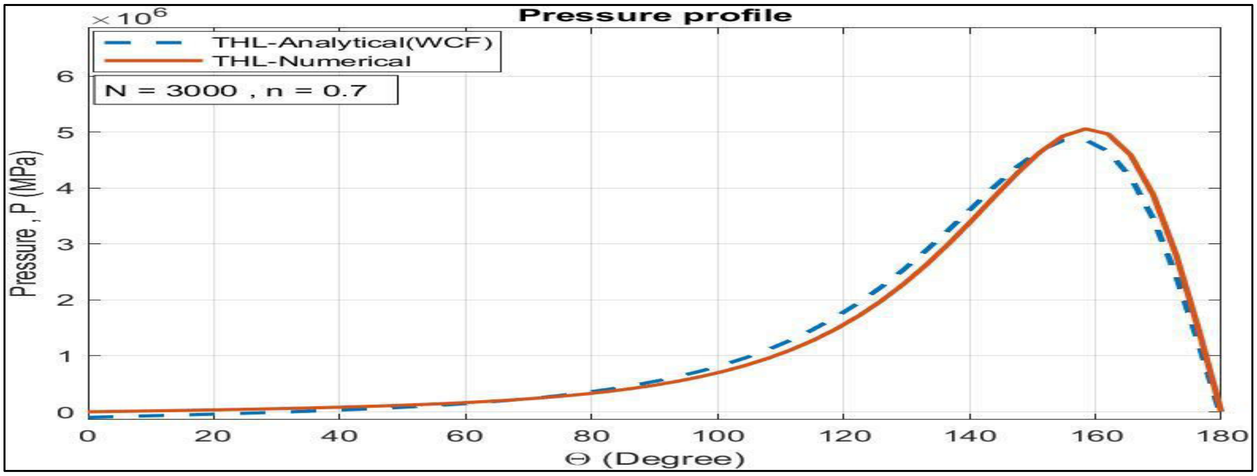

Figure (10) Pressure Profile (THL– using correction factor).

Nomenclature

D	Bearing Diameter (m)	E_{AL}	Modules of Elasticity for the Light Metal (GPa)
d	Journal Diameter (m)	T_i	The temperature at any radial section on the bearing surface (°C)
L	Bearing Length (m)	$T_\circ$	Inlet oil temperature(°C)
N	Rotational speed (rpm)	$\beta_\circ$	Temperature coefficient of viscosity (calculated from the temperature/viscosity graph) (1/ ºC)
C_D	Diametric Clearance (m)	α_{AL}	Thermal expansion coefficient of the bearing surfacem/(m. ºC)
C_r	Radial Clearance (m)	δ_{th}	Thermal distortion due to temperature rise effect (m)
h	Oil Film Thickness (m)	ΔT	The temperatures difference between any section and the inlet section.(ºC)
O_b	Bearing center	ρ	Density(Kg/m^3)
O_j	Journal center	μ	Kinematic Viscosity (m^2/Sec)
e	Eccentricity (m)	**η**	Dynamic Viscosity (Pas *Sec)
n	Eccentricity Ratio	h_{th}	Oil film thickness (considering the thermal distortion effect)(m)
Φ	Attitude Angle (Degree)	U	Linear Speed of the rotating Surface (m/sec)
P	Oil Film Pressure (Pas)	m	Number of nodes on x direction
t	Light metal Thickness(m)	k	Number of nodes on z direction
ω	Angular velocity (rad/sec)	Θ_{max}	The Angle of max Pressure (Degree)
h_{max}	Max Oil Film Thickness (m)	h_{min}	Min Oil Film Thickness (m)