Relative efficiency assessment without slacks in BCC DEA-based model

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Abstract: In data envelopment analysis (DEA) models, weak efficient units with zero weights are almost appeared as reference points in the models. To avoid zero weights or equivalently non-zero slacks in DEA evaluation, weights restrictions are used frequently. In this paper, a one-step method is presented based on the restricting input/output weights in the BCC DEA model for evaluating performance that guarantees non- zero weights and prevents weights dissimilarity.

Keywords: Data envelopment analysis, weight restriction, efficiency, input/output weights.

1. Introduction

Studies on how to improve the relative performances of production firms are one of the most frequently studied problem in the field of performance management and benchmarking. Data envelopment analysis (DEA) is one of the most representative methods of efficiency analysis. DEA is a linear programming based approach to assess the relative performance of homogeneous decision making units (DMUs), which use multiple in commensurate resources to produce multiple in commensurate outputs. This performance measurement tool is used in many contexts including health care units, agricultural productions, military and many other. The problem of determining optimal weights in performance measurement models is an important subject in the field of benchmarking and DEA that has attracted considerable attention among DEA researchers. See, for instances, Cooper et al. (2007), Lins et al. (2007), Ramon et al. (2010), Lam and Bai (2011) ,Cooper et al. (2011), Kiani Mavi et al.(2019), Omrani et al.(2019) and Ghazi(2019). Because of the piecewise linear nature of DEA frontier, the optimal multiplier of the DEA models may not be unique. Specially, extreme efficient units usually have alternative optimal solutions for their weights in the conventional DEA models. It is well known that alternative optimal solutions of the basic DEA models produces zero weights when units are evaluated. So, we need a procedure to choose the favorable weights from among alternative optimal solutions. The problem of alternative optimal solutions in DEA has been tackled mainly by the techniques of weights restriction. Although, we can incorporate very small lower bounds on the input and output weights, but computational problems in weights determinations in such a cases make trouble to decision makers. The Restrict weights in DEA models are discussed from different viewpoints. There are a number of methods such as cone-ratio or assurance region models that impose some restrictions on the weights and some researcher have mainly used these methods. These restrictions are refining the alternative optimal so that the resulting weights are strictly positive and thereby from the duality theory of linear programming, the corresponding slacks are zero but such restrictions need information or value judgments in the analysis. In the absence of experts or cost/price information, these models face serious problems. In some models, the existence of alternative optimal has led to the use of secondary goals some DEA researcher to choose a set of favorable weights; see Sexton et al. (1986), Doyle and Green (1994), Liang et al.(2008) , Wu et al.(2009), Hatami-Marbini et al.(2018) and Rezaeiani et al. (2018) among the others. In order to reduce the weights flexibility in DEA models, Lin (2014) applied the assurance region approach. He proposed a methodology for a fuzzy two-stage DEA model in which the input/output data are treated as fuzzy numbers. Wu et al. (2016) proposed an extended secondary goal model for weight selection DEA cross efficiency evaluation.

There are no restrictions on the input/output weights in traditional DEA models. A simple extension on the traditional DEA model was to incorporate a very small lower bound on the input and output weights. Podinovski and Chameeva (2016) identified theoretical problems that may arise by incorporating very small lower bounds on the weights. They showed that the use of infinitesimal lower bounds may lead to an efficient target with negative inputs. More Extensions on original DEA models, with bounds on the weights, are proposed to refine the alternative optimal weights in DEA models. These approaches are refining the alternative optima so that the resulting weights are strictly positive and thereby from the duality theory of linear programming, the corresponding slacks are zero. Podinovski and Chameeva (2016) stated that incorporation of weight restrictions in DEA models may lead to free or unlimited production of outputs. They have developed analytical conditions for a large class of linked and unlinked weight restrictions. Podinovski (2016) proved that in multiplier model of DEA, for any weight restrictions, the optimal weights show the unit under evaluation in the best light in comparison to the expanded technology with weight restrictions. Amirteimoori et al.(2018) proposed a model that their goal is to analyze the relative performance of Iranian electricity distribution companies using a new CCR DEA-based model Their model produce positive weights and avoids large differences in weights. Ramon et al. (2010) have proposed a two-step approach to determine the non-zero slacks in DEA assessment which is based on a weight restriction in the multiplier form of the CCR model of Charnes et al. (1978). Ramon et al. (2010) approach for efficiency evaluation in DEA proceeds in two steps. In the first step, they use a linear programs (LP) model for extreme efficient units (E) determine some weight bound that to reach a positive lower bound for the ratios of variation for each input/output weight. Then, in the second step, the lower bound calculated in a weight restriction of the CCR DEA model derived from the first step use for the evaluation of inefficient units (F ∪ NF) without slacks. Before using their models, they need to apply a procedure, like the one proposed by Charnes et al.(1991) or Thrall (1996), to partition DMUs into the sets E, E', F, NE, NE' and, NF that this procedure increase the complexity of the model. (Note that DMUs in E and E' are Pareto-efficient. E consists of the extreme efficient units, whereas those in E' are non-extreme Pareto-efficient units that can be expressed as linear combinations of DMUs in E. F is the set of weakly efficient units. The DMUs in NE, NE' and NF are inefficient and are projected onto points that are E, E' and F respectively). Therefore, in general, the aim of the two-step model Ramon et al. (2010) is to produce a weights restricted CCR model in efficiency evaluation of units, which ensures positive weights and prevents dissimilar weights However, the two-step model Ramon et al. (2010) needs to determine all extreme efficient units of E for the step one to achieve the above results and this increases the complexity of their model.

This paper, proposes a one-stage approach in DEA assessment of relative efficiency that it ensures the production of strictly positivity of the weights (and equivalently, zero slacks), and it avoids as much as possible the similarity between weights. The infeasibility problem and generating free and unlimited production are potentially troubling because, even if the infeasibility does not occur, the efficiency scores obtained may still be assessed in an erroneous model of production technology. The proposed one-stage approach in this paper does not have the infeasibility problem and since we minimize the deviations of input and output weights, we clearly avoid unrealistic and unfavorable weights and in this sense, we did not face erroneous model of production technology. Our multiplier bound approach also does not require any prior information on the weights and especially unit's classification and this will substantially reduce the computational efforts.

The structure of the paper is organized as follows. In section 2 the proposed approach in the DEA efficiency assessment without slacks is given. The applicability of the proposed one-stage approach is shown by numerical examples in section 3. In Section 4, an example is given . The paper concludes in section 5.

2. Preliminaries

We assume the problem of evaluating performance of n DMUs which use m inputs to produce s outputs. It is also supposed that input and output vectors are semi-positive. The basic BCC model for evaluating the efficiency of under evaluation unit, DMU_o , introduced by **B**amker, **C**harnes and **C**ooper (1984)(input oriented DEA model) is as follow:

$$
\begin{aligned}
Max \quad & \sum_{r=1}^{s} u_r^o y_{ro} + u_o \\
s.t. \quad & \sum_{i=1}^{m} v_i^o x_{io} = 1, & (1-1) \\
& -\sum_{i=1}^{m} v_i^o x_{ij} + \sum_{r=1}^{s} u_r^o y_{rj} + u_o \le 0, \quad j = 1, \cdots, n, & (1-2) \\
& u_r^o, v_i^o \ge 0, \quad for\ all\ i\ and\ r & (1-3)
\end{aligned}
$$

Variables v_i and u_r are weights of ith input and rth output, respectively. The above model finds the optimal multipliers or optimal weights and hence is called multiplier form of the BCC model and its dual envelopment program is as follow:

$$Min \quad h$$
$$s.t. \quad x_o h - \sum_{j=1}^{n} \lambda_j x_{ij} \geq o, \quad i = 1,2,...m; \qquad (2-1)$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_o, \quad r = 1,\cdots,s; \qquad (2-2)$$
$$\sum_{j=1}^{n} \lambda_j = 1; \qquad (2-3)$$
$$\lambda_j \geq 0, \; j = 1,2,...,n; \qquad (2-4)$$

The $n \times 1$ vector λ is an intensity vector, $\sum_{j=1}^{n} \lambda_j = 1$ is the convexity restriction whose dual free variable is u_o.

3. New one-stage approach

In this section, we are looking for a weight restricted BCC model, which ensures zero slacks or strictly positive weights and avoids dissimilar weights without need to classify the units in subsets *E, E', F, NE, NE'* and *NF*, all in one step. The following model selects the optimal weights for the under evaluation unit.

$$Min \quad \frac{s_o * \beta}{\alpha}$$
$$s.t. \quad \sum_{i=1}^{m} v_i^o x_{io} = 1, \qquad (3-1)$$
$$\sum_{r=1}^{s} u_r^o y_{ro} + s_o + u_o = 1, \qquad (3-2)$$
$$-\sum_{i=1}^{m} v_i^o x_{ij} + \sum_{r=1}^{s} u_r^o y_{rj} + u_o \leq 0, \quad j = 1,\cdots,n, \; j \neq o, \qquad (3-3)$$
$$\alpha \leq v_i^o x_{io} \leq \beta, \quad i = 1,\cdots,m, \qquad (3-4)$$
$$\alpha \leq u_r^o y_{ro} \leq \beta, \quad r = 1,\cdots,s, \qquad (3-5)$$
$$u_r^o, v_i^o, \alpha, \beta, s_o \geq 0, \quad for\ all\ i\ and\ r \qquad (3-6)$$

The first and second constraints (3-1) and (3-2) guarantee the full-efficiency of DMU_o and the third constraint (3-3) preserves the feasibility of the weights. (3-4), (3-5) guarantees choice of DEA profile of weights without zeros. Here, we don't need any information about the unit under evaluation, DMU_o, and the class it belongs to. The objective function in (3) is the fraction $\frac{s_o * \beta}{\alpha}$. The numerator, s_o, is the deviation from the full-efficient status for DMU_o, and the denominator, α, is the lower bound of the weights and β is the upper bound of the weights . By minimizing $\frac{s_o * \beta}{\alpha}$ in the objective function, we look for optimal input/output weights for minimizing s_o and β and simultaneously maximizing α. Also, with increasing α we look for a positive lower bound for input/output weights among all feasible multipliers of the observed units and with decreasing β, we avoids dissimilar weights among all feasible multipliers of the observed units

Now we find a method for solve the above model. We propose a two-step method to solve this model.

In the first step, we will minimize the inefficiency score of the under evaluation unit. Subsequently, by replacing the optimal inefficiency score of the under evaluation unit, in the second step, we minimize the upper bound of the weights and simultaneously maximize the lower bound of the weights which can be stated as follows:

Stage 1: The first, our proposed approach for efficiency evaluation in DEA minimize the inefficiency score of the under evaluation unit as follows:

$$
\begin{aligned}
Min \quad & s_o \\
s.t. \quad & \sum_{i=1}^{m} v_i^o x_{io} = 1, && (4-1) \\
& \sum_{r=1}^{s} u_r^o y_{ro} + s_o + u_o = 1, && (4-2) \\
& -\sum_{i=1}^{m} v_i^o x_{ij} + \sum_{r=1}^{s} u_r^o y_{rj} + u_o \le 0, \quad j = 1,\cdots,n,\ j \ne o, && (4-3) \\
& \alpha \le v_i^o x_{io} \le \beta, \quad i = 1,\cdots,m, && (4-4) \\
& \alpha \le u_r^o y_{ro} \le \beta, \quad r = 1,\cdots,s, && (4-5) \\
& u_r^o, v_i^o, \alpha, \beta, s_o \ge 0, \quad for\ all\ i\ and\ r && (4-6)
\end{aligned}
$$

We assume that the optimal value the objective function is $s^* = (s_1^*,...,s_n^*)'$.

Stage 2: In the second step, we minimize the upper bound of the weights and simultaneously maximize the lower bound of the weights by replacing the calculated inefficiency score from the first stage, in the second constraint. As a result, we look for a positive lower bound for input/output weights and with decreasing β, we avoids dissimilar weights among all feasible multipliers of the observed units.

$$
\begin{aligned}
Min \quad & \frac{\beta}{\alpha} \\
s.t. \quad & \sum_{i=1}^{m} v_i^o x_{io} = 1, && (5-1) \\
& \sum_{r=1}^{s} u_r^o y_{ro} + s_o^* + u_o = 1, && (5-2) \\
& -\sum_{i=1}^{m} v_i^o x_{ij} + \sum_{r=1}^{s} u_r^o y_{rj} + u_o \le 0, \quad j = 1,\cdots,n,\ j \ne o, && (5-3) \\
& \alpha \le v_i^o x_{io} \le \beta, \quad i = 1,\cdots,m, && (5-4) \\
& \alpha \le u_r^o y_{ro} \le \beta, \quad r = 1,\cdots,s, && (5-5) \\
& u_r^o, v_i^o, \alpha, \beta, s_o \ge 0, \quad for\ all\ i\ and\ r && (5-6)
\end{aligned}
$$

Clearly, model (5) is a non-linear programming problem and it is easy to use the usual Charnes and Cooper (1962) transformation for fractional programming problems to convert it into the following linear equivalent form:

$$
\begin{aligned}
& Min \quad \overline{\beta}_o \\
& s.t. \quad \sum_{i=1}^{m} \overline{v}_i^o x_{io} = t, && (6-1) \\
& \sum_{r=1}^{s} \overline{u}_r^o y_{ro} + ts^*_o + \overline{u_o} = t, && (6-2) \\
& -\sum_{i=1}^{m} \overline{v}_i^o x_{ij} + \sum_{r=1}^{s} \overline{u}_r^o y_{rj} + \overline{u_o} \le 0, \quad j = 1,\cdots,n, j \ne o, && (6-3) \\
& 1 \le \overline{v}_i^o x_{io} \le \overline{\beta}, \quad i = 1,\cdots,m, && (6-4) \\
& 1 \le \overline{u}_r^o y_{ro} \le \overline{\beta}, \quad r = 1,\cdots,s, && (6-5) \\
& \overline{u}_r^o, \overline{v}_i^o, \mathrm{t}, \overline{\beta_o} \ge 0, \quad for\ all\ i\ and\ r && (6-6)
\end{aligned}
$$

In which $\frac{1}{\alpha} = t, \overline{u_o} = t u_o, \overline{v}_i = t v_i, \overline{u}_r = t u_r.$ Now, we can use any linear programming approaches such as simplex algorithm or interior point algorithm to solve the LP model 6.

Now we want to prove by the following theorem that we can use model (3) to discriminate efficient and inefficient units.

Theorem 1. $\mathrm{DMU_o}$ is BCC-efficient if and only if we have $s_o^* = 0$ in model (3).

Proof. Supposing DMU_o is BCC efficient, then there exists an optimal solution $u_r^*, v_i^*, u_o^* \quad (i = 1,\cdots,m, \quad r = 1,\cdots,s)$ in the BCC model, model 1, such that $\sum_{r=1}^{s} u_r^* y_{ro} + u_o = 1$. Clearly $u_r^*, v_i^* u_o^* \quad (i = 1,\cdots,m, \quad r = 1,\cdots,s)$, $s_o^* = 0$ and $\alpha = \underset{r,i}{Min} \{u_r^*, v_i^*\}, \beta = \underset{r,i}{Max} \{u_r^*, v_i^*\}$ is a feasible and optimal solution for model (3).

The following theorem shoes that Model (3) is feasible and produces positive weights and this is an important issue for our one-model approach. Because of the fact that we need our model always to be feasible and the positivity of the weights is required.

Theorem 2. Model (3) is feasible and it produces in optimality $\alpha > 0$.

Proof. Assuming DMUp is a BCC reference unit in evaluating $\mathrm{DMU_o}$. There exists an optimal weight $u_r^p > 0, v_i^p > 0 (i = 1,\cdots,m: \quad r = 1,\cdots,s), \mathrm{u}_o$ in evaluating $\mathrm{DMU_p}$ with the BCC model. Therefore, $u_r^p, v_i^p, \alpha = \underset{i,r}{Min} \{u_r^p, v_i^p\} > 0, \beta = \underset{i,r}{Max} \{u_r^p, v_i^p\} > 0$ with $s_o = 1 - \sum_{r=1}^{s} u_r^o y_{ro} - u_o$ is a feasible solution in evaluating $\mathrm{DMU_o}$ in model (3).

The above theorem clearly shows that there is no need to partition the DMUs into two groups, efficient and inefficient.

The following theorem guarantees that the discrimination power of the new proposed approach is better than the BCC model.

Theorem 4. The relative efficiency score in our proposed model is less than or equal to the BCC model.

Proof. It is clear that maximizing $\sum_{r=1}^{s} u_r y_{ro}$ in the BCC model is equivalent to minimizing s_o in the objective function of model (3). Suppose $\left(u^*, v^*, \alpha^*, \beta^*, S^*, u_o^*\right)$ with $\alpha = \underset{i,r}{Min}\left\{u_r, v_i\right\} > 0, \beta = \underset{i,r}{Max}\left\{u_r, v_i\right\} > 0$ be an optimal solution to model (3). It is easy to show that $\left(u^*, v^*, \mathrm{u}_o^*\right)$ is a feasible solution the BCC model ,model 1, and this completes the proof.

The strength of our proposed model is that it ensures the production of strictly positivity of the weights (and equivalently, zero slacks), and it avoids as much as possible the similarity between weights. Moreover, the infeasibility problem does not occur in their models. In addition to, our approach does not require any prior information on the weights and especially unit's classification and this will substantially reduce the computational efforts.

4. An illustrative example

For a better comparison of the proposed model with the BCC model, here, we use our new weights selection procedure in a small-scale example containing four units. Four units are considered with one input and one output. The associated production set T_c in one input and one output space is depicted in figure 1.

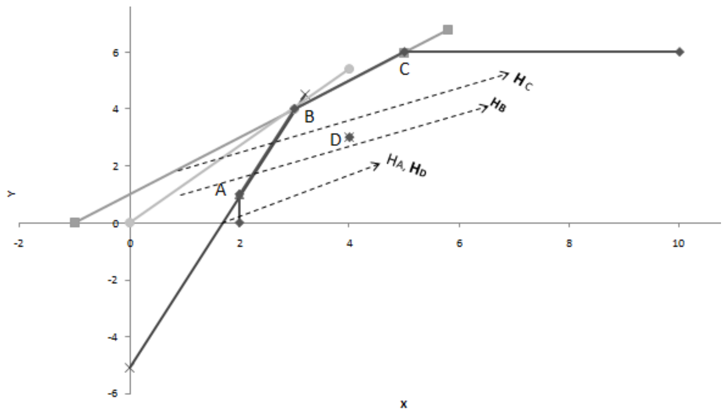

Figure 1. The production set for simple example.

The traditional BCC model identifies that DMUs A, B and C are extreme efficient and D is inefficient. Table1includesthe associated input/output BCC weights in parenthesis.

Table 1: The data for simple example

DMU	x	y	$u^*_{o\ BCC}$	Eff_o^{new}	u_o^*	v^*	u^*
A	2(0.5*)	1(0)	1	1	0.833	0.5	0.166
B	3(0.33)	4(0.11)	0.55	1	0	0.33	0.25
C	5(0.2)	6(0.2)	0.2	1	0.2	0.2	0.2
D	4(0.25)	3(0.08)	0.41	0.66	0.417	0.25	0.083

*The values in parenthesis show the BCC weights

Model (3) is also applied for this data set. The columns 6, 7 and 8 of Table 1 show new results of input/output weights in the restricted model, which yields the following efficient surfaces of the production set:

$$H_A = \{ (x, y) : y - 3x_1 + 5 = 0 \} \cap T_c$$
$$H_B = \{ (x, y) : y - 1.32x = 0 \} \cap T_c$$
$$H_C = \{ (x, y) : y - x - 1 = 0 \} \cap T_c$$
$$H_D = \{ (x, y) : y - 3x + 5 = 0 \} \cap T_c$$

Model 3 not only determined the most favorable weights to each DMU, but also, it gave the relative efficiency of the DMUs as shown with Eff_o^{new} in the Table 1. Based on the results, the BCC efficiency and in our new approach of DMU_d is $Eff_d^{new} = 0.66$.

5. A real application

After formulating the methodological framework and applying it to one simple example, its applicability has been illustrated through a real case on forest districts in Taiwan. Consider the sample from Kao and Hung (2005).There are 17 forest districts with four inputs: budget (x_1) (in US dollars), initial stocking (x_2) (in cubic meters), labor (x_3) (in number of employees) and land (x_4) (in hectares) and three outputs: main product (y_1) (in cubic meters), soil conservation (y_2) (in cubic meters) and recreation (y_3) (in number of visits). The input/output data set are listed in Table 2. The BCC model for this dataset is used. The BCC efficiency scores and it slacks are recorded in Table 3. The results of Table 3 are shown that the set of efficient districts are {D1, D2, D3, D4, D5, D6, D7, D8, D9, D11}. As the last seven columns of Table 3 shows, some of the input/output slacks in all inefficient DMUs are positive. This means that the corresponding weights in multiplier form of BCC model are zero and in this case, the corresponding supporting hyperplane is a weakly efficient hyperplane. As columns 4-8 of Table 3 show, all DMUs are projected on weakly efficient supporting hyperplane. This is not a good result.

Table 2. Data set

DMU	x_1	x_2	x_3	x_4	y_1	y_2	y_3
D1	51.6	11.2	49.2	33.2	40.4	14.8	3166.7
D2	85.7	123.9	55.1	108.4	43.5	173.5	6.4
D3	66.6	104.1	257	13.6	139.7	115.9	0
D4	27.8	107.6	14	146.4	25.4	131.7	0
D5	51.2	117.5	32	84.5	46.2	144.9	0
D6	36.0	193.3	59.5	8.2	46.8	190.8	822.9
D7	25.8	105.8	9.5	227.2	19.4	120	0
D8	123	82.4	87.3	98.8	43.3	125.8	404.6
D9	61.9	99.7	33	86.3	45.4	79.6	1252.6
D10	80.3	104.6	53.3	79	27.2	132.4	42.6
D11	205.9	183.4	144.1	59.6	14	196.2	16.1
D12	82.0	104.9	46.5	127.2	44.8	108.5	0
D13	202.2	187.7	149.3	93.6	44.9	184.7	0
D14	67.5	82.8	44.3	60.8	26	85	23.9
D15	72.6	132.7	44.6	173.4	5.5	135.6	24.1

D16	84.8	104.2	159.1	171.1	11.5	110.2	49
D17	71.7	88.1	69.1	123.1	44.8	74.5	6.1

Table 3. The BCC results and slacks in the BCC model

DMU	BCC efficiency	s_1^-	s_3^-	s_4^-	s_1^+	s_2^+	s_3^+
D1	**1**	0	0	0	0	0	0
D2	**1**	0	0	0	0	0	0
D3	**1**	0	0	0	0	0	0
D4	**1**	0	0	0	0	0	0
D5	**1**	0	0	0	0	0	0
D6	**1**	0	0	0	0	0	0
D7	**1**	0	0	0	0	0	0
D8	**1**	0	0	0	0	0	0
D9	**1**	0	0	0	0	0	0
D10	**0.97**	9.46	0	0	16.1	0	847.86
D11	**1**	0	0	0	0	0	0
D12	**0.83**	15.7	0	35.4	0	0	907.11
D13	**0.89**	123	73.9	38	0	0	509.53
D14	**0.93**	12.3	0	0	17.5	0	1521.5
D15	**0.794**	0	0	0	29.6	0	292.92
D16	**0.778**	0	76.6	36.9	27.9	0	1089.4
D17	**0.695**	0	0	18.4	0	0.29	1622.5

Our new approach is also applied to this data set. The new efficiency scores, Eff_o^{new}, and the optimal weights from our proposed approach is listed in Table 4 . The third column in this table shows the new efficiency scores of the DMUs .We can see that units D1, D2, D3, D4, D6, D7, D8. D9 and D11 are efficient, as determined by traditional BCC model. From looking at the columns 5-11, one can see that all weights of the DMUs are strictly positive and this means that inefficient DMUs are projected to efficient supporting surfaces of production technology set. The last column shows the reference set corresponding to each inefficient DMU. The results of this column indicate that DMU1 is benchmark to all DMUS.

Table 4. Result for our proposed approach

DMU	s_o stage1	Eff	u_1	u_2	u_3	v_1	v_2	v_3	v_4	References In our model
D1	0	1	0.006174	0.01679	0.00007895	0.004849	0.022262	0.005079	0.007458	-

D2	0	1	3.89E-05	0.008893	0.000262 09	1.97E-05	0.00481	0.007279	1.56E-05	-
D3	0.000 25	1	7.16E-06	0.009657	0.000067 82	0.000015	0.009051	3.89E-06	0.004033	-
D4	0	1	3.93E-05	0.006414	0.000000 01	1.73E-05	0.008873	0.003093	6.83E-06	-
D5	0	1	0.003737	0.005298	0.000000 2	1.95E-05	0.007354	0.004172	1.18E-05	-
D6	0	1	0.005333	0.00131	0.000303 8	0.006935	0.001293	0.0042	0.030377	-
D7	0	1	0.001214	0.002738	0.000045 05	0.011875	0.003108	0.003749	0.001447	-
D8	0	1	0.004028	0.009595	0.000088 73	0.000292	0.010823	0.000411	0.000363	-
D9	0	1	0.0126	0.00179	0.000115 92	0.002301	0.001428	0.017347	0.00165	-
D10	0.298 6	0.97 00	3.67E-05	0.005817	0.000000 57	1.25E-05	0.005931	0.002312	0.003227	{1,2,5, 6}
D11	0	1	0.000174	0.014652	0.000152 15	1.19E-05	0.00541	1.7E-05	4.12E-05	-
D12	0.168 93	0.83 00	0.009448	0.005539	0.000000 03	1.22E-05	0.007879	0.003681	7.86E-06	{1, 2,3,5}
D13	0.106 03	0.89 82	0.005512	0.021463	0.000000 43	4.95E-06	0.005311	6.69E-06	1.07E-05	{1,2,6, 11}
D14	0.072 22	0.87 09	3.84E-05	0.000808	0.000041 75	1.48E-05	1.21E-05	0.01591	0.0048	{1, 5, 7}
D15	0.207 26	0.77 69	0.00018	0.005306	0.000041 44	0.001783	0.005559	0.002288	0.000176	{1,2,4, 5,7}
D16	0.223 88	0.75 36	8.67E-05	0.00597	0.000020 37	0.003353	0.006843	6.28E-06	5.84E-06	{1, 2, 7}
D17	0.330 69	0.40 82	0.00962	0.008369	0.000162 87	0.003063	0.006412	0.003092	8.12E-06	{1,3,5, 7}

As we can see from tables 3 and 4, our new score is less than both, the BCC efficiency scores for all districts. The average scores of efficiency are 0.9353and 0.9123 in the BCC and our proposed approach models respectively. The result implies that the average of efficiency scores in our proposed approach is less than BCC model. Therefore our approach can produce weights that avoid large differences in weights. Another point is that if we use the very small lower bounds on the weights in BCC model, there is no guarantee that we could project inefficient DMUs to an efficient supporting surface.

GAMS software on a machine with CPU: Intel Pentium 4 at 2 GHz, RAM: 512 MB is used to our calculation.

6. Conclusions

Unrealistic and unfavorable weights in DEA models may lead to incorrect assessment in efficiency analysis. The flexibility of DEA in weights selection, often leads to zero and unrealistic weights. Avoiding zero weights and large differences in weights is an important subject in the field of benchmarking that has attracted considerable attention among researchers. In this paper, we introduced a one-model approach to assess the relative performances of the DMUs in DEA that guarantees the positivity of the weights and avoids the large differences in weights. In our approach, there is no need to prior information on the input/output weights. The computational effort in the developed model is substantially less than the other approaches without any infeasibility issues.

The classic and traditional DEA models and the proposed approach classify the DMUs into two groups: efficient and inefficient groups. Moreover, the input and output data are deterministic and crisp. Future studies can be extended in cases in which data set are stochastic and a more powerful discrimination analysis is recommended.

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